Introduction to DYNAMICS

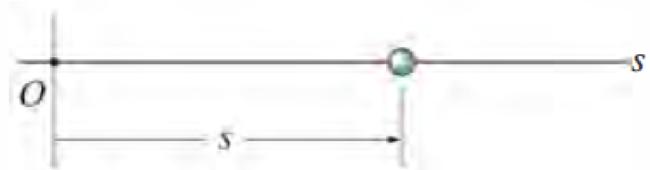
Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies subjected to the action of forces. Engineering mechanics is divided into two areas of study, namely, statics and dynamics. *Statics* is concerned with the equilibrium of a body that is either at rest or moves with constant velocity. Here we will consider *dynamics*, which deals with the accelerated motion of a body. The subject of dynamics will be presented in two parts: *kinematics*, which treats only the geometric aspects of the motion, and *kinetics*, which is the analysis of the forces causing the motion. To develop these principles, the dynamics of a particle will be discussed first, followed by topics in rigid-body dynamics in two and then three dimensions.

Rectilinear Kinematics: Continuous Motion

We will begin our study of dynamics by discussing the kinematics of a particle that moves along a rectilinear or straight line path. Recall that a *particle* has a mass but negligible size and shape. Therefore we must limit application to those objects that have dimensions that are of no consequence in the analysis of the motion. In most problems, we will be interested in bodies of finite size, such as rockets, projectiles, or vehicles. Each of these objects can be considered as a particle, as long as the motion is characterized by the motion of its mass center and any rotation of the body is neglected.

Rectilinear Kinematics. The kinematics of a particle is characterized by specifying, at any given instant, the particle's position, velocity, and acceleration.

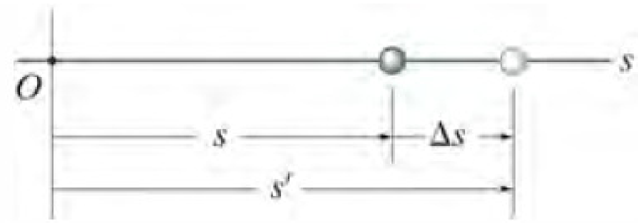
Position. The straight-line path of a particle will be defined using a single coordinate axis s, The origin O on the path is a fixed point, and from this point the *position coordinate* s is used to specify the location of the particle at any given instant. The magnitude of s is the distance from O to the particle, usually measured in meters (m) or feet (ft), and the sense of direction is defined by the algebraic sign on s. Although the choice is arbitrary, in this case s is positive since the coordinate axis is positive to the right of the origin. Likewise, it is negative if the particle is located to the left of O. Realize that position is a vector quantity since it has both magnitude and direction. Here, however, it is being represented by the algebraic scalar s since the direction always remains along the coordinate axis.



Position

Displacement. The *displacement* of the particle is defined as the *change* in its *position*. For example, if the particle moves from one point to another, the displacement is

$$\Delta s = s' - s$$



Displacement

In this case Δs is *positive* since the particle's final position is to the *right* of its initial position, i.e., $s' > s$. Likewise, if the final position were to the *left* of its initial position, Δs would be *negative*.

The displacement of a particle is also a *vector quantity*, and it should be distinguished from the distance the particle travels. Specifically, the *distance traveled* is a *positive scalar* that represents the total length of path over which the particle travels.

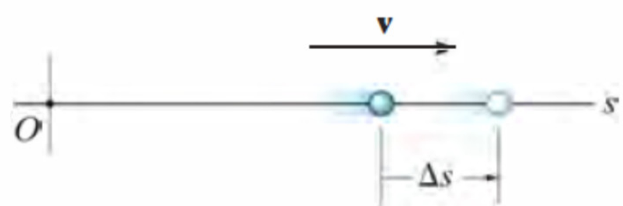
a vector quantity since it has both magnitude and direction. Here, however, it is being represented by the algebraic scalar s since the direction always remains along the coordinate axis.

Velocity. If the particle moves through a displacement Δs during the time interval Δt, the *average velocity* of the particle during this time interval is

$$v_{avg} = \frac{\Delta s}{\Delta t}$$

If we take smaller and smaller values of Δt, the magnitude of Δs becomes smaller and smaller. Consequently, the *instantaneous velocity* is a vector defined as $v = \lim_{\Delta t \to 0} (\Delta s / \Delta t)$, or

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt}$$



Velocity

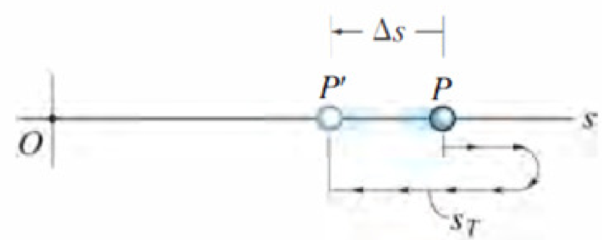
Since Δt or dt is always positive, the sign used to define the *sense* of the velocity is the same as that of Δs or ds. For example, if the particle is moving to the *right*, Fig. 12–1c, the velocity is *positive;* whereas if it is moving to the *left*, the velocity is *negative*.

The *magnitude* of the velocity is known as the *speed*, and it is generally expressed in units of m/s or ft/s.

Occasionally, the term "average speed" is used. The *average speed* is always a positive scalar and is defined as the total distance traveled by a particle, s_T, divided by the elapsed time Δt; i.e.,

$$(v_{sp})_{avg} = \frac{s_T}{\Delta t}$$

For example, the particle in the figure travels along the path of length s_T in time Δt, so its average speed is $(v_{sp})_{avg} = s_T/\Delta t$, but its average velocity is $v_{avg} = -\Delta s/\Delta t$.



Average velocity and Average speed

Acceleration. Provided the velocity of the particle is known at two points, the *average acceleration* of the particle during the time interval Δt is defined as

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

Here Δv represents the difference in the velocity during the time interval Δt, i.e., $\Delta v = v' - v$,

The *instantaneous acceleration* at time t is a vector that is found by taking smaller and smaller values of Δt and corresponding smaller and smaller values of Δv, so that $a = \lim_{\Delta t \to 0} (\Delta v / \Delta t)$, or

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt}$$

Finally, an important differential relation involving the displacement, velocity, and acceleration along the path may be obtained by eliminating the time differential dt

$$(\overset{+}{\rightarrow}) \qquad a\,ds = v\,dv$$

Constant Acceleration, $a = a_c$. When the acceleration is constant, each of the three kinematic equations $a_c = dv/dt$, $v = ds/dt$, and $a_c\,ds = v\,dv$ can be integrated to obtain formulas that relate a_c, v, s, and t.

Velocity as a Function of Time. Integrate $a_c = dv/dt$, assuming that initially $v = v_0$ when $t = 0$.

$$\int_{v_0}^{v} dv = \int_0^t a_c\,dt$$

$(\overset{+}{\rightarrow})$

$$v = v_0 + a_c t$$

Constant Acceleration

Position as a Function of Time. Integrate $v = ds/dt = v_0 + a_c t$, assuming that initially $s = s_0$ when $t = 0$.

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + a_c t)\, dt$$

$(\overset{+}{\rightarrow})$

$$s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

Constant Acceleration

Velocity as a Function of Position.

integrate $v\, dv = a_c\, ds$, assuming that initially $v = v_0$ at $s = s_0$.

$$\int_{v_0}^{v} v\, dv = \int_{s_0}^{s} a_c\, ds$$

$(\overset{+}{\rightarrow})$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Constant Acceleration

Important Points

- Dynamics is concerned with bodies that have accelerated motion.
- Kinematics is a study of the geometry of the motion.
- Kinetics is a study of the forces that cause the motion.
- Rectilinear kinematics refers to straight-line motion.
- Speed refers to the magnitude of velocity.
- Average speed is the total distance traveled divided by the total time. This is different from the average velocity, which is the displacement divided by the time.
- A particle that is slowing down is decelerating.
- A particle can have an acceleration and yet have zero velocity.
- The relationship $a\,ds = v\,dv$ is derived from $a = dv/dt$ and $v = ds/dt$, by eliminating dt.

Kinetics of a Particle: Force and Acceleration

Kinetics is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change. The basis for kinetics is Newton's second law, which states that when an *unbalanced force* acts on a particle, the particle will *accelerate* in the direction of the force with a magnitude that is proportional to the force.

This law can be verified experimentally by applying a known unbalanced force $\vec{\mathbf{F}}$ to a particle, and then measuring the acceleration $\vec{\mathbf{a}}$. Since the force and acceleration are directly proportional, the constant of proportionality, m, may be determined from the ratio $m = F/a$. This positive scalar m is called the *mass* of the particle. Being constant during any acceleration, m provides a quantitative measure of the resistance of the particle to a change in its velocity, that is its inertia.

If the mass of the particle is m, Newton's second law of motion may be written in mathematical form as

$$\vec{\mathbf{F}}_{net} = m\vec{\mathbf{a}}$$

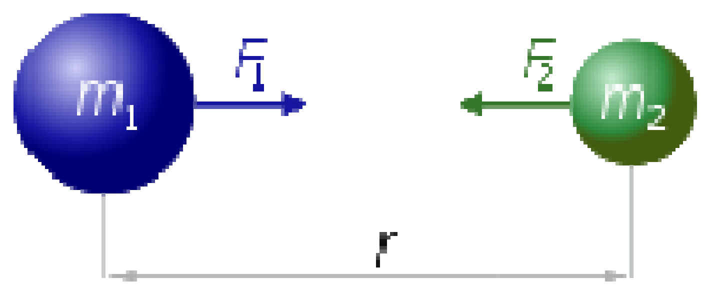
The above equation, which is referred to as the *equation of motion*, is one of the most important formulations in mechanics.* As previously stated, its validity is based solely on *experimental evidence.* In 1905, however, Albert Einstein developed the theory of relativity and placed limitations on the use of Newton's second law for describing general particle motion. Through experiments it was proven that *time* is not an absolute quantity as assumed by Newton; and as a result, the equation of motion fails to predict the exact behavior of a particle, especially when the particle's speed approaches the speed of light (0.3 Gm/s). Developments of the theory of quantum mechanics by Erwin Schrödinger and others indicate further that conclusions drawn from using this equation are also invalid when particles are the size of an atom and move close to one another. For the most part, however, these requirements regarding particle speed and size are not encountered in engineering problems

Newton's law of universal gravitation

- Every particle attracts every other particle in the universe with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. This is a general physical law derived from empirical observations by what Isaac Newton.

•F is the force between the particles;
•G is the gravitational constant (6.674×10^{-11} N · (m/kg)^2);
•m_1 is the first mass;
•m_2 is the second mass;
•r is the distance between the centers of the masses.

R: Radius of the Earth≅$6.38*10^6$
m_E:mass of the Earth≅$5.98*10^{24}$
W=(Gm_E/R^2) m → $W = mg$



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

Important Points

- The equation of motion is based on experimental evidence and is valid only when applied within an inertial frame of reference.
- The equation of motion states that the *unbalanced force* on a particle causes it to *accelerate*.
- An inertial frame of reference does not rotate, rather its axes either translate with constant velocity or are at rest.
- Mass is a property of matter that provides a quantitative measure of its resistance to a change in velocity. It is an absolute quantity and so it does not change from one location to another.
- Weight is a force that is caused by the earth's gravitation. It is not absolute; rather it depends on the altitude of the mass from the earth's surface.

Planar Kinematics of a Rigid Body

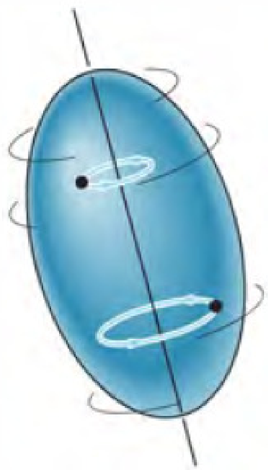
The *planar motion* of a body occurs when all the particles of a rigid body move along paths which are equidistant from a fixed plane. There are three types of rigid body planar motion, in order of increasing complexity, they are

Path of rectilinear translation

(a)

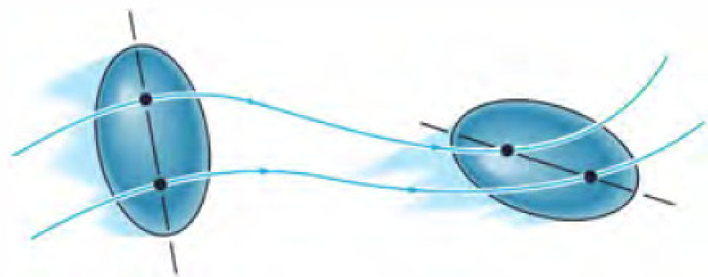
Path of curvilinear translation

(b)



Rotation about a fixed axis

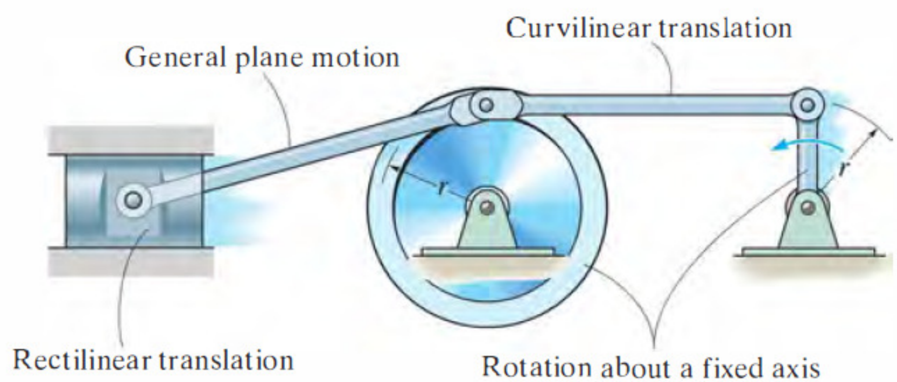
(c)



General plane motion

(d)

- *Translation.* This type of motion occurs when a line in the body remains parallel to its original orientation throughout the motion. When the paths of motion for any two points on the body are parallel lines, the motion is called *rectilinear translation,* Fig. *a.* If the paths of motion are along curved lines which are equidistant, the motion is called *curvilinear translation,* Fig. *b.*
- *Rotation about a fixed axis.* When a rigid body rotates about a fixed axis, all the particles of the body, except those which lie on the axis of rotation, move along circular paths, Fig. *c.*
- *General plane motion.* When a body is subjected to general plane motion, it undergoes a combination of translation *and* rotation, Fig. *d.* The translation occurs within a reference plane, and the rotation occurs about an axis perpendicular to the reference plane.



TRANSLATION of a Rigid Body

Consider a rigid body which is subjected to either rectilinear or curvilinear translation in the x–y plane, Fig.

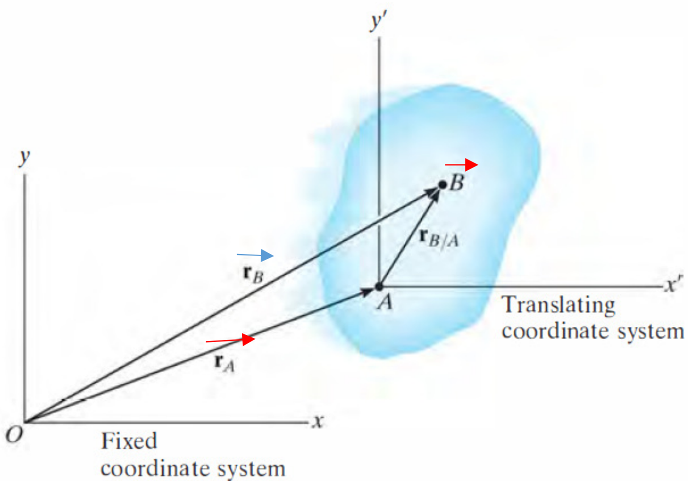


Fig.

Position. The locations of points A and B on the body are defined with respect to fixed x, y reference frame using *position vectors* $\mathbf{r}_A$ and $\mathbf{r}_B$. The translating x', y' coordinate system is *fixed in the body* and has its origin at A, hereafter referred to as the *base point*. The position of B with respect to A is denoted by the *relative-position vector* $\mathbf{r}_{B/A}$ ("$\mathbf{r}$ of B with respect to A"). By vector addition,

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

Velocity. A relation between the instantaneous velocities of A and B is obtained by taking the time derivative of this equation, which yields $\mathbf{v}_B = \mathbf{v}_A + d\mathbf{r}_{B/A}/dt$. Here $\mathbf{v}_A$ and $\mathbf{v}_B$ denote *absolute velocities* since these vectors are measured with respect to the x, y axes. The term $d\mathbf{r}_{B/A}/dt = \mathbf{0}$, since the *magnitude* of $\mathbf{r}_{B/A}$ is *constant* by definition of a rigid body, and because the body is translating the *direction* of $\mathbf{r}_{B/A}$ is also *constant*. Therefore,

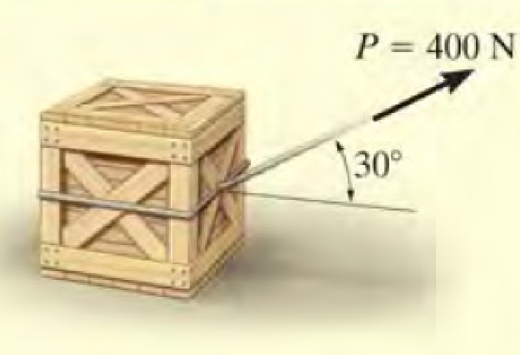
$$\mathbf{v}_B = \mathbf{v}_A$$

Acceleration. Taking the time derivative of the velocity equation yields a similar relationship between the instantaneous accelerations of A and B:

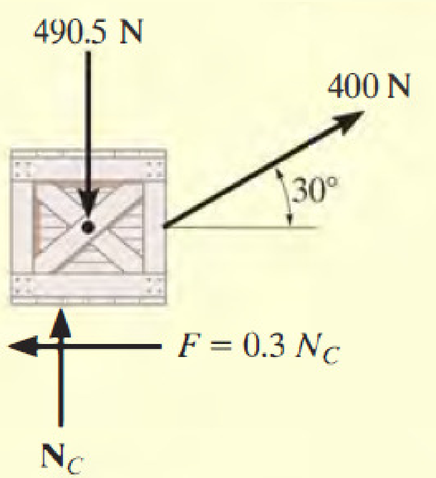
$$\mathbf{a}_B = \mathbf{a}_A$$

The above two equations indicate that *all points in a rigid body subjected to either rectilinear or curvilinear translation move with the same velocity and acceleration.* As a result, the kinematics of particle motion, can also be used to specify the kinematics of points located in a translating rigid body.

Example:



(a)



(b)

The 50-kg crate shown in Fig. *a* rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from rest.

SOLUTION

Using the equations of motion, we can relate the crate's acceleration to the force causing the motion. The crate's velocity can then be determined using kinematics.

Free-Body Diagram. The weight of the crate is $W = mg = 50\ \text{kg}\ (9.81\ \text{m/s}^2) = 490.5\ \text{N}$. As shown in Fig. *b*, the frictional force has a magnitude $F = \mu_k N_C$ and acts to the left, since it opposes the motion of the crate. The acceleration **a** is assumed to act horizontally, in the positive x direction. There are two unknowns, namely N_C and a.

Equations of Motion. Using the data shown on the free-body diagram, we have

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 400 \cos 30° - 0.3N_C = 50a \qquad (1)$$

$$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 490.5 + 400 \sin 30° = 0 \qquad (2)$$

Solving Eq. 2 for N_C, substituting the result into Eq. 1, and solving for a yields

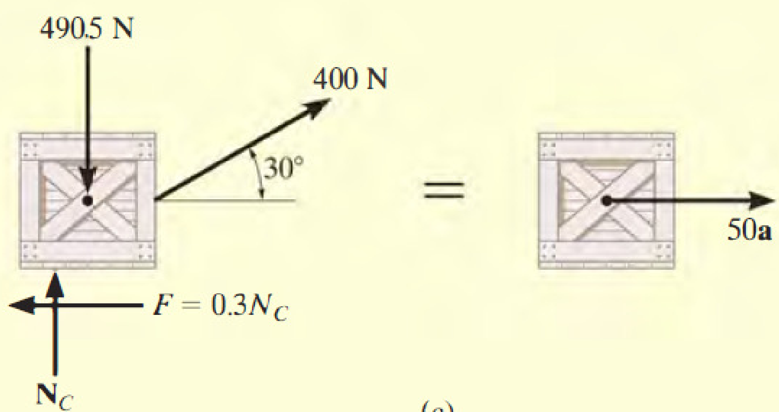
$$N_C = 290.5 \text{ N}$$

$$a = 5.185 \text{ m/s}^2$$

Kinematics. Notice that the acceleration is *constant*, since the applied force **P** is constant. Since the initial velocity is zero, the velocity of the crate in 3 s is

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t = 0 + 5.185(3)$$

$$= 15.6 \text{ m/s} \rightarrow \qquad \textit{Ans.}$$



(c)

Power. The term "power" provides a useful basis for chosing the type of motor or machine which is required to do a certain amount of work in a given time. For example, two pumps may each be able to empty a reservoir if given enough time; however, the pump having the larger power will complete the job sooner.

The *power* generated by a machine or engine that performs an amount of work dW within the time interval dt is therefore

$$P = \frac{dW}{dt}$$

If the work dW is expressed as $dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, then

$$P = \frac{dW}{dt} = \frac{\vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}}{dt} = \vec{\mathbf{F}} \cdot \frac{d\vec{\mathbf{r}}}{dt}$$

$$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

Efficiency. The *mechanical efficiency* of a machine is defined as the ratio of the output of useful power produced by the machine to the input of power supplied to the machine. Hence,

$$\eta = \frac{\text{power output}}{\text{power input}}$$

Since machines consist of a series of moving parts, frictional forces will always be developed within the machine, and as a result, extra energy or power is needed to overcome these forces. Consequently, power output will be less than power input and so *the efficiency of a machine is always less than 1*.

The power supplied to a body can be determined using the following procedure.

Procedure for Analysis

- First determine the external force $\vec{\mathbf{F}}$ acting on the body which causes the motion. This force is usually developed by a machine or engine placed either within or external to the body.
- If the body is accelerating, it may be necessary to draw its free-body diagram and apply the equation of motion ($\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$) to determine $\vec{\mathbf{F}}$.
- Once $\vec{\mathbf{F}}$ and the velocity $\vec{\mathbf{v}}$ of the particle where $\vec{\mathbf{F}}$ is applied have been found, the power is determined by multiplying the force magnitude with the component of velocity acting in the direction of $\vec{\mathbf{F}}$, (i.e., $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = Fv\cos\theta$).
- In some problems the power may be found by calculating the work done by $\vec{\mathbf{F}}$ per unit of time ($P_{avg} = \Delta W/\Delta t$,).

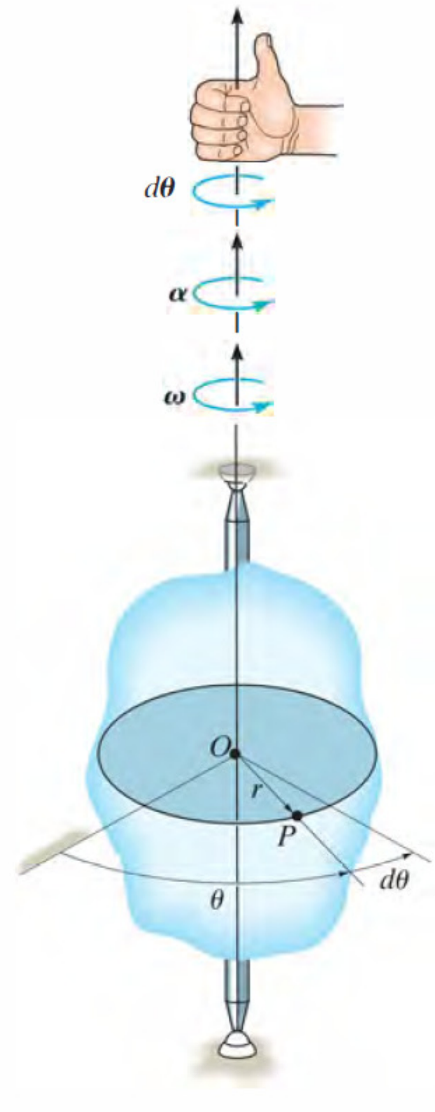
Rotation about a Fixed Axis

When a body rotates about a fixed axis, any point P located in the body travels along a *circular path*. To study this motion it is first necessary to discuss the angular motion of the body about the axis.

Angular Motion. Since a point is without dimension, it cannot have angular motion. *Only lines or bodies undergo angular motion.* For example, consider the body shown in Fig. *a* and the angular motion of a radial line r located within the shaded plane.

Angular Position. At the instant shown, the *angular position* of r is defined by the angle θ, measured from a *fixed* reference line to r.

Angular Displacement. The change in the angular position, which can be measured as a differential $d\boldsymbol{\theta}$, is called the *angular displacement*.* This vector has a *magnitude* of $d\theta$, measured in degrees, radians, or revolutions, where 1 rev $= 2\pi$ rad. Since motion is about a *fixed axis*, the direction of $d\boldsymbol{\theta}$ is *always* along this axis. Specifically, the *direction* is determined by the right-hand rule; that is, the fingers of the right hand are curled with the sense of rotation, so that in this case the thumb, or $d\boldsymbol{\theta}$, points upward, Fig. *a*. In two dimensions, as shown by the top view of the shaded plane, Fig. *b*, both θ and $d\theta$ are counterclockwise, and so the thumb points outward from the page.



(a)

Angular Velocity. The time rate of change in the angular position is called the *angular velocity* $\vec{\boldsymbol{\omega}}$ (omega). Since $d\vec{\boldsymbol{\theta}}$ occurs during an instant of time dt, then,

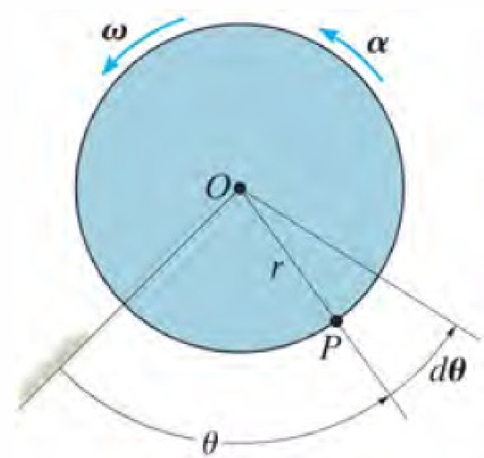
$$(\circlearrowleft +) \qquad \omega = \frac{d\theta}{dt}$$

This vector has a *magnitude* which is often measured in rad/s. It is expressed here in scalar form since its *direction* is also along the axis of rotation, Fig. *a*. When indicating the angular motion in the shaded plane, Fig. *b*, we can refer to the sense of rotation as clockwise or counterclockwise. Here we have *arbitrarily* chosen counterclockwise rotations as *positive* and indicated this by the curl shown in parentheses next to Eq. Realize, however, that the directional sense of $\vec{\boldsymbol{\omega}}$ is actually outward from the page.

Angular Acceleration. The *angular acceleration* $\vec{\boldsymbol{\alpha}}$ (alpha) measures the time rate of change of the angular velocity. The *magnitude* of this vector is

$$(\circlearrowleft +) \qquad \alpha = \frac{d\omega}{dt}$$

$$\alpha \, d\theta = \omega \, d\omega$$



(b)

Constant Angular Acceleration. If the angular acceleration of the body is *constant*, $\boldsymbol{\alpha} = \boldsymbol{\alpha}_c$, then Eqs. when integrated, yield a set of formulas which relate the body's angular velocity, angular position, and time. These equations are similar to Eqs. used for rectilinear motion. The results are

$$(\circlearrowleft +) \qquad \omega = \omega_0 + \alpha_c t$$

$$(\circlearrowleft +) \qquad \theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha_c t^2$$

$$(\circlearrowleft +) \qquad \omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

Constant Angular Acceleration

Here θ_0 and ω_0 are the initial values of the body's angular position and angular velocity, respectively.

Motion of Point *P*. As the rigid body in Fig. *c* rotates, point *P* travels along a *circular path* of radius *r* with center at point *O*. This path is contained within the shaded plane shown in top view, Fig *d*.

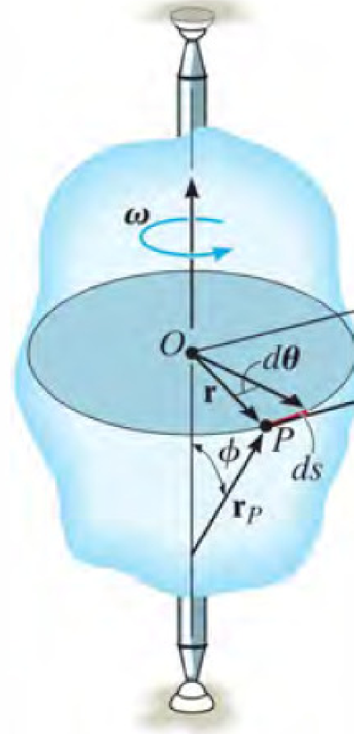
Position and Displacement. The position of *P* is defined by the position vector **r**, which extends from *O* to *P*. If the body rotates $d\theta$ then *P* will displace $ds = r\,d\theta$.

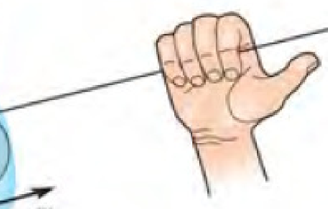
Velocity. The velocity of *P* has a magnitude which can be found by dividing $ds = r\,d\theta$ by dt so that

$$v = \omega r$$

$$\omega = \frac{2\pi n}{60}$$

where
n: number of turns per minute, [rpm]

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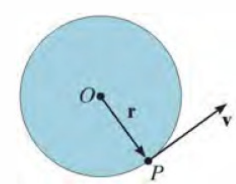
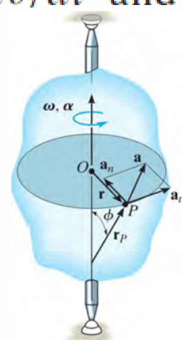


(d)

Acceleration. The acceleration of *P* can be expressed in terms of its normal and tangential components. Since $a_t = dv/dt$ and $a_n = v^2/\rho$, where $\rho = r$, $v = \omega r$, and $\alpha = d\omega/dt$, we have

$$a_t = \alpha r$$

$$a_n = \omega^2 r$$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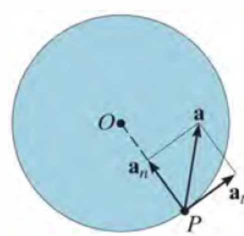

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