EE 524 April 4, 2012
Midterm Exam 1 Solutions

1. A geostationary satellite is 39000 km from an Earth station. It is transmitting a digital signal at a bit rate of 36 Mbit/s using a 40 dBm transmitter at a frequency of 11.2 GHz. The transmitting antenna is a parabolic dish with a diameter of 80 cm. Assume that the aperture efficiencies of both transmitting and receiving antennas are $\varepsilon_{ap} = 0.75$, and that the propagation is in free space.

   (a) Determine the minimum required receive power if the required E_b/N_0 ratio is 12 dB and the noise temperature of the receiving system is 160 K.

   (b) Determine the minimum size of the receiving antenna (If you cannot solve part (a), use $P_r = -90$ dBm).

SOLUTION :

(a) The noise energy at the receiver is

$$N_0 = kT = 1.380658 \times 10^{-23}\,\mathrm{J\,K^{-1}} \times 160\,\mathrm{K} = 2.209\,052\,8 \times 10^{-21}\,\mathrm{J}$$

and the required signal energy per bit becomes

$$E_b = 10^{12/10} N_0 = 3.5 \times 10^{-20}\ \mathrm{J/bit}.$$

The required receive power is then $P_r = 3.5 \times 10^{-20} \times 36 \times 10^6 = 1.26 \times 10^{-12}\,\mathrm{W} = -89$ dBm.

(b) The free space path loss can be calculated as

$$L = 20 \log_{10} \frac{4\pi R}{\lambda_0} = 205.2\ \mathrm{dB}$$

where $R = 39 \times 10^6\,\mathrm{m}$, $\lambda_0 = \frac{3\times10^8}{11.2\times10^9} = 2.68 \times 10^{-2}\,\mathrm{m} = 2.68\,\mathrm{cm}$. The gain of the transmitting antenna can be estimated from its diameter to be

$$G_t = \varepsilon_{ap} \left(\frac{\pi d}{\lambda_0}\right)^2 = 6595.86 = 38.2\ \mathrm{dB}.$$

Using Friis' transmission formula we get

$$P_r = P_t + G_t + G_r - L \geq -89 \Rightarrow 40 + 38.2 + G_r - 205.2 \geq -89$$

or

$$G_r \geq 38\ \mathrm{dB.} = 6309.6$$

Using the approximate formula for the gain of the dish, we find

$$G_t = \varepsilon_{ap} \left(\frac{\pi d}{\lambda_0}\right)^2 \Rightarrow d \geq 78.2\,\mathrm{cm}$$

would be required.

2. Consider a communication link operating at 300 MHz with two antennas at a height of 20 m above the Earth's surface.

   (a) Determine the maximum distance between the antennas so that the antennas remain above the horizon with respect to each other.

   (b) What would be the maximum distance between the two antennas so that the Earth does not protrude into the first Fresnel region. Consider a spherical Earth and make necessary approximations.

   SOLUTION :

   (a) The horizon distance for the antennas are

$$R = \sqrt{2a_e h_1} = 18.44\,\text{km}$$

   where $a_e = 8500\,\text{km}$, $h_1 = 20\,\text{m}$. Then, we get

$$R_{\text{max}} = 2 \times 18.44\,\text{km} = 36.88\,\text{km}$$

   (b) The maximum radius of the Fresnel ellipsoid occurs at the midpoint, say at d with a radius of

$$h_f = \frac{1}{2}\sqrt{d\lambda_0}.$$

   The height of the earth at a distance d can be written as

$$h_e = a_e - \sqrt{a_e^2 - d^2}.$$

   When the Fresnel ellipsoid touches the Earth's surface, we have

$$\frac{a_e + h}{a_e + h_f} = \frac{a_e}{a_e - h_e}.$$

   Using the expressions for h_f and h_e and rearranging, we get

$$\frac{a_e + h}{a_e + \frac{1}{2}\sqrt{d\lambda_0}} = \frac{a_e}{a_e - \left(a_e - \sqrt{a_e^2 - d^2}\right)}$$

   where d is the only unknown. To simplify the solution we can make the approximations

$$\begin{aligned} \frac{1}{\sqrt{a_e^2 - d^2}} &\approx \frac{1}{a_e}\left(1 + \frac{d^2}{2a_e^2}\right) \\ \frac{a_e + h}{a_e + \frac{1}{2}\sqrt{d\lambda_0}} &\approx 1 + \frac{1}{a_e}h - \frac{1}{2}\frac{a_e + h}{a_e^2}\sqrt{d\lambda_0} \end{aligned}$$

   giving

$$h - \frac{d^2}{2a_e} \approx \frac{1}{2}\sqrt{d\lambda_0}.$$

   Squaring both sides gives

$$\frac{1}{4}\frac{d^4}{a_e^2} - \frac{d^2}{a_e}h + h^2 \approx \frac{1}{4}d\lambda_0.$$

Finally, ignoring the first term gives

$$-\frac{d^2}{a_e} + h \approx \frac{1}{4}\frac{d\lambda_0}{h}.$$

Replacing the numerical values, $a_e = 8500\,\text{km}$, $h = 20\,\text{m}$, $\lambda_0 = 1\,\text{m}$, we get

$$-\frac{d^2}{8500000} + 20 \approx \frac{1}{4}\frac{d}{20}$$

which yields $d = 1576.6\,\text{m}$ (the solution of the original equation gives $d = 1576.7\,\text{m}$). Note that if we assumed flat Earth we would have

$$h = \frac{1}{2}\sqrt{d\lambda_0}$$

giving $d = 1600\,\text{m}$. The distance is twice the value of d, i.e.,

$$R_{\max} = 3153.2\,\text{m}.$$

3. Consider a communication link operating at a frequency of 3 GHz with horizontal polarization. The transmitting and receiving antennas are located over earth and separated by a horizontal distance of 50 km. The transmitting antenna is 200 m high and the receiving antenna is 100 m high. The earth's surface between the antennas is a rough surface having $\sigma_h = 1\,\text{m}$. The constitutive parameters of the earth are $\kappa' = 15$ and $\sigma = 10\,\text{mS/m}$.

   (a) Determine the horizon distances of the transmitter and receiver and the total horizon distance. Are the antennas within line-of-sight?

   (b) Determine the reflection coefficient using flat Earth model.

   (c) Determine the propagation factor for the two-ray model using flat Earth model.
   SOLUTION :

   (a)

$$\begin{aligned} R_{ht} &= \sqrt{2a_e h_t} = 58309.5\,\text{m} = 58.31\,\text{km}, \\ R_{hr} &= \sqrt{2a_e h_r} = 41231.1\,\text{m} = 41.23\,\text{km}. \end{aligned}$$

   Since $R_{ht} + R_{hr} = 99.54\,\text{km} > 50\,\text{km}$, the antennas are within line-of-sight.

   (b) We must first solve for the grazing angle

$$\psi = \tan^{-1}\frac{h_t + h_r}{R} = 0.344\,°.$$

   The surface roughness parameter is then

$$\frac{\sigma_h \sin\psi}{\lambda_0} = 0.06$$

   The complex dielectric constant of the Earth is

$$\kappa = \kappa' - j\kappa'' = \kappa' - j\sigma/\omega\epsilon_0 = 15 - j5.99 \times 10^{-2}$$

The smooth surface reflection coefficient is

$$\Gamma_h = \frac{\sin\psi - \sqrt{\kappa - \cos^2\psi}}{\sin\psi + \sqrt{\kappa - \cos^2\psi}} = -0.997\ + j6.84 \times 10^{-6}.$$

Referring to Fig. 3.X, we see that Ament's formula is sufficiently accurate for surface roughness parameter of 0.06. Hence,

$$\Gamma_r = \Gamma_h \exp\left(-2k_0^2\sigma_h^2 \sin^2\psi\right) = -0.75\ + j5.152 \times 10^{-6}.$$

(c) The path length difference is

$$\Delta R = \frac{2h_r h_t}{R} = 0.8\,\mathrm{m}$$

The attenuation factor is

$$F = \left|1 - \Gamma_r e^{-jk_0\Delta R}\right| = 1.75 \text{ or } 4.86 \text{ dB}.$$

The free space loss is

$$L = 20\log_{10}\frac{4\pi R}{\lambda_0} = 135.96 \text{ dB}.$$

The propagation factor is then

$$F - L = -131.1 \text{ dB}.$$

NOTE : Using spherical earth formulas would give

$$\psi_s = 4.421 \times 10^{-3}\,\mathrm{rad}; \qquad \Delta R_s = 0.45\,\mathrm{m}.$$

The smooth surface reflection coefficient can be calculated as

$$\Gamma_{h,s} = \frac{\sin\psi_s - \sqrt{\kappa - \cos^2\psi_s}}{\sin\psi_s + \sqrt{\kappa - \cos^2\psi_s}} = -0.998 + j5.048 \times 10^{-6}$$

and the rough surface reflection coefficient becomes

$$\Gamma_{r,s} = \Gamma_{h,s} \exp\left(-2k_0^2\sigma_h^2 \sin^2\psi\right) = -0.855\ + j4.33 \times 10^{-6}.$$

The attenuation factor would be

$$F_s = \left|1 - \Gamma_{r,s} e^{-j\frac{2\pi}{0.1}0.45}\right| = 0.145 \text{ or } -16.77 \text{ dB}.$$

The propagation factor is then

$$F - L = -152.73 \text{ dB}.$$

Obviously, the two results are quite different, basically due to the different values of the path length difference, ΔR. Using the formula given in 5.1.8, we see that the flat Earth model is valid only upto

$$d < \frac{\lambda_0 a_e (h_2 + h_1)}{10 h_1 h_2} = 1275.0\,\mathrm{m}.$$

4. A radar operating at 10 GHz with vertical polarization is sited at a height of $h_r = 50\,\text{m}$ above sea and observing a target at a distance of 150 km and a height of $h_t = 3000\,\text{m}$ above sea. The radar transmit power is 30 kW, and antenna gain is 30 dB. Using spherical Earth model, determine the electric field intensity at the location of the target. Assume a calm sea.

SOLUTION :Note that the total horizon distance is

$$R_{\max} = \sqrt{2a_e h_r} + \sqrt{2a_e h_t} = 255\,\text{km}$$

which shows that we may use interference region formulas. We first calculate the necessary parameters.

$$\begin{aligned}
p &= \frac{2}{\sqrt{3}}\left[a_e\left(h_1+h_2\right)+\frac{d^2}{4}\right]^{1/2} = 2.05\times 10^5\,\text{m} \\
\Phi &= \cos^{-1}\left(\frac{2a_e d\left(h_1-h_2\right)}{p^3}\right) = 2.63\,\text{rad} \\
d_1 &= \frac{d}{2}+p\cos\left(\frac{\Phi+\pi}{3}\right) = 4092\,\text{m}; \quad d_2 = d-d_1 = 145908\,\text{m} \\
S_1 &= \frac{d_1}{\sqrt{2a_e h_1}} = 0.140; \quad S_2 = \frac{d_2}{\sqrt{2a_e h_2}} = 0.646 \\
T &= \sqrt{h_1/h_2} = 0.129\,; \quad S = \frac{S_1 T + S_2}{1+T} = 0.588 \\
J\left(S,T\right) &= \left(1-S_1^2\right)\left(1-S_2^2\right) = 0.571; \quad K\left(S,T\right) = \frac{\left(1-S_2^2\right)+T^2\left(1-S_1^2\right)}{1+T^2} = 0.589 \\
\Delta R &= \frac{2h_1h_2}{d}J\left(S,T\right) = 1.142\,\text{m}; \quad \tan\psi = 1.198\times 10^{-2} \rightarrow \psi = 0.686\,^\circ \\
D &= \left[1+\frac{4S_1S_2^2T}{S\left(1-S_2^2\right)\left(1+T\right)}\right]^{-1/2} = 0.963
\end{aligned}$$

Since $\Delta R > \lambda_0/4$ we should include the divergence factor in the formula (although it will not effect the result too much). The reflection coefficients for vertical polarization is

$$\Gamma_v = \frac{\kappa\sin\psi - \sqrt{\kappa-\cos^2\psi}}{\kappa\sin\psi + \sqrt{\kappa-\cos^2\psi}} = -0.805 - j9.727\times 10^{-3}$$

Then

$$F = \left|1+\Gamma_v D e^{-jk_0\Delta R}\right| = 0.42 \text{ or } -7.52\text{ dB}.$$

The rms field at the location of the target is then

$$E_{rms} = \frac{\sqrt{30P_tG_t}}{R}F = 84\text{ mV/ m}.$$

5. An FOD (foreign object detector) radar is used to detect the objects on the runway of an airport that may be dangerous for planes. Assume that an FOD radar operating at 5 GHz observes the runway from a distance of 5 km and radiates just enough power to detect possibly dangerous objects under clear air conditions. Obviously, if there is fog the radar power should

be increased. Determine the excess power required (in dB) if there is a fog that reduces visibility to 3 m all over the airport. Assume a temperature of 0 °C.

SOLUTION : The water density is found as

$$W = 0.0156\,(V)^{-1.43}\ \mathrm{g/\,m^3} = 63.\,22\,\mathrm{g/\,m^3}.$$

Fog attenuation coefficient is given by

$$\begin{aligned} K_l &= 6.0826 \times 10^{-4} f^{1.8963} \theta^{\left(7.8087 - 0.01565 f - 3.073\times 10^{-4} f^2\right)}, \qquad 5\,\mathrm{GHz} \leq f \leq 150\,\mathrm{GHz} \\ &= 2.\,655 \times 10^{-2}\,(\mathrm{dB/\,km})\,/\,\left(\mathrm{g/\,m^3}\right) \end{aligned}$$

where $\theta = 300/273.15$, and $f = 5$. The fog attenuation is then

$$A = WK_l = 1.\,68\,(\mathrm{dB/\,km})\,.$$

This attenuation occurs along $R = 10\,\mathrm{km}$ two way path, i.e., the total attenuation due to fog is

$$L = AR = 16.\,8\ \mathrm{dB}$$

which is the excess power required. In other words, the transmitter power should be increased by a factor of $10^{1.68} \approx 48$.