Question 1

A **context-free grammar** G is a quadruple $\langle V_N, V_T, R, S\rangle$ where

V_N is the set of **non-terminal** symbols,

V_T is the set of **terminal** symbols,

R is the set of **rules** each of the form $A \to x$, where $A \in V_N$ and $x \in (V_N \cup V_T)^*$,

$S \in V_N$ is the **start** symbol.

A **step** in a **derivation** can be characterized as follows. For any $u, v \in (V_N \cup V_T)^*$, we write $u \Rightarrow v$ (and say v is derived from u in one step) if and only if there are strings $x, y \in (V_N \cup V_T)^*$ such that $u = xAy$, $v = xty$, and the rule $A \to t$ is in R.

We call a sequence of the form:

$$w_0 \Rightarrow w_1 \Rightarrow w_2 \Rightarrow \cdots \Rightarrow w_n$$

a **derivation** of w_n from w_0 in n **steps**, where $n \geq 0$. When $w_0 = S$, we say that G **generates** w_n in n steps.

The language of a grammar G, denoted as $L(G)$, is the set $\{w \in V_T^* \mid w \text{ is generated by } G\}$.

Given the context-free grammar

$$\begin{aligned} G = \langle \{S, A\}, \{a, b\}, S, \\ \{S &\to AA, \\ A &\to AAA, \\ A &\to a, \\ A &\to bA, \\ A &\to Ab\}\rangle \end{aligned}$$

Which strings of $L(G)$ can be generated in four or fewer steps?