MIDDLE EAST TECHNICAL UNIVERSITY
Graduate School of Natural and Applied Sciences
Department of Mechanical Engineering

NEW COURSE PROPOSAL

Course Code[1]

5690710

Course Title

Higher Order Methods for Partial Differential Equations

Catalog Description

Introduction to the concept of spectral methods. Fourier-collocation spectral methods. Polynomial-collocation spectral methods. Smoothness and accuracy. Boundary value problems. Polar coordinates. Time stepping. Initial value problems. Introduction to spectral element method and Discontinuous Galerkin method.

Background Requirements(s)

Basic Linear Algebra, Basic Numerical Methods

Course in relation to the programs

This course provides alternative tools for computational modeling of physical phenomena that are available in every program such as Finite Element Method, Finite Difference Method, Finite Volume Method.

Course Objectives

The student will be enable to understand the fundamental mathematical ideas as well as implementational ideas behind higher order numerical methods for PDEs. The student will then be able to shape the methods for a particular computational modeling practice.

Reference Material

- *Canuto, C., Hussaini, M. Y., Quarteroni, A, and Zang, T. A., Spectral Methods : Fundamentals in Single Domains, Springer Verlag, 2006.*
- *Canuto, C., Hussaini, M. Y., Quarteroni, A, and Zang, T. A., Spectral Methods : Evolution to Complex Geometries and Applications to Fluid Dynamics, Springer Series in Scientific Computation, 2007.*
- *Deville, M., Fischer, P. F., and Mund, E., High-Order Methods in Incompressible Fluid Flows, Cambridge University Press, 2002.*
- *Funaro, D., Spectral Elements for Transport-Dominated Equations, Springer, 1997.*
- *Hesthaven, J. S., Gottlieb, S. And Gottlieb, D., Spectral Methods for Time-Dependent Problems, Cambridge University Press, 2007.*
- *Hesthaven, J. S. and Warburton, T., Nodal Discontinuous Galerkin Methods: Algorithms, Analysis, and Applications, Springer 2008.*
- *Kopriva, D. A., Implementing Spectral Methods for Partial Differential Equations: Algorithms for Scientists and Engineers, Springer, 2009.*
- *Karniadakis, G. E. and Sherwin, S. J., Spectral/hp Element Methods for Computational Fluid Dynamics, Oxford Universiry Press, 2005.*
- *Peyret, R., Spectral Methods for Incompressible Viscous Flow, Applied Mathematical Sciences Vol. 148, Springer, 2002.*
- *Trefethen, L. N., Spectral Methods in Matlab, SIAM, 2000.*
- *Weideman, J. A. C. and Reddy, S. C., A Matlab Differentiation Matrix Suite, ACM TOMS, Vol. 26, pp. 465--519 (2000).*

Course Outline

Week ***Lecture***

1 Introduction

- *Spectral differentiation versus Finite Differences*
- *MATLAB as a tool in problem solving*
- *Basic layout of Spectral methods*

2-3 Fourier Spectral Differentiation

- *Fourier approximation*
- *Fourier Spectral differentiation via differentiation matrices*
- *Fourier Spectral differentiation via FFT*
- *Smoothness and accuracy*
- *Aliasing and aliasing removal*
- *MATLAB demonstrations*

4-5 Polynomial Spectral Differentiation

- *Polynomial approximation; Jacobi polynomials*
- *Chebyshev Spectral differentiation via Differentiation Matrices*
- *Chebyshev Spectral differentiation via FFT*
- *Smoothness and accuracy*
- *MATLAB demonstrations*

6-7 Boundary Value Problems

- *Treatment of problems Dirichlet/Neumann/Robin type boundary conditions*
- *Eigen boundary value problems*
- *Boundary value problems in Polar coordinates*
- *Differential eigen problems.*
- *Case studies and MATLAB demonstrations*

8 Time Stepping

- *Linear multistep and multistage methods*
- *Stability and convergence criterions*
- *The concept of stability regions*
- *Stiffness*

9-10 Initial Value Problems

- *Method of lines treatment of problems with mixed initial/boundary conditions*
- *Semi-implicit methods*
- *Method of integrating factors*
- *Case studies and MATLAB demonstrations*

11-13 Introduction to spectral element method

- *Weak variational formulation*
- *Elemental representation and parametric mapping*
- *Legendre Spectral differentiation and integration*
- *Local elemental operations*
- *Global operations*
- *Boundary representation*
- *Case studies and MATLAB demonstrations*

14 Introduction to discontinuous Galerkin method

- *The Discontinuous Galerkin formulation*
- *The Mass, Stiffnes integrals and the jump terms*
- *Nodal Discontinuous Galerkin Method*
- *Case studies and MATLAB demonstrations*

Course Conduct

This course will be presented by formal lectures that will be supported with visual aids such as PowerPoint presentation for a better comprehension of some subjects. Matlab will be the main computational tool. Also students will be given regular homework assignments and a term paper on a selected topic.

Grading

Results of homework assignments and one term paper will be used in grading.