MIDDLE EAST TECHNICAL UNIVERSITY
Graduate School of Natural and Applied Sciences
Department of Engineering Sciences

Course Syllabus

Course Code Title and Credit: ES 516 Spectral Methods (3-0)3

Catalog Description: Introduction to the concept of spectral methods. Fourier-collocation spectral methods. Chebyshev-collocation spectral methods. Smoothness and accuracy. Boundary value problems. Polar coordinates. Time stepping. Initial value problems. Introduction to spectral element method.

Background Requirement(s): Basic Linear Algebra and Numerical Analysis

Course Outline

Week Lecture

1 Introduction
- Spectral differentiation versus Finite Differences
- MATLAB as a tool in problem solving
- Basic layout of Spectral methods

2-4 Fourier Spectral Differentiation
- Fourier approximation
- Fourier Spectral differentiation via differentiation matrices
- Fourier Spectral differentiation via FFT
- Smoothness and accuracy
- Aliasing and aliasing removal
- MATLAB demonstrations

5-6 Chebyshev Spectral Differentiation
- Polynomial approximation; Jacobi polynomials
- Chebyshev Spectral differentiation via Differentiation Matrices
- Chebyshev Spectral differentiation via FFT
- Smoothness and accuracy
- MATLAB demonstrations

7-8 Boundary Value Problems
- Treatment of problems Dirichlet/Neumann/Robin type boundary conditions
- Eigen boundary value problems
- Boundary value problems in Polar coordinates
- Differential eigen problems.
- Case studies and MATLAB demonstrations

9 Time Stepping
- Linear multistep and multistage methods
- Stability and convergence criterions
- The concept of stability regions
- Stiffness

10-11 Initial Value Problems
- Method of lines treatment of problems with mixed initial/boundary conditions
- Semi-implicit methods
- Method of integrating factors
- Case studies and MATLAB demonstrations

12-14 Introduction to spectral element method

- Weak variational formulation
- Elemental representation and parametric mapping
- Legendre Spectral differentiation and integration
- Local elemental operations
- Global operations
- Boundary representation
- Case studies and MATLAB demonstrations

Textbook:

- L. N. Trefethen, Spectral Methods in Matlab, SIAM, 2000.

Reference Materials:

- Gottlieb, D. and Orszag, S. A., Numerical Analysis of Spectral Methods: Theory and Applications, CBMS-NSF 26, Philadelphia: SIAM, 1977.
- Canuto, C., Hussaini, M. Y., Quarteroni, A, and Zang, T. A., Spectral Methods in Fluid Dynamics, Springer Series in Computational Physics, 1988.
- Funaro, D., Polynomial Approximation of Differential Equations, Lecture Notes in Physics, 8, Berlin: Springer-Verlag, 1992.
- Fornberg, B., A practical guide to pseudospectral methods, Cambridge University Press, 1996.
- Funaro, D., Spectral Elements for Transport-Dominated Equations, Springer, 1997.
- Guo, B-Y, Spectral methods and their applications, World Scientific, 1998
- Karniadakis, G. E. and Sherwin, S. J., Spectral/hp Element Methods for CFD, Oxford Universiry Press, 1999.
- Boyd, J. P., Chebyshev and Fourier Spectral Methods, Dover, 2000.
- Weideman, J. A. C. and Reddy, S. C., A Matlab Differentiation Matrix Suite, ACM TOMS, Vol. 26, pp. 465--519 (2000). http://dip.sun.ac.za/~weideman/research/differ.html
- Deville, M., Fischer, P. F., and Mund, E., High-Order Methods in Incompressible Fluid Flows, Cambridge University Press, 2002.
- Peyret, R., Spectral Methods for Incompressible Viscous Flow, Applied Mathematical Sciences Vol. 148, Springer, 2002.
- Karniadakis, G. E. and Sherwin, S. J., Spectral/hp Element Methods for Computational Fluid Dynamics, Oxford Universiry Press, 2005.
- Canuto, C., Hussaini, M. Y., Quarteroni, A, and Zang, T. A., Spectral Methods : Fundamentals in Single Domains, Springer Verlag, 2006.
- Canuto, C., Hussaini, M. Y., Quarteroni, A, and Zang, T. A., Spectral Methods : Evolution to Complex Geometries and Applications to Fluid Dynamics, Springer Series in Scientific Computation, 2007.
- Hesthaven, J. S., Gottlieb, S. And Gottlieb, D., Spectral Methods for Time-Dependent Problems, Cambridge University Press, 2007.
- Kopriva, D. A., Implementing Spectral Methods for Partial Differential Equations: Algorithms for Scientists and Engineers, Springer, 2009.

Course Conduct: This course will be presented by formal lectures that will be supported with visual aids such as PowerPoint presentation for a better comprehension of some subjects. Matlab will be the main computational tool. Also students will be given regular homework assignments and a term paper on a selected topic.

Grading :

Results of homework assignments and one term paper will be used in grading.