Ultra-Low Sidelobes from Time-Modulated Arrays*

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Summary—Antenna patterns with ultra-low sidelobes have been obtained for an experimental receiving array designed with time-modulated slot radiators. The technique utilizes on-off RF switches which are programmed in a predetermined sequence to produce the desired pattern.

The results of tests on an experimental eight-element slot array designed for sidelobe reduction are presented. Two examples are considered: 1) an initial static pattern with 30-db sidelobes which are reduced to about 40 db by sequential switching and 2) an initial static pattern with 13-db sidelobes which are reduced directly to about 40 db by a different sequence of switching. Measured patterns illustrating the sidelobe reduction are included.

The applicability of the sidelobe reduction technique to large electronically steerable phased arrays is considered briefly and it is concluded that it is compatible with such systems.

INTRODUCTION

THE USE OF TIME as a fourth dimension in the design of antennas with improved performance was proposed in an article published several years ago,[1] and the possible applications of these "time-modulated" antennas were discussed. Considerable study has since been directed toward specific use of the time-modulation technique for the reduction of sidelobes and many facets of the problem have been considered. The time modulation consists of simple on-off switching of antenna elements in a predetermined sequence, so that, after the antenna output has been filtered, the resulting pattern will have reduced sidelobes.[2]

Time modulation is introduced in an effort to reduce the effects of mechanical tolerances which tend to degrade sidelobe levels from their design values. For example, in a standing-wave array of waveguide longitudinal shunt slots, the on-off switching may be used to reduce the effects of errors in the position of the slots from the centerline of the waveguide.

The time-modulated arrays have great flexibility in the control of the aperture excitation since the time parameter which tapers the distribution is easily, rapidly and accurately adjusted. The RF portion requires only two discrete states, on and off, instead of the continuous amplitude control required in conventional arrays.

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[1] H. E. Shanks and R. W. Bickmore, "Four-dimensional electromagnetic radiators," *Canad. J. Phys.*, vol. 37, pp. 263 March, 1959.

[2] For multiple beam applications of this technique see W. H. Kummer, A. T. Villanueve and F. G. Terno, "Scanning Without Phase Shifters," *Electronics*, vol. 36, pp. 27–32; March, 1963.

THEORY OF SIDELOBE REDUCTION

In this section a brief outline is presented of the technique of sidelobe reduction as applied to linear arrays. The technique uses periodic on-off modulation of the microwave energy received by each element and subsequent filtering of the antenna output to achieve the desired results.

A linear array of equally spaced identical elements is considered (Fig. 1). If a plane wave of angular frequency ω is incident at an angle θ with the array axis, the output signal from the array may be written (in complex notation)

$$v(\theta, \varphi, t) \propto e(\theta, \phi) \sum_{n=0}^{N-1} a_n \exp\left(j[\omega t + nkd \cos\theta]\right), \quad (1)$$

where a_n is the relative excitation of the nth element $k=2\pi/\lambda$ and $e(\theta, \phi)$ is the element factor. Eq. (1) expresses the receiving pattern of the array. If the a_n's are now made periodic functions of time whose period T is much greater than the RF period, the same equation holds at any instant of time and the pattern changes with time periodically. The resulting output signal has a line spectrum centered about ω with the lines separated in frequency by the repetition frequency of the antenna modulation.

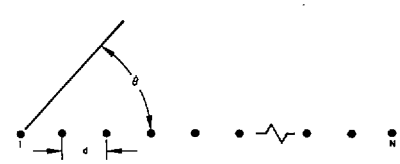

Fig. 1—Linear array of n equally spaced elements.

The signal may be written in terms of its frequency components as

$$v(\theta, \varphi, t) \propto e(\theta, \phi)$$

$$\sum_{m=-\infty}^{\infty} \cdot \exp\left[j(\omega + m\omega_0)t\right] \sum_{n=0}^{N-1} a_{mn} \exp\left(jnkd \cos\theta\right) \quad (2a)$$

where

$$a_{mn} = \frac{1}{T}\int_0^T a_n(t) \exp\left[-jm\omega_0 t\right] dt \quad (2b)$$

and

$$\omega_0 = \frac{2\pi}{T}. \quad (2c)$$

The terms at the incident frequency, that is, for $m=0$, give the following expressions:

$$v_0(\theta, \varphi, t) = e(\theta, \phi) \exp(j\omega t) \sum_{n=0}^{N-1} a_{0n} \exp(jnkd \cos\theta) \quad (3a)$$

where

$$a_{0n} = \frac{1}{T}\int_0^T a_n(t)dt. \quad (3b)$$

Thus the pattern at the incident frequency is the pattern of an array whose excitation coefficients are the time averages of the time varying coefficients.

The simplest type of time variation to realize is on-off switching of the excitations of the elements. With this type of switching (if interactions between elements are negligible), the excitation coefficients may be written

$$a_n(t) = A_n[U(t) - U(t-\tau_n)] \quad (4)$$

where $U(t)$ is the unit step function and $0 \leq \tau_n \leq T$. The A_n are constants. Thus the A_n's are the excitations of the static elements. Then (3a) becomes

$$v_0(\theta,\varphi, t) = e(\theta, \phi) \exp(j\omega t) \sum_{n=0}^{N-1} \frac{\tau_n}{T} A_n \exp(jnkd \cos\theta). \quad (5)$$

It can be seen from (5) that the pattern of the modulated array at the carrier frequency has an aperture distribution controlled by the ON time of the array elements. Adjustment of the fractional time ON of each element provides electronic control of the effective amplitude of its excitation. *If the A_n are all real and positive, then any pattern that can be realized with a unifrom phase aperture distribution can be electronically synthesized.* In particular ultra-low sidelobe patterns may be realized in this manner.

Experimental Array and Associated Equipment

Experiments have been performed with an X-band eight-element array of collinear slots to obtain sidelobe reduction by time modulation of the aperture.

The experimental system consisted of the antenna, diode switches, modulation circuits and the transmitting and detection apparatus. The antenna will be discussed first.

Antenna

The experimental antenna was a corporate-fed array of eight collinear slots and contained three diode switches as shown schematically in Fig. 2. The center two elements (numbers 4 and 5) were always on while each of the three switches controlled a pair of elements located symmetrically about the array center. Isolators were located at the inputs and outputs of each switch to prevent impedance changes as the switches were turned on and off. Static aperture control was achieved by inserting a precision phase shifter and attenuator in each element feed line. The size of the ground plane was 3 by 4 feet. Layers of absorbent material attenuated the energy reflected from the area immediately below the antenna.

The switching technique used was as follows. All array elements were turned on simultaneously; then the outer pair (element numbers 1 and 8) was turned off; then the next outer pair (element numbers 2 and 7) and the next (element numbers 3 and 6) until only the center pair (4 and 5) remained on, as shown in Fig. 3. This sequence was repeated periodically at a frequency of 10 kc. The various ON times were selected to give the correct low sidelobe pattern after the signal was filtered. The relationship between the array coefficients is[3]

$$a_{0n} = A_n \frac{\tau_n}{T}. \quad (6)$$

The ratio of time-averaged excitation coefficients to static coefficients is then directly proportional to the ratio τ_n/T.

Switches

The on-off switching was accomplished with waveguide diode switches which were controlled by timing

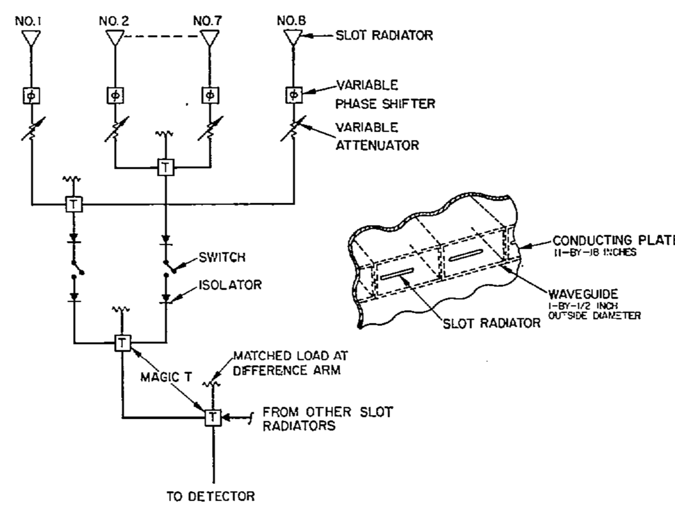

Fig. 2—Experimental array.

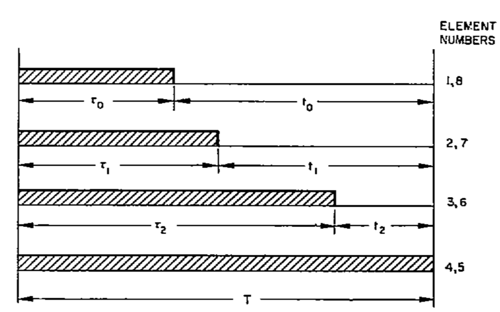

Fig. 3—ON times of switches.

[3] W. H. Kummer, *et al.*, "Time Domain Antenna Techniques," Rome Air Dev. Ctr., AFSC, Rpt. RADC-TDR-61-290; p. 11; October, 1961.

pulses and variable time delay generators. All switches were turned on together and the time delay generators were used to adjust the times at which they were turned off.

Each switch employed two microwave diodes, mounted directly across a section of X-band waveguide and separated along the guide axis by three-fourths a guide wavelength. These units have a switching ratio of better than 45 db with a change of about $1\frac{1}{2}$ volts in bias. The switching times of the switches themselves have not been measured, but, based on available literature, should be about a few nanoseconds. Thus, in any particular application, switching times will be determined by the external control circuitry which is discussed below.

Switching Circuits

Proper switching requires the generation of positive current pulses with widths variable over the required range of τ_n. For the array described, the values of τ_n/T ranged from about 0.1 to 0.95. Three switches with individually controlled pulse widths were required for the experimental array. A block diagram of the control circuitry is shown in Fig. 4. The repetition rate was furnished by a 10-kc crystal controlled oscillator, which drove a pulse generator producing 10-kc negative spikes. Beyond this stage three parallel channels were used, one for each switch. In each channel the 10-kc negative timing spikes entered a pulse width control in which the spikes became flat pulses with duty cycles adjustable from 0.03 to 0.98. Each generator had a memory flip-flop which turned on with the application of an input spike and turned off when the delayed pulse left. The variable width output pulse was derived from this memory circuit through a buffer circuit. The negative rectangular pulses were passed through an inverting current amplifier to both the diode switches and on-off indicators. Each switch could be turned on or off or be programmed. The resultant microwave energy transmitted through the switch consisted of rectangular pulses which had duration times adjustable from 3 to 98 μseconds and a 100-μsecond repetition period; rise and fall times were less than 0.1 μsecond.

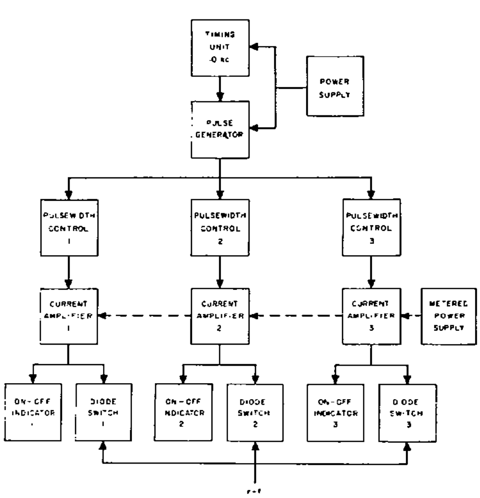

Fig. 4—Block diagram of control circuitry.

Transmitter and Receiver

The transmitter consisted of a crystal controlled multiplier chain with a power output of 14 dbm at 9.375 Gc; the frequency stability was 1 part in 10^8 per hour.

A superheterodyne receiver with quadruple detection was used; the band-pass filter had a 6-kc bandwidth at the lowest intermediate frequency. A tunable local oscillator permitted selection of the proper frequency component for detection and recording. The dynamic range of the receiver was about 60 db; its frequency stability was the same as that of the transmitter. It should be pointed out that the requirements for stability were dictated by the 10-kc modulation frequency. This frequency was chosen for convenience and is not a limiting factor in the technique.

A higher modulation frequency would ease the tolerances in the frequency stability of the transmitter and receiver; however, the driving circuitry would have to be designed for higher frequencies. If the transmitter and receiver can be in close proximity to each other, the local oscillator power could be derived from the transmitter. Thus the receiver and transmitter would track each other and the requirements on the frequency stability of the RF sources could be further relaxed.

Pattern Range

The measured pattern of any antenna includes the characteristics of the range on which it is measured as well as those of the antenna itself. Thus any pattern measurement will contain a certain error, particularly in the measured sidelobe levels. For example, measurement of sidelobes 40 db below the main beam to an accuracy of ± 1 db requires that stray energy from the pattern range be 60 db below the direct signal. The particular pattern range used was selected because of its suitable topography. The transmitter and receiver are located 350 feet apart on small hills separated by a natural gully. The closest obstruction behind the receiver location is five miles away. Extensive tests showed that extraneous reflections in the significant areas between transmitter and receiver are 60 db below the direct signal except for two isolated points at which they are only 56 db. Energy is also scattered into the receiving antenna from the receiver building and antenna mount since the transmitter main beam illuminates these structures as well as the receiving antenna. Layers of absorbent material placed on an apron in front of the receiving antenna, and rotated with it, were found to be an adequate shield against this source of stray reflection. Tests showed that the characteristics of the pattern range were satisfactory for the measurement of 40-db sidelobes to within ± 1.5 db.

Experimental Results

Static Aperture Distributions

The static (unswitched) aperture distributions were set up with a null bridge technique. The signal from one slot element was arbitrarily used as a reference. The amplitude and phase of the signal from each of the remaining slots were compared with the signal from the reference slot, one at a time, and the attenuator and phase shifter of each were adjusted for a null at the bridge output. A uniform aperture distribution was achieved with the reversal of the phase of the reference element. The required static aperture taper was then obtained by the addition of the proper amount of attenuation to the attenuator settings.

The attenuator calibrations were not used in setting up the uniform distribution; the only dial reading actually used was that of the phase shifter of the reference element. However, the attenuator settings were used in achieving the desired aperture taper. The addition of attenuation to each setting involved direct use of the dial reading of each attenuator. Any error in the dial readings become part of the tapered distribution. Because of the possibility of error in the tapered distribution, the uniform static distribution seemed to be the more practical reference.[4] However, effective system gain would be lower with the uniform static distribution than it would be with the tapered distribution. Gain considerations are discussed in detail in a subsequent section.

Static patterns were recorded for both a 30-db Chebyshev aperture distribution and for a uniform aperture distribution. These patterns are shown in Figs. 5 and 6. Some of the irregularities in the 30-db pattern were apparently due to effects of the ground plane. They did not appear to degrade antenna performance significantly. The ripples on the sidelobes close to ± 90 degrees from broadside are attributed to interference between the incident signal and leakage signals in the antenna system.

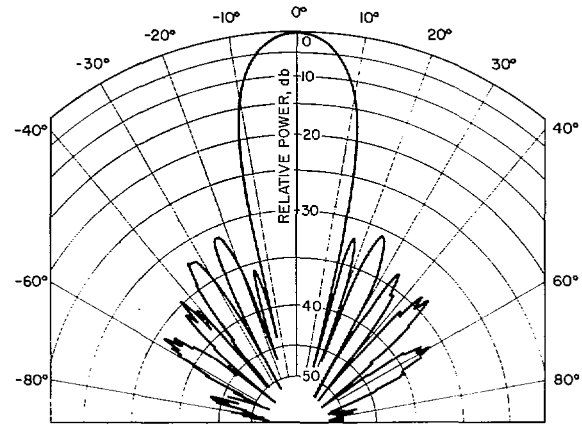

Fig. 5—Measured static 30-db pattern.

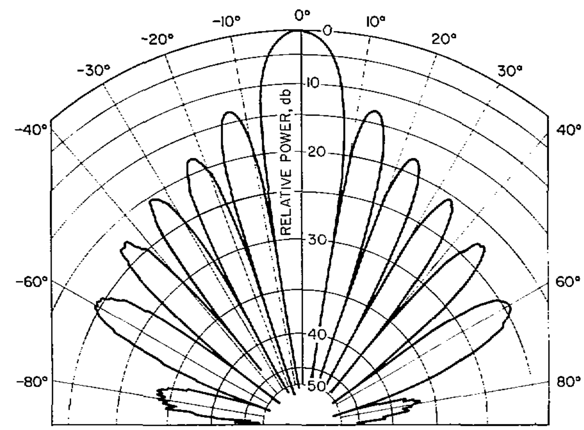

Fig. 6—Measured static 13-db pattern (uniform aperture illumination).

Time-Modulated Distributions

A low sidelobe radiation pattern was obtained from a given static distribution by a controlled modulation of the aperture excitation. The switches which controlled the slots were turned on and off electronically in a predetermined periodic sequence. The switching time for each slot was selected to give a reduction in sidelobe level at the carrier frequency.

The required switching times, calculated using (6), were set with the aid of an oscilloscope; no further adjustments were made. A brief summary of the results is given in Table I.[5]

Measured patterns are presented in Figs. 7–11. Figs. 7, 9 and 11 show the static patterns of a uniform aperture distribution electronically switched to give 30-db, 40-db and 50-db Chebyshev patterns. Figs. 8 and 10 show the patterns of a tapered static aperture distribution (30-db Chebyshev) switched to give 40-db and 50-db Chebyshev patterns.

The uniform to 30-db pattern was measured for comparison with the static 30-db pattern. It can be seen from Figs. 5 and 7 that the agreement is good. Further comparison of Fig. 8 with Fig. 9 shows that modulation which theoretically results in 40-db sidelobes comes very close to accomplishing its goal. Attempts to obtain the actual 40-db level by striving for theoretical 45- to 50-db sidelobes do not result in much more sidelobe improvement than was obtained with the switched 40-db pattern. (See Figs. 10 and 11.) The lack of further sidelobe improvement is attributed to the effects of

[4] It should be pointed out that the ON times of the switched elements would be reduced when they were switched from a uniform static distribution, as compared with the ON times involved when the switching occurred from a tapered static distribution. Any errors in the static distribution would therefore be reduced in the same proportion as the reduction in ON times.

[5] The values given are uncorrected values. If the ripple on the pattern due to interference between direct and leakage signals were corrected, even lower sidelobe levels would result. The leakage signals are about 53 db below the main peak.

TABLE I

Summary of Results of Sidelobe Suppression Experiments

Static Distribution	Intended Sidelobe Level by Modulation	Result Obtained (Sidelobe Level)
30-db sidelobe Chebyshev	40-db sidelobe Chebyshev	38 db
30-db sidelobe Chevyshev	45-db sidelobe Chebyshev	39 db
30-db sidelobe Chebyshev	50-db sidelobe Chebyshev	39.5 db
Uniform amplitude aperture distribution (13-db sidelobe)	30-db sidelobe Chevyshev	31 db
Uniform amplitude aperture distribution (13-db sidelobe)	40-db sidelobe Chebyshev	38 db
Uniform amplitude aperture distribution (13-db sidelobe)	50-db sidelobe Chebyshev	39 db

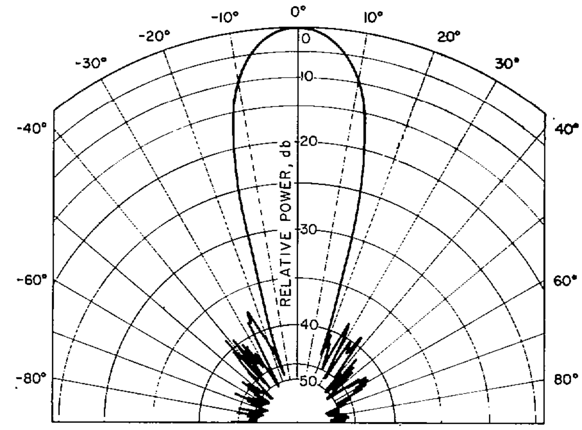

Fig. 9—Measured pattern of a uniform 13-db static distribution switched to give a 40-db Chebyshev pattern.

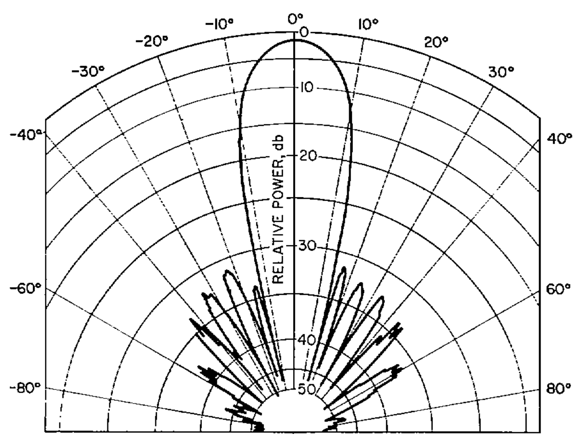

Fig. 7—Measured pattern of a uniform 13-db static distribution switched to give a 30-db Chebyshev pattern.

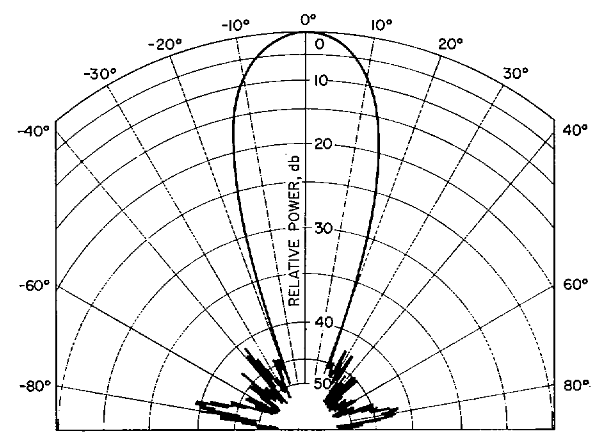

Fig. 10—Measured pattern of 30-db Chebyshev static distribution switched to give a 50-db Chebyshev pattern.

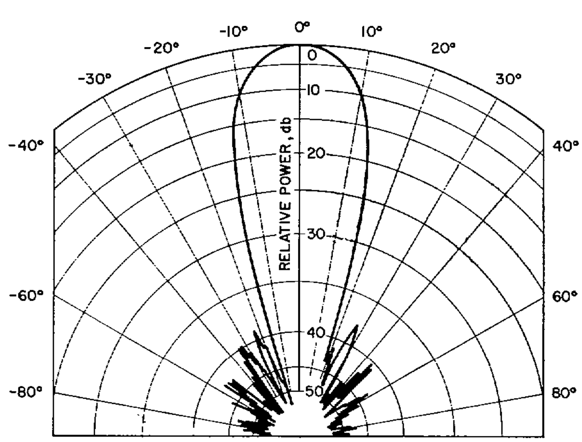

Fig. 8—Measured pattern of 30-db Chebyshev static distribution switched to give a 40-db Chebyshev pattern.

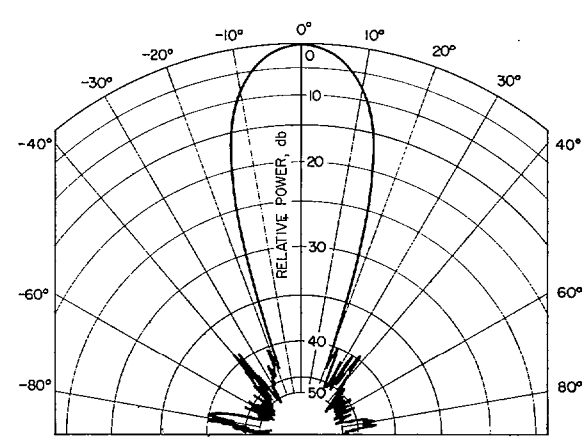

Fig. 11—Measured pattern of a uniform 13-db static distribution switched to give a 50-db Chebyshev pattern.

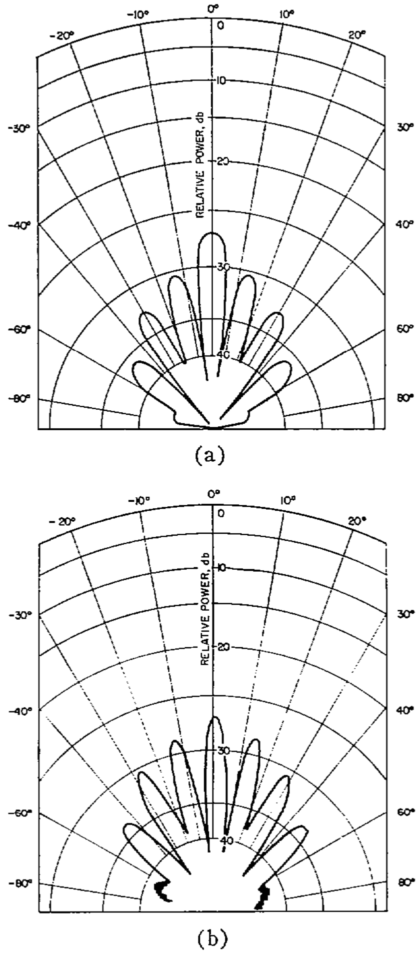

Fig. 12—(a) Computed sideband pattern for $m = \pm 3$, 30-db static—40-db switched. (b) Measured sideband pattern for $m = -3$, 30-db static—40-db switched.

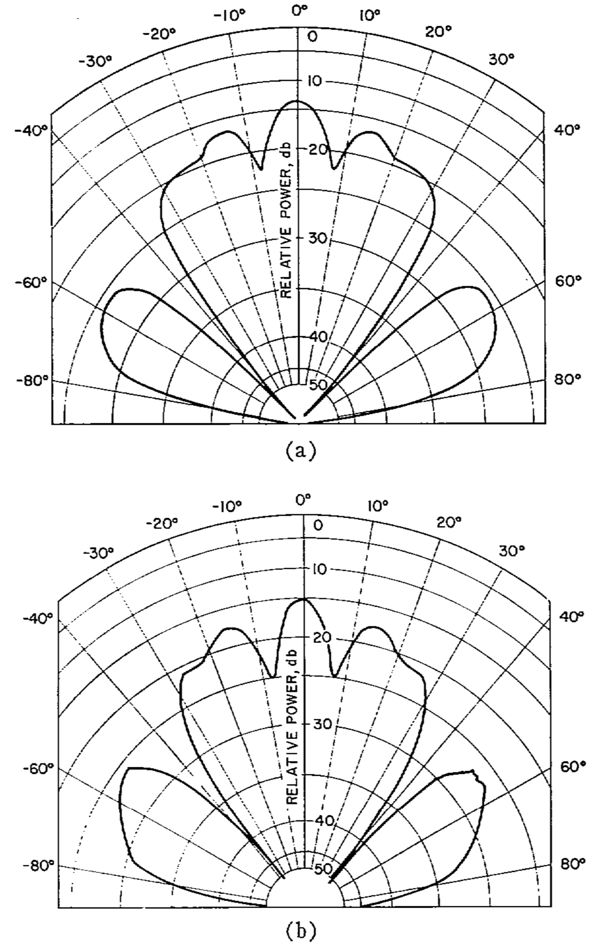

Fig. 13—(a) Computed sideband pattern for $m = \pm 1$, 13-db static—50-db switched. (b) Measured sideband pattern for $m = -1$, 13-db static—50-db switched.

leakage signals (ripples on the low sidelobe patterns) in the system and to extraneous reflections from the pattern range.

Sideband Patterns

The modulation of the aperture excitation of the antenna produces energy in sidebands which are spaced at multiples of the modulation frequencies as shown in (2b). There is a pattern at every one of these frequencies. Patterns were taken for the first three upper and lower sidebands. The theoretical and measured patterns for two of the sidebands are shown in Figs. 12 and 13, respectively. The agreement between theory and experiment was equally good for the other sidebands.

Gain Considerations

In any antenna, whose characteristics are not constant with respect to time, there is a question as to its gain or the signal-to-noise ratio of the received signal as compared to that of a static antenna whose characteristics are identical in some parameter such as pattern, area, functional operation or equivalent information at output terminals.

Calculations were made of the gain typical of the time-modulated arrays and of a reference antenna. For the purposes of the discussion, the reference antenna was assumed to be an array with the same aperture size and receiving pattern as that of the experimental array but with an amplitude taper achieved statically. Qualitatively it can be seen that the output at the carrier frequency of the modulated antenna should be less than that of the reference antenna for the same incident signal, since, due to the modulation of the array, some energy is converted to the sideband frequencies. The comparative loss in gain of the two antennas is presented for a switched array with isolators and for a switched array without isolators.

The first comparison was made between the reference antenna and a switched array with isolators. The isolators were used to prevent impedance changes with switching but their presence did cause power absorption when the switches were off and resulted in lower efficiency. For simplicity, the analysis was carried out for a linear array of isotropic radiators spaced one-half wavelength apart, but it should apply approximately to the antenna used in the experiment.

The ratio of the static gain of the reference antenna to the gain of the switched array with the same aperture size and with isolators is[6]

[6] W. H. Kummer, *et al.*, "Time Domain Antenna Techniques," Rome Air Development Ctr., AFSC Rept. RADC-TDR-62-153, p. 42; 1961.

$$\frac{g}{g_0} = \frac{\sum_{n=0}^{N-1} |A_n|^2}{\sum_{n=0}^{N-1} \left| A_n \frac{\tau_n}{T} \right|^2} \tag{7}$$

where A_n is the amplitude of the static excitation of the nth element of the time-modulated array, τ_n is the ON time of the nth element and T is the modulation period.

For an array with 30-db sidelobes switched to 40 db, the gain is found to be only 0.51 db less than for a reference antenna with 40-db sidelobes. For a static uniform distribution (13-db sidelobes) switched to 40 db, however, the comparative loss in gain becomes 3.53 db. It is apparent that to prevent excessive loss in gain due to switching, it is desirable to design a static pattern with as low sidelobes as possible by conventional techniques. Additional reduction of the sidelobe level can then by obtained by the switching technique.

Further decrease in the loss of effective gain is possible with a switched array designed without isolators around the switches. As an example, the effective gains of the reference antenna and the switched antenna will again be compared. For this calculation it was assumed that the experimental array did not use isolators. The relation between the gains of the two antennas becomes more complicated than it was for the array with isolators, since the feed line now looks into an impedance that varies with time. The ratio of the gain of the reference antenna to that of the switched array without isolators is[7]

$$\frac{g}{g_0} = \frac{\left| \frac{a_{0((N/2-1))}}{A_{((N/2)-1)}} \right| \sum_{n=0}^{N-1} |A_n{}^2|}{\sum_{n=0}^{N-1} (|a_n A_n|)} \tag{8}$$

where A_n is the static amplitude of the nth element for the *switched* array and a_n is the amplitude of the nth element for the reference *static* array. Eq. (8) was evaluated for two cases analogous to the ones used with (7). If the switched array is modulated from a uniform distribution to one with 40-db sidelobes, the loss in gain due to switching is now found to be 2.4 db when the switched array has been optimized. This degradation compares with the 3.53-db ratio calculated for the switched array with isolators. If the switched array is modulated from 30 db to 40 db, the loss in gain due to switching is 0.3 db as compared with the 0.51 db calculated for similar modulation of the switched array with isolators.

Thus, the loss of system gain is decreased if isolators are not used and the resulting system is adjusted for optimum gain. A reasonable degree of sidelobe reduction gives a modest loss in system gain.

[7] *Ibid.*, see p. 47.

Application of Sidelobe Reduction to Phased Arrays

One important consideration in the use of time-modulated antennas is their compatibility with other requirements imposed by the system to which they are connected. A brief discussion of the compatibility of the sidelobe-reduction technique as applied to phased arrays will be given.

A conventional electronically steerable array accomplishes beam steering by electronic control of the phase of the signal from element to element. Two general configurations are most commonly used. One configuration incorporates the phase shifters in the branch lines, while the second places them in the feed lines.

Either of these configurations appears to be compatible with the time-modulation technique. This technique merely introduces switches into the lines feeding each element. The switches would have no direct effect upon the operation of the phase shifters used in scanning the beam. Therefore, the filtered signal at the carrier frequency would retain the phased array characteristics upon which the reduced sidelobe characteristics would be superimposed.

The switching frequency of the array must be compatible with the bandwidth of the received signal. In practice, significantly higher switching rates than those used in the experimental array would be employed.

Conclusions

It has been shown that through periodic switching of the elements of an array, sidelobe levels of nearly 40 db are attainable. Measured array patterns were presented which illustrate this sidelobe reduction for two aperture distributions, one for which the static sidelobe level was 30 db Chebyshev and one for which the static sidelobe level was 13 db (uniform). For both aperture distributions, sidelobes were reduced to levels of nearly 40 db. Once the static pattern was set up, only ON-OFF switching of the array elements was used to obtain electronic control of the array pattern.

It appears that the technique has possible applications to a number of situations which call for low sidelobes. For example, the technique could be applied to parabolic reflector antennas in which the beam shaping in one plane is done by control of the aperture distribution on the primary feed array. In fact, the technique permits any pattern to be obtained electronically that could ordinarily be obtained from a conventional array through amplitude control of the excitation. It has also been determined that the technique is compatible with the operation of electronically controlled phased arrays.

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