M E T U
Department of Metu Mathematics

	Elementary Number Theory II **Midterm 1**	

Code : *Math 366* Acad. Year : *2018-2019* Semester : *Spring* Instructor : *Tolga Karayayla* Date : *02.04.2019* Time : *17.40* Duration : *120 minutes*	Last Name : First Name : Student ID : Department : Signature : **7** Questions on **4** Pages SHOW DETAILED WORK!

1	2	3	4	5	6	7		

1. (15 pts.) Find all solutions $(x, y, z) \in \mathbb{Z}^3$ of the equation $2x^2 + 3y^2 = 8z^2$.

2. (14 pts.) Find all solutions $(x, y, z) \in \mathbb{Z}^3$ of the linear Diophantine equation $24x + 14y + 63z = 1$.

3. (2×7 pts.) For each integer n below, express n as a sum of two squares if it is possible. If not, express n as a sum of four squares:

a) $n = 2^3 \cdot 7^2 \cdot 29 \cdot 73$

b) $n = 13 \cdot 43$

4. (14 pts.) Find all $(x, y, z) \in \mathbb{Z}^3$ such that $x > 0, y > 0, z > 0$, $x^2 + y^2 = z^2$ and $x + z = 150$.

5. (14 pts.) Let p and q be two distinct primes such that $p \equiv q \equiv 1 (\mod 4)$. Show that pq can be expressed as a sum of two squares in at least two distinct ways (that is, $pq = x^2 + y^2 = s^2 + t^2$ f0r positive integers x, y, s, t such that $(x, y) \neq (s, t)$ and $(x, y) \neq (t, s)$).

6. (14 pts.) For the elliptic curve C given by the equation $y^2 = x^3 - 2x + 1$, find all rational points of finite order on C (Discriminant of $x^3 + bx + c$ is $D = -4b^3 - 27c^2$).

7. (2×7 pts.) a) Show that $\frac{1}{x^4} + \frac{1}{y^4} = \frac{1}{z^4}$ has no solution in integers.

b) Show that any $n \in \mathbb{Z}$ can be written as $n = a^2 + b^2 - c^2$ for some integers a, b and c.