M E T U
Department of Mathematics

	Elementary Number Theory II **Midterm 2**		
Code : *Math 366* Acad. Year : *2018-2019* Semester : *Spring* Instructor : *Tolga Karayayla* Date : *24.04.2019* Time : *17.40* Duration : *120 minutes*	Last Name : First Name : Department : Signature :	Student ID :	
	6 Questions on **4** Pages SHOW DETAILED WORK!		

1	2	3	4	5	6		

1. (10+10 pts.) a) Calculate $x = [1; 3, \overline{5, 2}] = [1; 3, 5, 2, 5, 2, 5, 2, ...]$.

b)Find the infinite continued fraction representation of $x = \dfrac{13 + \sqrt{5}}{4}$.

2. (10+10 pts.) a) Using the information $\sqrt{22} = [4; \overline{1, 2, 4, 2, 1, 8}]$ find the fundamental solution of the equation $x^2 - 22y^2 = 1$.

b) Find 3 solutions $(x, y) \in \mathbb{Z}_+^2$ of the equation $x^2 - 27y^2 = 1$ (Hint: $\sqrt{27} = [5; \overline{5, 10}]$).

3. (15 pts.) Let d be a positive integer which is not a square, and let $k \in \mathbb{Z}$. Show that if $x^2 - dy^2 = k$ has a solution $(x, y) \in \mathbb{Z}^2$, then there are infinitely many solutions $(x, y) \in \mathbb{Z}^2$. (Hint: Use the properties of the norm function $N(x+y\sqrt{d}) = (x+y\sqrt{d})(x-y\sqrt{d})$ on $\mathbb{Z}[\sqrt{d}]$. Consider the numbers which have norm 1).

4. (15 pts.) Find $gcd(10 + 16i, 5 + i)$ in $\mathbb{Z}[i]$ using the Euclidean algorithm.

5. (15 pts.) Find a prime factorization of $21 - 27i$ in Gaussian integers $\mathbb{Z}[i]$.

6. (15 pts.) Show that there are infinitely many odd integers n such that n and $\dfrac{n-1}{366}$ are both perfect squares.