EE 524 May 10, 2012
Midterm Exam 2

1. Determine the electric field strength at a distance of 100 km away from a transmitter and 2 m above the ground. Suppose that the transmitter is operating at 30 cm, radiates 5 kW power, and uses an antenna with a gain of 10 located 2 m above the ground. Assume vertical polarization and propagation over wet ground with $\kappa' = 30$ and $\sigma = 10^{-2}$ S/ m.

   Do you think it is possible to establish a communication between two such points?

   SOLUTION: Since the antennas are very close to the ground, we should consider ground wave propagation. In fact the total horizon distance is

$$R_h = \sqrt{2a_e h_t} + \sqrt{2a_e h_r} = 11.7\,\text{km}$$

   and therefore we are well beyond the interference region. We also find that ($f_{\text{MHz}} = 1000$)

$$80/f_{\text{MHz}}^{1/3} = 8\,\text{km}$$

   which shows that earth's sphericity should be taken into account. Therefore, we use the single term expansion.

   The normalized surface impedance and the q value can be calculated as ($a_e = 8500\,\text{km}$ is used)

$$\begin{aligned} \kappa &= \kappa' + \frac{\sigma}{j\omega\varepsilon_0} = 30 - j0.179, \\ \Delta &= \frac{\sqrt{\kappa - 1}}{\kappa} = 0.179 + j5.17 \times 10^{-4}, \\ q &= -j\left(\frac{k_0 a_e}{2}\right)^{1/3} \Delta = 0.231 - j79.9. \end{aligned}$$

   From the given link parameters we also find

$$x = \left(\frac{k_0 a_e}{2}\right)^{1/3} \left(\frac{R}{a_e}\right) = 5.25 > 10/3$$

   which justifies the use of single term expansion. We also find

$$y_t = y_r = \left(\frac{2}{k_0 a_e}\right)^{1/3} k_0 h = 9.38 \times 10^{-2}.$$

   Since these are much smaller than unity, we can write (since q is large)

$$U(y_t) = U(y_r) = 9.38 \times 10^{-2} = -20.6\ \text{dB}.$$

   Finally, we can use the approximation

$$V(x) = -17x + 14 = -75.25\ \text{dB}.$$

If we use (5.2.5), we get

$$
\begin{aligned}
V(x) &= 20\log_{10}(2) + 10\log_{10}(\pi) + 10\log_{10}(x) + 20\left(\frac{-2.02x}{\ln 10}\right) \\
&= -73.9 \text{ dB}
\end{aligned}
$$

and the difference is less than 1.5 dB. The ground wave attenuation factor is then

$$F_g = -75.25 - 20.6 - 20.6 = -116.45 \text{ dB}.$$

The free space loss for this path would be

$$F_{fs} = -20\log_{10}\frac{4\pi R}{\lambda_0} = -142 \text{ dB}.$$

The power density at the receiver location is then

$$
\begin{aligned}
P_r &= P_t + G_t + F_{fs} + F_{fs} \\
&= 67 \text{ (dBm)} + 10 - 116.45 - 142 = -181.45 \text{ (dBm)}
\end{aligned}
$$

from which we can calculate the field strength as

$$E = \sqrt{120\pi P_r} = \sqrt{120\pi 1000 \times 10^{-18.145}} = 0.58\,\text{nV/m}.$$

The received power is very small and is below the typical value of $kT = -173$ dBm. However, if the bandwidth is made sufficiently small (e.g. below 1 Hz), and using a suitable antenna, it would still be possible to establish communication, although that would not be practical.

2. An FM radio broadcast station located 15 km inside the shore is operating at 90 MHz, with a transmitter power of 1 kW. The transmitter antenna is a vertical dipole array located 10 m above the ground, and its gain may be assumed to be 10. Assume that the region between the transmitter and the shore is wet ground.

   (a) Suppose that an FM radio is available in a ship 10 km away from the shore with an antenna located on a 5 m pole. The FM bandwidth is 250 kHz. If the receiver noise figure is 10, and the antenna gain is 1.5, what will be the SNR at the receiver?

SOLUTION: The wavelength at this frequency is $\lambda_0 = c/f = 3.33$ m. So the antennas are above the ground and the height-gain functions should be incorporated. We have

$$
\begin{aligned}
80/f_{\text{MHz}}^{1/3} &= \frac{80}{(90)^{1/3}} = 17.85\,\text{km}, \\
610/f_{\text{MHz}}^{2/3} &= \frac{610}{(90)^{2/3}} = 30.37\,\text{m},
\end{aligned}
$$

which means that we can use flat earth formulas (at least for 10 km, and 15 km paths). For 25 km paths, we will also use the flat earth formulas, however the small curvature approximations are also given below and the results are not too different. We will use Millington's approach,

$$
\begin{aligned}
F_{tr} &= F_{land}(15\,\text{km}) - F_{sea}(15\,\text{km}) + F_{sea}(25\,\text{km}), \\
F_{rt} &= F_{sea}(10\,\text{km}) - F_{land}(10\,\text{km}) + F_{land}(25\,\text{km}).
\end{aligned}
$$

For land and sea surfaces we have

$$\begin{aligned}
\kappa_{land} &= 30 + \frac{10^{-2}}{j2\pi \times 90 \times 10^6 \times 8.85 \times 10^{-12}} = 30 - j2, \\
\kappa_{sea} &= 80 + \frac{5}{j2\pi \times 90 \times 10^6 \times 8.85 \times 10^{-12}} = 80 - j999.1
\end{aligned}$$

Now we need to calculate the following numerical distances ($\Omega = -jk_0 R\,(\kappa - 1)\,/2\kappa^2$):

$$\begin{aligned}
\Omega_{10\,\mathrm{km},land} &= 19.\,44 - 302.\,44j = 303.\,1\exp\left(-j86.\,3^0\right) \\
\Omega_{15\,\mathrm{km},land} &= 29.\,16 - 453.\,66j = 454.\,6\exp\left(-j86.\,3^0\right) \\
\Omega_{25\,\mathrm{km},land} &= 48.\,60 - 756.\,10j = 757.\,7\exp\left(-j86.\,3^0\right) \\
\Omega_{10\,\mathrm{km},sea} &= 9.\,38 - 0.76j = 9.\,4\exp\left(-j4.\,6^0\right) \\
\Omega_{15\,\mathrm{km},sea} &= 14.\,06 - 1.\,14j = 14.\,1\exp\left(-j4.\,6^0\right) \\
\Omega_{25\,\mathrm{km},sea} &= 23.\,44 - 1.\,90j = 23.\,5\exp\left(-j4.\,6^0\right)
\end{aligned}$$

Now we calculate the attenuation factors

$$F = 1 - j\sqrt{\pi\Omega}e^{-\Omega}\operatorname{erfc}\left(j\sqrt{\Omega}\right)$$

which gives

$$\begin{aligned}
F_{sea}\,(10\,\mathrm{km}) &= -23.7\text{ dB};\ F_{land}\,(10\,\mathrm{km}) = -55.6\text{ dB}; \\
F_{sea}\,(15\,\mathrm{km}) &= -27.9\text{ dB};\ F_{land}\,(15\,\mathrm{km}) = -59.2\text{ dB}; \\
F_{sea}\,(25\,\mathrm{km}) &= -32.8\text{ dB};\ F_{land}\,(25\,\mathrm{km}) = -63.6\text{ dB}; \\
F_{sea,sph}\,(25\,\mathrm{km}) &= -34.7\text{ dB};\ F_{land,sph}\,(25\,\mathrm{km}) = -66\text{ dB}.
\end{aligned}$$

Note that $F_{sea,sph}\,(25\,\mathrm{km})$, and $F_{land,sph}\,(25\,\mathrm{km})$ are small curvature approximations which are quite close to flat earth values. We may also use the approximate formula

$$F = \frac{2 + 0.3p}{2 + p + 0.6p^2} - \sqrt{\frac{p}{2}}\exp\left(-\frac{5p}{8}\right)\sin b$$

which gives

$$\begin{aligned}
F_{sea,app}\,(10\,\mathrm{km}) &= 22.6\text{ dB};\ F_{land,app}\,(10\,\mathrm{km}) = 55.5\text{ dB}; \\
F_{sea,app}\,(15\,\mathrm{km}) &= 26.7\text{ dB};\ F_{land,app}\,(15\,\mathrm{km}) = 59.1\text{ dB}; \\
F_{sea,app}\,(25\,\mathrm{km}) &= 31.9\text{ dB};\ F_{land,app}\,(25\,\mathrm{km}) = 63.6\text{ dB}.
\end{aligned}$$

Note that the approximate formula is quite good, especially over land region. We can now write

$$\begin{aligned}
F_{tr} &= 59.2 - 27.9 + 32.8 = 64.\,1\text{ dB}, \\
F_{rt} &= 23.7 - 55.6 + 63.6 = 31.\,7\text{ dB}, \\
F_x &= \frac{F_{tr} + F_{rt}}{2} = 47.\,9\text{ dB}.
\end{aligned}$$

We now need to add the antenna height-gains. For vertical polarization we have ($\Delta = \sqrt{\kappa - 1}/\kappa$)

$$\begin{aligned}
\Delta_{land} &= \left[\frac{\sqrt{\kappa - 1}}{\kappa}\right]_{\kappa=30-j2} = 0.179 + j5.762 \times 10^{-3} \\
\Delta_{sea} &= \left[\frac{\sqrt{\kappa - 1}}{\kappa}\right]_{\kappa=80-j999.1} = 2.321 \times 10^{-2} + j2.141 \times 10^{-2}
\end{aligned}$$

and

$$\begin{aligned}
|U_t| &= |1 + jk_0 h_t \Delta_{land}| = 3.49, \\
|U_r| &= |1 + jk_0 h_r \Delta_{sea}| = 0.83.
\end{aligned}$$

Now the attenuation factor is

$$F = 47.9 - 20\log_{10}(3.49 \times 0.83) = 38.7 \text{ dB}.$$

The free space loss for this path is

$$F_{fs} = 20\log_{10}\frac{4\pi R}{\lambda_0} = 99.5 \text{ dB}$$

The received power is then

$$P_r = 60\,(\text{dBm}) - 99.5 - 38.7 + 10\,(G_t) + 1.8\,(G_r) = -66.4 \text{ dBm}.$$

The receiver noise power is

$$N = kTBF = -173.8 + 54 + 10 = -109.8 \text{ dBm}$$

and the SNR is

$$SNR = 43.4 \text{ dB}.$$

So it should be fairly easy for the sailors to listen this radio.

3. Assume that a tropospheric scatterer link operates at 1 GHz. The propagation path length is 350 km, and both transmitting and receiving antennas have a gain of 10 dB looking towards the radio horizon. The path is in Continental sub-tropical climate zone. ITU-R Rec. 617-1 gives $Y(90)$ for this climate zone as:

$$Y(90) = -2.2 - (8.1 - 2.3 \times 10^{-4} f)\exp(-0.137h) \text{ dB},$$

and $C(99.9) = 2.41$, $C(99.99) = 2.90$, where f is in MHz.

   (a) Determine the average annual median transmission loss for this path.

   (b) Assume that the transmitter power is initially adjusted to secure communication for 99.9% of the time. Determine the required increase in the transmitter power (dB) to secure communication for 99.99% of the time.

NOTE : Use ITU-R Rec. 617-1 formulation.

SOLUTION:

(a) The climate zone is 2 and we have $M = 29.73$, $\gamma = 0.27$ for this zone. The antennas are looking towards the horizon and $\theta_t = \theta_r = 0$. Then, $\theta = \theta_e = d \times 10^3/a_e = 41.2\,\text{mrad}$. Also we have

$$
\begin{aligned}
H &= \frac{10^{-3}\theta d}{4} = 3.6\,\text{km};\ h = \frac{10^{-6}\theta^2 a_e}{8} = 1.8\,\text{km}; \\
L_N &= 20\log_{10}(5+\gamma H) + 4.34\gamma h = 17.63 \text{ dB}, \\
L_c &= 0.07\exp\left[0.055\left(G_t + G_r\right)\right] = 0.21 \text{ dB}, \\
L_{50} &= M + 30\log_{10} f + 10\log_{10} d + 30\log_{10}\theta + L_N + L_c - G_t - G_r \\
&= 190.79 \text{ dB}.
\end{aligned}
$$

(b) We have

$$
\begin{aligned}
L(99.9) &= L(50) - Y(99.9), \\
L(99.99) &= L(50) - Y(99.99).
\end{aligned}
$$

Obviously, the required increase in the transmitter power is $Y(99.9) - Y(99.99)$. From the given formula we have

$$Y(90) = -8.35$$

so

$$
\begin{aligned}
Y(99.9) &= Y(90)\,C(99.9) = -20.12 \\
Y(99.99) &= Y(90)\,C(99.99) = -24.22
\end{aligned}
$$

which gives

$$Y(99.9) - Y(99.99) = 4.1 \text{ dB}.$$

This is the required increase in the transmitter power.

4. Design an ionospheric radio link over a distance of 2500 km given that the ionosphere effective height is

275 km and the electron density $N = 2 \times 10^{11}\,\text{m}^{-3}$, namely find:

(a) the angle of incidence on the ionospheric layer.

(b) Find the maximum usable frequency (MUF) and the optimum working frequency (OWF). At night, N is reduced to $5 \times 10^{10}\,\text{m}^{-3}$, find the new operating frequency.

SOLUTION:

(a) The angle of incidence is solved by

$$\sin\psi_i = \frac{\sin\frac{d}{2a_e}}{\sqrt{\left(\frac{h'}{a_e}\right)^2 + 4\left(1+\frac{h'}{a_e}\right)\sin^2\frac{d}{4a_e}}} = 0.959$$

or

$$\psi_i = 73.5^0$$

(b) The critical frequency is

$$\begin{aligned} f_c &= 9\sqrt{N} = 4.02\,\text{MHz (daytime)} \\ &= 2.01\,\text{MHz (at night)} \end{aligned}$$

and the MUF is

$$\begin{aligned} MUF &= f_c \sec\psi_i = 14.18\,\text{MHz (daytime)} \\ &= 7.09\,\text{MHz (at night).} \end{aligned}$$

ITU reccomends to use 90% of the MUF,

$$\begin{aligned} OWF &= 12.76\,\text{MHz (daytime)} \\ &= 6.38\,\text{MHz (at night).} \end{aligned}$$