EE 524 May 16, 2011
Midterm Exam 1 Solutions

1. Determine how far below the free-space level the signal will be on a 10 cm, 130 km link between terminals at 60 and 90 m? Assume vertical polarization and propagation over wet ground with $\kappa' = 30$ and $\sigma = 10^{-2}$ S/m.

SOLUTION : The total horizon distance for this geometry is

$$R_h = \sqrt{2a_e h_1} + \sqrt{2a_e h_2} = 71.052\,\text{km}$$

which shows that we are in the diffraction zone. The normalized surface impedance and the q value can be calculated as

$$\begin{aligned} \kappa &= 30 - j\frac{10^{-2}}{\omega\epsilon_0} = 30 - j5.\,99 \times 10^{-2}, \\ \Delta &= \frac{\sqrt{\kappa - 1}}{\kappa} = 0.18 + 1.\,73 \times 10^{-4} j, \\ q &= -j\left(\frac{k_0 a_e}{2}\right)^{1/3} \Delta = 0.11 - j115.\,9. \end{aligned}$$

From the given link parameters we find

$$x = \left(\frac{k_0 a_e}{2}\right)^{1/3} \left(\frac{R}{a_e}\right) = 9.\,75 > 10/3$$

which implies that single term expansion will be accurate, along with the fact that q is large. We can determine

$$\begin{aligned} y_1 &= \left(\frac{2}{k_0 a_e}\right)^{1/3} k_0 h_1 = 5.\,85, \\ y_2 &= \left(\frac{2}{k_0 a_e}\right)^{1/3} k_0 h_2 = 8.\,78. \end{aligned}$$

From the height-gain function plot (Ch. 5, Fig. 5.5) we find

$$U(y_1) = 26.9 \text{ dB}; \quad U(y_1) = 36.2 \text{ dB}.$$

We can also calculate

$$V(x) = 2\sqrt{\pi x}\exp(-2.02x) = 3.\,071 \times 10^{-8} = -150.3 \text{ dB}.$$

Thus the attenuation factor is

$$F = -150.3 + 26.9 + 36.2 = -87.\,2 \text{ dB}.$$

The signal will be 87. 2 dB below the free space level.

NOTE : If we use flat earth ground wave propagation, we would obtain

$$\begin{aligned} \Omega &= -j\frac{2\pi}{0.1}130000\frac{(30-j5.99\times 10^{-2}-1)}{2\left(30-j5.99\times 10^{-2}\right)^2} = 253.7 - 1.316\times 10^5 j, \\ F &= \left|1 - j\sqrt{\pi\Omega}e^{-\Omega}\operatorname{erfc}\left(j\sqrt{\Omega}\right)\right| = 3.799\times 10^{-6} = -108.4\text{ dB}. \end{aligned}$$

However, flat earth surface wave model is valid up to $80/f_{\mathrm{MHz}}^{1/3}$ km which is $80/\left(3000\right)^{1/3} = 5.55$ km. So this result is not valid. Furthermore, antenna height-gains cannot be included in this case. Similarly, small curvature approximation is not appropriate since antenna height-gains cannot be included.

2. Small sailing yachts are equipped with a radio transmitter operating at about 150 MHz. Assume that the transmitter antenna gain is 1.5, the transmitter power is 1 W, and vertical polarization is used.

   (a) If the ship is at 10 km away from the shore, determine the signal level at a receiving station 5 km inside the shore. Assume very dry ground.

   (b) If the communication bandwidth is 25 kHz, and the receiver noise figure is 10, with an antenna gain of 1.5, what will be the SNR at the receiver?

   You may assume that both antennas are on the ground.

SOLUTION : At 150 MHz, we have $\lambda = 2$ m, and

$$\frac{80}{f_{\mathrm{MHz}}^{1/3}} = 15.06\text{ km}.$$

Therefore, we can use flat earth surface wave formulas. Since the propagation path is mixed, we will use Millington's formula.

(a) For sea surface we have

$$\kappa_{sea} = 80 - j\frac{5}{\omega\epsilon_0} = 80 - j599.21.$$

which, for $d_1 = 10$ km, gives

$$\begin{aligned} \Omega_{10\,\mathrm{km}} &= -j\frac{2\pi}{2}10000\frac{(80-j599.21-1)}{2\left(80-j599.21\right)^2} = 25.74 - j3.48, \\ p &= |25.74 - j3.48| = 25.97; \qquad b = \tan^{-1}\frac{3.48}{25.74} = 0.134\text{ rad} = 7.7^\circ, \\ F_{sea}\left(10\text{ km}\right) &= \left|1 - j\sqrt{\pi\Omega_{10\,\mathrm{km}}}e^{-\Omega_{10\,\mathrm{km}}}\operatorname{erfc}\left(j\sqrt{\Omega_{10\,\mathrm{km}}}\right)\right| = 2.05\times 10^{-2} = -33.8\text{ dB}. \end{aligned}$$

If we use the approximate formula, we find

$$F_{sea,approx}\left(10\text{ km}\right) = \frac{2+0.3p}{2+p+0.6p^2} - \sqrt{\frac{p}{2}}\exp\left(-5p/8\right)\sin b = 2.26\times 10^{-2} = -32.9\text{ dB}.$$

Similarly,

$$\begin{aligned}
\Omega_{15\,\mathrm{km}} &= -j\frac{2\pi}{2}15000\frac{(80-j599.21-1)}{2\,(80-j599.21)^2} = 38.62-j5.22, \\
p &= |38.62-j5.22| = 38.97; \quad b=\tan^{-1}\frac{5.22}{38.62} = 0.134\text{ rad} = 7.7^\circ, \\
F_{sea}\,(15\,\mathrm{km}) &= \left|1-j\sqrt{\pi\Omega_{15\,\mathrm{km}}}e^{-\Omega_{15\,\mathrm{km}}}\operatorname{erfc}\left(j\sqrt{\Omega_{15\,\mathrm{km}}}\right)\right| = 1.335\times10^{-2} = -37.5\text{ dB}, \\
F_{sea,approx}\,(15\,\mathrm{km}) &= \frac{2+0.3p}{2+p+0.6p^2}-\sqrt{\frac{p}{2}}\exp\left(-5p/8\right)\sin b = 1.438\times10^{-2} = -36.8\text{ dB}.
\end{aligned}$$

Proceeding in a similar way, we find

$$F_{sea}\,(5\,\mathrm{km}) = -27.1\text{ dB}; \quad F_{sea,approx}\,(5\,\mathrm{km}) = -25.9\text{ dB}.$$

For land, we have

$$\kappa_{land} = 3-j\frac{10^{-4}}{\omega\epsilon_0} = 3-1.198\times10^{-2}j$$

which gives

$$\begin{aligned}
F_{land}\,(5\,\mathrm{km}) &= -70.9\text{ dB}; \quad F_{land,approx}\,(5\,\mathrm{km}) = -70.8\text{ dB}; \\
F_{land}\,(10\,\mathrm{km}) &= -76.9\text{ dB}; \quad F_{land,approx}\,(10\,\mathrm{km}) = -76.9\text{ dB}; \\
F_{land}\,(15\,\mathrm{km}) &= -80.4\text{ dB}; \quad F_{land,approx}\,(15\,\mathrm{km}) = -80.4.
\end{aligned}$$

These values can also be determined from the graph of Fig. 4.2, or F (dB) $= 29.1 - 20\log_{10}(R)$, (see example 22), since p is large in all cases. Now, using the Millington formula

$$\begin{aligned}
F_{tr} &= F_{sea}\,(10\,\mathrm{km}) - F_{land}\,(10\,\mathrm{km}) + F_{land}\,(15\,\mathrm{km}) = -41.0\text{ dB} \\
F_{rt} &= F_{land}\,(5\,\mathrm{km}) - F_{sea}\,(5\,\mathrm{km}) + F_{sea}\,(15\,\mathrm{km}) = -81.3\text{ dB} \\
F &= \frac{F_{tr}+F_{rt}}{2} = -61.15\text{ dB}.
\end{aligned}$$

The free space signal at the location of the receiver would be

$$E_{rms,fs} = \frac{\sqrt{30P_tG_t}}{R} = 447\,\mu\mathrm{V/m}.$$

The signal level at a receiving station will then be

$$E_{rms} = E_{rms,fs}\sqrt{F} = 0.39\,\mu\mathrm{V/m}.$$

(b) The free space loss is

$$L_{fs} = 20\log_{10}\frac{4\pi\times15000}{2} = 99.5\text{ dB}.$$

The received power will be

$$P_r = P_t+G_r+G_t-L_{fs}+F = 30+3-99.5-61.15 = -127.65\text{ dBm}.$$

At 25 kHz bandwidth, the noise power is

$$P_n = kTBF = -120 \text{ dBm}.$$

Thus, the SNR is $-127.65 + 120 = -7.65$ dB. Communication would not be possible with standard techniques.

3. Assume that a tropospheric scatterer link operates at 500 MHz. The propagation path length is 300 km, and both transmitting and receiving antennas have a gain of 10 dB looking towards the radio horizon. The path is in Mediterranean climate zone. ITU-R Rec. 617-1 gives $Y(90) = -9$ dB and $C(99.9) = 2.41$.

   (a) Determine the average annual median transmission loss for this path.

   (b) If the communication bandwidth is 250 kHz, receiver noise figure is 10, and required SNR is 17 dB, determine the transmitter power required assuming the loss is constant at the median level.

   (c) Determine the required increase in the transmitter power to secure communication for 99.9% of the time.

   SOLUTION :

   (a) The climate zone is 5 for which we have $M = 38.5$ and $\gamma = 0.27$. Since the antennas are looking towards the horizon, we have $\theta_t = \theta_r = 0$. Then,$\theta = \theta_e = 35.3$ mrad. Also,

$$\begin{aligned} H &= 10^{-3}\theta d/4 = 2.65\,\text{km}; \quad h = 10^{-6}\theta^2 a_e/8 = 1.32\,\text{km}, \\ L_N &= 20\log_{10}(5+\gamma H) + 4.34\gamma h = 16.7 \text{ dB}, \\ L_c &= 0.07\exp\left[0.055\left(G_t + G_r\right)\right] = 0.21 \text{ dB}, \\ L(50) &= M + 30\log_{10} f + 10\log_{10} d + 30\log_{10}\theta + L_N + L_c - G_t - G_r = 187.6 \text{ dB}. \end{aligned}$$

   (b) The noise power is

$$P_n = 10\log_{10} 250000 - 174 + 10 = -110 \text{ dBm}.$$

   For 17 dB SNR we must have $P_r = P_n + 17 = -93$ dBm. Then,

$$P_t = P_r - G_t - G_r + L(50) = 74.6 \text{ dBm} = 44.6 \text{ dBW} = 28.84\,\text{kW}$$

   transmit power is required.

   (c) We have

$$Y(99.9) = C(99.9)\,Y(90) = 9 \times 2.41 = 21.7 \text{ dB}.$$

   This means the transmitter power must be increased by 21.7 dB to secure communication for 99.9% of the time. This corresponds to a factor of 148, i.e., the required transmitter power will be $28.84 \times 148 = 4268.32$ kW.

4. Sky wave communication between two points on earth's surface at a distance of 2500 km is to be established using reflection from $F1$ layer at a virtual height of 200 km. The critical frequency is 4 MHz at the reflection point.

(a) Determine the electron density at the reflection point.

(b) Find the maximum usable frequency (MUF) and the optimum working frequency (OWF).

(c) Ignoring collisions, determine the refractive index of ionosphere at 200 km virtual height for the OWF. Discuss this result.

SOLUTION :

(a) From the equation $f_c = 9\sqrt{N}$, we find $N = \left(\frac{4\times10^6}{9}\right)^2 = 1.975 \times 10^{11}\,\mathrm{m}^{-3}$.

(b) For distance d and a virtual height h', the incidence angle is given by

$$\sin\psi_i = \frac{\sin\frac{d}{2a_e}}{\sqrt{\left(\frac{h'}{a_e}\right)^2 + 4\left(1+\frac{h'}{a_e}\right)\sin^2\frac{d}{4a_e}}}.$$

Using $h' = 200\,\mathrm{km}$, $a_e = 8500\,\mathrm{km}$, and $d = 2500\,\mathrm{km}$, we find

$$\sin\psi_i = 0.974 \rightarrow \psi_i = 1.341\,\mathrm{rad}$$

The MUF is then given by

$$\mathrm{MUF} = f_c \sec\psi_i = 17.6\,\mathrm{MHz}.$$

The OWF is chosen as 10% below MUF, i.e., OWF $= 0.9 \times 17.6 = 15.84\,\mathrm{MHz}$.

(c) The refraction index is given by

$$n = \sqrt{1 - 81\frac{N}{f^2}} = 0.968 \text{ at } f = 15.84\,\mathrm{MHz}.$$

For reflection we should have $n = \sin\psi_i$. However, for OWF $n > \sin\psi_i$. This means that we require higher electron density for reflection. That is, the OWF will be reflected back from a higher layer in the ionosphere.

5. Suppose that the electron density in the ionosphere varies linearly with height by the formula

$$N(h) = (49.5h - 4850) \times 10^9\,\mathrm{m}^{-3}$$

where h is in km. Given that the cyclotron frequency is $f_c = 1.2\,\mathrm{MHz}$. Determine the heights at which the ordinary and extraordinary waves will be reflected for a wave of frequency 6 MHz. Assume propagation perpendicular to the magnetic field of earth.

SOLUTION : For propagation perpendicular to the magnetic field of earth, the propagation factors for the two waves are

$$\begin{aligned} k_1 &= \sqrt{\kappa_3}k_0, \\ k_2 &= \sqrt{\frac{(\kappa_1-\kappa_2)(\kappa_1+\kappa_2)}{\kappa_1}}k_0, \end{aligned}$$

where

$$\kappa_1 = 1 + \frac{1}{-\omega^2 + \omega_c^2}\omega_p^2,$$
$$\kappa_2 = \frac{\omega_p^2 \omega_c}{\omega\left(\omega^2 - \omega_c^2\right)},$$
$$\kappa_3 = 1 - \frac{\omega_p^2}{\omega^2}$$

and k_1 corresponds to the ordinary wave, while k_2 corresponds to the extraordinary wave. The critical frequencies for each wave is determined by equating the corresponding refractive index to zero. For the ordinary wave we have

$$\sqrt{\kappa_3} = 0 \to 1 - \frac{\omega_p^2}{\omega^2} = 0 \to f_{crit}^o = f_p = 9\sqrt{N}.$$

For 6 MHz, we find $N = 4.44 \times 10^{11}$ and the corresponding height is $h_o' = 106.9\,\mathrm{km}$.

For the extraordinary wave we must have

$$\sqrt{\frac{\left(\kappa_1 - \kappa_2\right)\left(\kappa_1 + \kappa_2\right)}{\kappa_1}} = 0$$

which gives

$$1 + \frac{f_p^2}{-f^2 + f_c^2} + \frac{f_p^2 f_c}{f\left(f^2 - f_c^2\right)} = 0.$$

Replacing $f_p = 9\sqrt{N}$ we find

$$\frac{f f_c - 81N + f^2}{\left(f_c + f\right) f} = 0$$

which yields

$$N = \frac{1}{81}\left(f_c + f\right) f.$$

For the given values, we calculate N as $5.333 \times 10^{11}\,\mathrm{m}^{-3}$, which corresponds to a height of $h_x' = 108.8\,\mathrm{km}$.