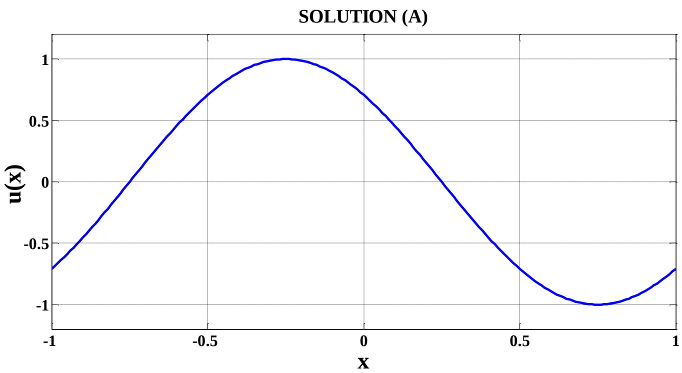
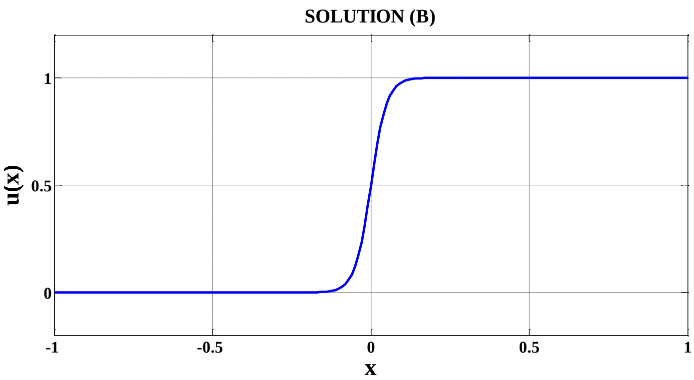
1. Consider the problems on $[-1,1]$:

(A) $u'' = -\pi^2 \cos \pi(x+\frac{1}{4})$ with $u(-1) = u(1) = -\sqrt{2}/2 \quad \Rightarrow \quad u(x) = \cos \pi(x+\frac{1}{4})$

(B) $u'' = -20^2 \tanh(20x)\,\mathrm{sech}^2(20x)$ with $u(-1) = 0$ and $u(1) = 1 \quad \Rightarrow \quad u(x) = \frac{1}{2}[\tanh(20x)+1]$





Use the following elemental decompositions :

Problem (A) : **(a)** $[-1,1]$,

(b) $[\ -1\ ,\ 0\ ]\,[\ 0\ ,\ 1\ ]$,

(c) $[\ -1\ ,\ -0.2\ ],[\ -0.2\ ,\ 0.2\ ],[\ 0.2\ ,\ 1\ ]$

(d) $[\ -1\ ,\ -0.5\ ],[\ -0.5\ ,\ 0\ ],[\ 0\ ,\ 0.5\ ],[\ 0.5\ ,\ 1\ ]$

Problem (B) : **(a)** $[-1,1]$,

(b) $[\ -1\ ,\ 0\ ],[\ 0\ ,\ 1\ ]$

(c) $[\ -1\ ,\ -0.2\ ],[\ -0.2\ ,\ 0.2\ ],[\ 0.2\ ,\ 1\ ]$

(d) $[\ -1\ ,\ -0.1\ ],[\ -0.1\ ,\ 0.1\ ],[\ 0.1\ ,\ 1\ ]$

in an application of Spectral Element method. Use Legendre-Lobatto pseudospectral discretization with the same <u>total number</u> of collocation points N in each case to facilitate comparison. Plot error versus N for various N and discuss (I suggest $N = 24, 36, 48$). Modify **Helmholtz1D.m**.

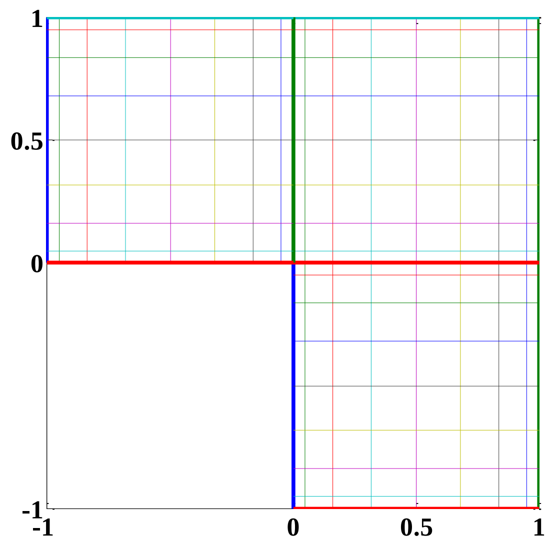
2. Solve Poisson equation on the L-shaped rectangular geometry

$$-\nabla^2 u = 1 \quad \text{in}\,\Omega$$

subject to

$$u = 0 \quad \text{on}\ \partial\Omega$$

using Spectral Element Method with three elements (see **p16_sem.m**). Assess the accuracy of the resulting solution by comparing it with a higher resolution solution



3. Solve Poisson equation on the quarter annular geometry

$$-\nabla^2 u = f \quad \text{in}\, \Omega$$

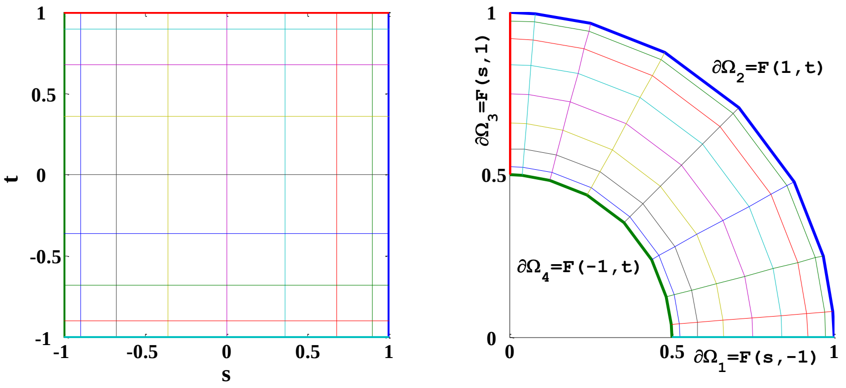
subject to

$$u_y = 0 \ \text{ on } \ \partial\Omega_1$$
$$u = 0 \ \text{ on the rest of } \ \partial\Omega$$

using Spectral Element Method with single element and compare with the exact solution

$$u(x,y) = 10xy^2(1-x^2-y^2)(0.25-x^2-y^2)\,.$$

(see **p29_sem.m**)



Notes

(1) Determine the forcing function $f(x,y)$ so that given $u(x,y)$ is the exact solution (may use Matlab Symbolic Math Toolbox).

(2) Generate the map $(x,y) = F(s,t)$ by using Gordon & Hall procedure

$$\begin{aligned} F(s,t) = {} & F(s,-1)L_1(t) + F(s,1)L_2(t) \\ & + F(-1,t)L_1(s) + F(1,t)L_2(s) \\ & - F(-1,-1)L_1(s)L_1(t) - F(1,-1)L_2(s)L_1(t) \\ & - F(-1,1)L_1(s)L_2(t) - F(1,1)L_2(s)L_2(t) \end{aligned}$$

where L_1, L_2 are linear cardinal functions. So, modify **map2.m** for testing.

4. Bonus Problem: Solve Poisson equation on the unit circular geometry

$$-\nabla^2 u = r^2 \sin^4(\theta/2) - \sin(6\theta)\cos^2(\theta/2) \quad \text{in} \quad \Omega$$

subject to

$$u = 0 \ \text{ on } \ \partial\Omega$$

using Spectral Element Method with the five-element configuration given in the figure. Assess the accuracy of the resulting solution by comparing it with **p29**.