**EE 524 April 4, 2011**
**Midterm Exam 1 Solutions**

1. For what distances is the two ray flat earth model valid for a link using antenna heights $h_t = 50\,\text{m}$ and $h_r = 50\,\text{m}$. Determine also the range $d_F$ at which the first Fresnel ellipsoid just touches the earth. Assume a frequency of 1 GHz.

   **SOLUTION** : The total horizon distance for this geometry is

$$R_h = \sqrt{2a_e h_1} + \sqrt{2a_e h_2} = 58.5\,\text{km}.$$

   We also have

$$\frac{\lambda_0 a_e (h_1 + h_2)}{10 h_1 h_2} = 10.2\,\text{km}.$$

   If we accept an error of $\lambda_0/10$ in path length difference calculations, we can use the flat earth formulas up to 10.2 km.

   Since the antenna heights are equal, the maximum radius of Fresnel ellipsoid occurs at the midpoint. Equating this to antenna heights we get

$$\frac{1}{2}\sqrt{d_F \lambda_0} = 50$$

   which gives $d_F = 33.3\,\text{km}$.

2. A building 20 m high obstructs the line of sight path between a transmitting antenna, 13 m high and a mobile, 2 m high. The distance between the transmitter and the building is 500 m and that between the building and the receiver is 300 m. Ignoring the presence of ground, determine the total path loss at a frequency of 1 GHz. Approximate the slant distances with horizontal distances at appropriate places.

   **SOLUTION** : The radius of the first Fresnel ellipsoid at the location of obstruction is given by the formula

$$h_1 = \sqrt{\frac{d_1 d_2 \lambda_0}{d_1 + d_2}}$$

   where $d_1 = 500\,\text{m}$, $d_2 = 300\,\text{m}$, $\lambda_0 = 0.3\,\text{m}$. Then we get

$$h_1 = 7.5\,\text{m}.$$

   The height of the path joining the transmitter and the receiver at this point is

$$H = 2 + \frac{300}{800}11 = 6.125\,\text{m}.$$

   So the obstruction penetrates into the first Fresnel zone by $H - h_1 = 1.375\,\text{m}$. Actually, we must calculate the orthogonal distance which would be $(H - h_1)\cos\psi$ where $\psi = \tan^{-1}(11/800)$. This would give $H - h_1 = 6.124\,\text{m}$, the difference is negligible. The parameter $v$ is then

$$v = \frac{\sqrt{2}\,(H - h_1)}{h_1} = 0.259 > -0.7.$$

Using the approximate formula for diffraction loss we get

$$
\begin{aligned}
20\log_{10}|F_d(v)| &= -6.9 - 20\log_{10}\left(\sqrt{1+(v-0.1)^2}+v-0.1\right) \\
&= -8.3 \text{ dB}
\end{aligned}
$$

The free space loss is

$$F_{fs} = 20\log_{10}\frac{\lambda_0}{4\pi d} = -90.5 \text{ dB}$$

giving a total loss of

$$F_T = -90.5 - 8.3 = -98.8 \text{ dB}.$$

3. Consider a communication link operating at a frequency of 1 GHz with vertical polarization. The transmitting and receiving antennas are located over earth and separated by a horizontal distance of 20 km. The transmitting antenna is 15 m high and the receiving antenna is 20 m high. The earth's surface between the antennas covered with shrubs and grass which may be characterized as a rough surface having $\sigma_h = 1$ m. The constitutive parameters of the earth are $\kappa' = 15$ and $\sigma = 10$ mS/m.

   (a) Determine the horizon distances of the transmitter and receiver and the total horizon distance.

   (b) Are the antennas within line-of-sight?

   (c) Can the earth be regarded as smooth? If no, determine the normalized reflection coefficient.

   (d) Determine the propagation factor for the two-ray model.

   **SOLUTION** :

   (a)

$$
\begin{aligned}
R_{ht} &= \sqrt{2a_e h_t} = 15968.7194 = 15.97 \text{ km}, \\
R_{hr} &= \sqrt{2a_e h_r} = 18439.0889 = 18.44 \text{ km}.
\end{aligned}
$$

   (b) Since $R_{ht} + R_{hr} = 34.41$ km $> 20$ km, the antennas are within line-of-sight.

   (c) We must first solve for the grazing angle. Since

$$\frac{\lambda_0 a_e (h_1 + h_2)}{10 h_1 h_2} = 29.75 \text{ km} > 20 \text{ km}$$

   we may use the flat earth approximation. However, spherical earth formulas are used in this solution.

$$
\begin{aligned}
p &= 23022 \text{ m}; \quad \Phi = 1.711; \quad d_t = 8928 \text{ m}; \quad d_r = 11072 \text{ m}; \\
S_t &= 0.559; \quad S_r = 0.600; \quad T = 0.866; \quad K(S,T) = 0.660; \\
\psi &= 1.155 \times 10^{-3} \text{ rad}.
\end{aligned}
$$

The surface roughness parameter is then

$$\frac{\sigma_h \sin\psi}{\lambda_0} = 3.85 \times 10^{-3}$$

which a very small value. Therefore, the earth surface can be regarded as smooth.

(d) The path length difference is

$$\Delta R = 1.318 \times 10^{-2}\,\text{m} = 1.318\,\text{cm}.$$

Since this value is less than $\lambda_0/4$, we will take $D = 1$. Also, since the grazing angle is very small we can take the reflection coefficient as $-1$. Then the attenuation factor is

$$F = \left|1 - e^{-jk_0\Delta R}\right| = 0.275 \text{ or } -11.2 \text{ dB}.$$

**NOTE** : Using flat earth formulas would give

$$\psi_f = 1.750 \times 10^{-3}\,\text{rad}; \quad \Delta R_f = 0.03\,\text{m}$$

and the attenuation factor would be

$$F_f = \left|1 - e^{-j\frac{2\pi}{0.3}0.03}\right| = 0.618 \text{ or } -4.2 \text{ dB}.$$

The difference is due to the fact that we are very close to a minimum and the attenuation factor changes very rapidly.

The reflection coefficient can be calculated as

$$\Gamma_v = \frac{\kappa\sin\psi - \sqrt{\kappa - \cos^2\psi}}{\kappa\sin\psi + \sqrt{\kappa - \cos^2\psi}} = -0.991 - 5.1 \times 10^{-5} j$$

which justifies the use of $-1$ for the reflection coefficient.

4. A radar operating at 3 GHz is sited at a height of $h_r = 10\,\text{m}$ above sea and observing a target at a distance of 100 km and a height of $h_t = 2000\,\text{m}$ above sea. The radar transmit power is 1 kW, and antenna gain is 30 dB. Determine the electric field intensity at the location of the target.

**SOLUTION** : At such a large distance, we may not expect the flat earth model to be valid. Indeed, the formula gives

$$\frac{\lambda_0 a_e (h_1 + h_2)}{10 h_1 h_2} = 8.5\,\text{km}$$

for the validity range of flat earth model.

Since it is not specified, we will assume a calm sea. Using the spherical earth formulas we find

$$\begin{aligned} p &= 1.616 \times 10^5\,\text{m}; \quad \Phi = 2.501; \quad d_t = 697.3\,\text{m}; \quad d_r = 99302.66\,\text{m}; \\ S_t &= 5.348 \times 10^{-2}; \quad S_r = 0.539; \quad T = 7.071 \times 10^{-2}; \quad K(S,T) = 0.711; \\ \psi &= 1.430 \times 10^{-2}\,\text{rad}; \quad \Delta R = 0.283\,\text{m}; \quad D = 0.994. \end{aligned}$$

Since $\Delta R > \lambda_0/4$ we should include the divergence factor in the formula (although it will not effect the result too much). The grazing angle is very small and we can assume $\Gamma = -1$ and the polarization is not important. Then

$$F = \left|1 - De^{-jk_0\Delta R}\right| = 1.006.$$

The rms field at the location of the target is then

$$E_{rms} = \frac{\sqrt{30P_tG_t}}{R}F = 55.1\,\mathrm{mV/m}.$$

**NOTE** : Using $\kappa' = 80$ and $\sigma = 5\,\mathrm{S/m}$ for sea, the reflection coefficient for vertical polarization is

$$\Gamma_v = \frac{\kappa\sin\psi - \sqrt{\kappa - \cos^2\psi}}{\kappa\sin\psi + \sqrt{\kappa - \cos^2\psi}} = -0.768 - 3.66 \times 10^{-2}j$$

and for horizontal polarization

$$\Gamma_h = \frac{\sin\psi - \sqrt{\kappa - \cos^2\psi}}{\sin\psi + \sqrt{\kappa - \cos^2\psi}} = -0.997 + 5.59 \times 10^{-4}j.$$

Obviously, for horizontal polarization the assumption that $\Gamma = -1$ is more appropriate. If we consider vertical polarization with the correct value of $\Gamma_v$, we would get

$$F = \left|1 + \Gamma_v De^{-jk_0\Delta R}\right| = 0.947$$

which changes the result by only 0.05 dB.

5. The distance between the transmitter and receiver of a radio link operating at $f = 1\,\mathrm{GHz}$ is $12\,\mathrm{km}$. Both antennas have a gain of 50, and the receiver is located at a height of $h_r = 30\,\mathrm{m}$. The receiver noise figure is 10 and the bandwidth of the link system is 1 MHz. The propagation is over smooth wet ground so that you may assume that the reflection coefficient is $-1$. Also assume that the flat earth model is valid.

   (a) What should be the height of the transmitter antenna for maximum signal strength at the receiver?

   (b) If the receiver requires 20 dB SNR for reliable communication, what should be the transmitter power?

   (c) Assume that the specific attenuation due to rain is given by $A = aR^b$ dB/km, where $a = 6 \times 10^{-2}$, $b = 0.6$ and $R$ is the rain rate in mm/h. The probability that there will be a rain with a rate exceeding $R$ is given by

   $$P\left(\tilde{R} \geq R\right) = 1 - e^{-0.005R}$$

   Assume that when it rains, it rains all over the propagation path. Determine the excess power required (in dB) if the communication link is desired to operate at least 99% of the time.

**SOLUTION** :

1. (a) For maximum signal strength at the receiver, the direct and reflected signals must add in phase, or equivalently, the phase difference between them must be equal to $2n\pi$ where $n$ is an integer. Since the reflection coefficient is $-1$, this means that the path length difference must satisfy

$$\Delta R = (2n+1)\frac{\lambda_0}{2}.$$

Obviously, one would like to keep the transmitter as low as possible. Then, we must have

$$\frac{2h_t h_r}{d} = \frac{\lambda_0}{2} \rightarrow h_t = 30.0\,\mathrm{m}.$$

(b) It is reasonable to assume that the receiver operates at ambient temperature and hence the noise power can be calculated as

$$P_n = kTBF = 4.141\,974 \times 10^{-14} \text{ or } -103.8 \text{ dBm}.$$

Since we require 20 dB SNR at the receiver, the received signal must be greater than $-83$ dBm. The transmitter is located at a maximum hence the attenuation factor must be 2 or 6 dB. The free space loss is

$$L_{fs} = 20\log_{10}\frac{\lambda_0}{4\pi d} = -114 \text{ dB}.$$

Then, using Friis' transmission formula we get

$$P_t = -83.8 - 17 - 17 + 114 - 6 = -9.8 \text{ dBm}.$$

(c) Equating the given cumulative probability distribution to 0.99 we get

$$1 - e^{-0.005R} = 0.99 \rightarrow R = 921.$$

This means that the rain rate will not exceed this rate for 99% of the time. We must increase the power to compensate the attenuation due to this rain rate. The specific attenuation for this rate is

$$A = 6 \times 10^{-2}\,(921)^{0.6} = 3.6 \text{ dB/km}$$

yielding total rain attenuation of

$$L_r = 12 \times 3.6 = 43.2 \text{ dB}.$$

We must increase the transmit power by 43.2 dB to guarantee communication for 99% of the time.

**NOTE** : The numerical values for this problem are chosen for easy solution and do not reflect real situations. However, the final result may be typical for many communication links due to other reasons.