Question 1 (40%)

Given the language $L = \{w \in \{a,b\}^* \mid w = w^R\}$, namely the set of palindromes over the alphabet $\{a,b\}$, is it possible to show that L is *not* a regular language by using the Pumping Theorem for FALs? Explain.

Solution: No. First observe that it is possible to choose some x, some non-empty y and some z such that $xy^nz \in L$ for $n \geq 0$. Take x and z to be the empty string and y as a.

The theorem says that if a language is a FAL, then there must be such x, y and z. It's in the form of $P \rightarrow Q$. Showing that Q in such a situation does not take you anywhere. You cannot conclude whether the language of palindromes is a FAL or whether it is not a FAL. If you had succeeded in showing that there cannot be any such x, y and z, then you would have been able to show that the language of palindromes is not a FAL; because from $-Q$ and $P \rightarrow Q$, you can deduce $-P$.

Question 2 (30%)

Given the context-free grammar

$$
\begin{aligned}
G = \langle \{S,A\}, \{a,b\}, S,& \\
\{S &\rightarrow aAa, \\
S &\rightarrow bAb, \\
S &\rightarrow \varepsilon, \\
A &\rightarrow SS\}\rangle
\end{aligned}
$$

(a) give a derivation of the string *abbbaaba*;

Solution:

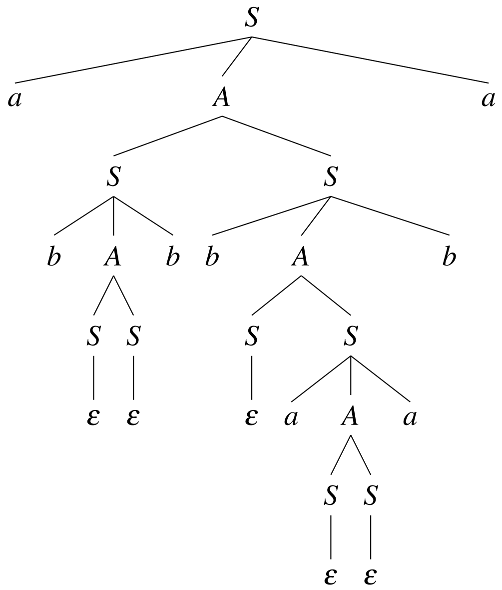
$$S \Rightarrow aAa \Rightarrow aSSa \Rightarrow aSbAba \Rightarrow aSbSSba \Rightarrow aSbSba \Rightarrow$$

$$aSbaAaba \Rightarrow aSbaSSaba \Rightarrow aSbaSaba \Rightarrow aSbaaba \Rightarrow$$

$$abAbbaaba \Rightarrow abSSbbaaba \Rightarrow abSbbaaba \Rightarrow abbbaaba$$

(b) draw the parse tree for the same string.

Solution:



Question 3 (30%)

Given $\Sigma = \{a, b, (,), \cup, *, \varepsilon\}$, construct a context-free grammar for the language

$L = \{w \in \Sigma^* \mid w \text{ is a regular expression}\}$.

Solution:

You need to distinguish between the ε in the language you are trying to characterize and the empty string you will use in your grammar. One way to do this is to designate the empty string with another Greek letter, say μ.

$S \rightarrow R$
$R \rightarrow RR$
$R \rightarrow (R)$
$R \rightarrow R^*$
$R \rightarrow R \cup R$
$R \rightarrow a$
$R \rightarrow b$