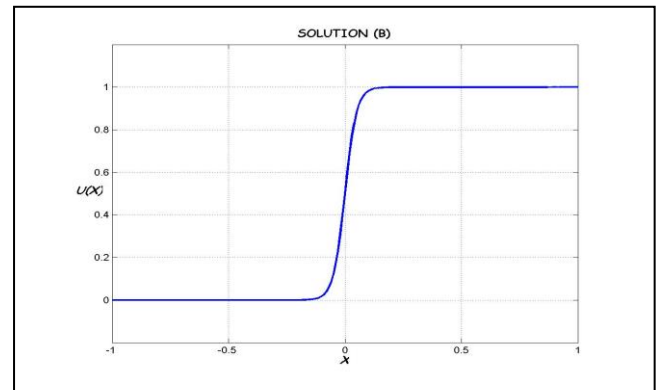
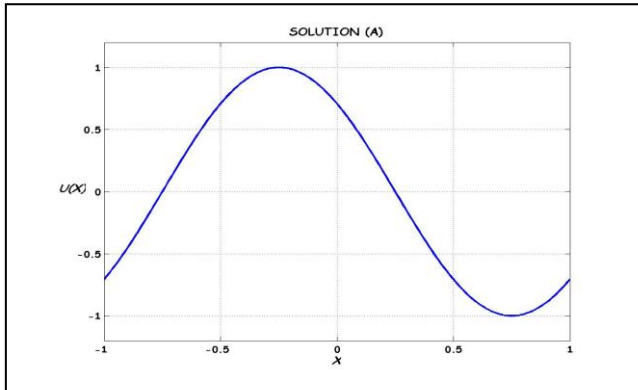


In your solutions, provide details such as formulation, implementation and results together with relevant figures, tables, computer codes written.

1. Consider the problems on  $[-1, 1]$ :

(A)  $u'' = -\pi^2 \cos \pi(x + \frac{1}{4})$  with  $u(-1) = u(1) = -\sqrt{2}/2 \Rightarrow u(x) = \cos \pi(x + \frac{1}{4})$

(B)  $u'' = -20^2 \tanh(20x) \operatorname{sech}^2(20x)$  with  $u(-1) = 0$  and  $u(1) = 1 \Rightarrow u(x) = \frac{1}{2} [\tanh(20x) + 1]$



Use the following elemental decompositions :

- Problem (A) :**
- (a)  $[-1, 1]$ ,
  - (b)  $[-1, 0]$   $[0, 1]$ ,
  - (c)  $[-1, -0.2]$ ,  $[-0.2, 0.2]$ ,  $[0.2, 1]$
  - (d)  $[-1, -0.5]$ ,  $[-0.5, 0]$ ,  $[0, 0.5]$ ,  $[0.5, 1]$
- Problem (B) :**
- (a)  $[-1, 1]$ ,
  - (b)  $[-1, 0]$ ,  $[0, 1]$
  - (c)  $[-1, -0.2]$ ,  $[-0.2, 0.2]$ ,  $[0.2, 1]$
  - (d)  $[-1, -0.1]$ ,  $[-0.1, 0.1]$ ,  $[0.1, 1]$

in an application of Spectral Element method. Use Legendre-Lobatto pseudospectral discretization with the same total number of collocation points  $N$  in each case to facilitate comparison. Plot error versus  $N$  for various  $N$  and discuss (I suggest  $N = 24, 36, 48$ ). Modify **Helmholtz1D.m**.

2. Solve Poisson equation on the L-shaped rectangular geometry

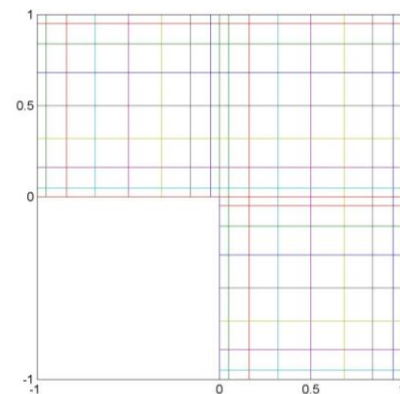
$$-\nabla^2 u = 1 \quad \text{in } \Omega$$

subject to

$$u = 0 \quad \text{on } \partial\Omega$$

using Spectral Element Method with three elements (see **p16\_sem.m**).

Assess the accuracy of the resulting solution by comparing it with a higher resolution solution



3. Solve Poisson equation on the quarter annular geometry

$$-\nabla^2 u = f \quad \text{in } \Omega$$

subject to

$$u_y = 0 \quad \text{on } \partial\Omega_1$$

$$u = 0 \quad \text{on the rest of } \partial\Omega$$

using Spectral Element Method with single element and compare with the exact solution

$$u(x, y) = 10xy^2(1 - x^2 - y^2)(0.25 - x^2 - y^2).$$

(see **p29\_sem.m**)

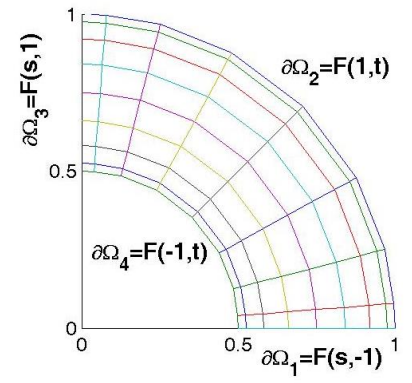
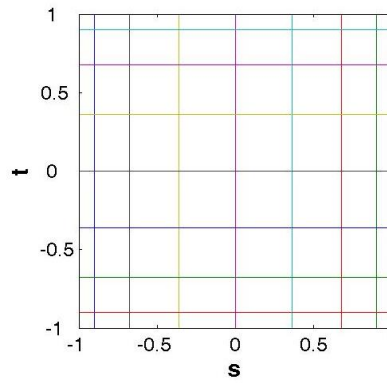
### Notes

(1) Determine the forcing function  $f(x, y)$  so that given  $u(x, y)$  is the exact solution (may use Matlab Symbolic Math Toolbox).

(2) Generate the map  $(x, y) = F(s, t)$  by using Gordon & Hall procedure

$$\begin{aligned} F(s, t) = & F(s, -1)L_1(t) + F(s, 1)L_2(t) \\ & + F(-1, t)L_1(s) + F(1, t)L_2(s) \\ & - F(-1, -1)L_1(s)L_1(t) - F(1, -1)L_2(s)L_1(t) \\ & - F(-1, 1)L_1(s)L_2(t) - F(1, 1)L_2(s)L_2(t) \end{aligned}$$

where  $L_1, L_2$  are linear cardinal functions. So, modify **map2.m** for testing.



4. **Bonus Problem:** Solve Poisson equation on the unit circular geometry

$$-\nabla^2 u = r^2 \sin^4(\theta/2) - \sin(6\theta) \cos^2(\theta/2) \quad \text{in } \Omega$$

subject to

$$u = 0 \quad \text{on } \partial\Omega$$

using Spectral Element Method with the five-element configuration given in the figure.

Assess the accuracy of the resulting solution by comparing it with **p29**.

