EE 524 June 4, 2012
Final Exam Solutions

1. EME or moonbouncing is a mode of creating radio links at two far away points on earth. The moon can be considered as a scatterer with a radar cross-section equal to 7% of its physical cross-section. The radius of moon is $\rho \approx 3500\,\text{km}$, and its average distance to earth is $d \approx 384000\,\text{km}$.

   (a) VHF frequencies are preferred for EME. Discuss possible reasons for this choice.

   (b) Assuming transmitter and receiver antennas have a gain of 10, and that the transitter power is 10 W, determine the power received. The operating frequency is 200 MHz.

   SOLUTION:

   (a) At frequencies below HF, the antennas are too large to be practical. HF frequencies cannot penetrate the ionosphere. So we must use frequencies above VHF. As the frequency is increased, the equipment cost also increases. Therefore, the VHF band is a good compromise.

   (b) With the given values, the back scattering cross-section of the moon is

$$\sigma_m = 0.07 \times \pi\rho^2 = 2.7 \times 10^{12}\,\text{m}^2$$

   The formula for the received power is

$$P_r = \frac{P_t G_t G_r \sigma_m \lambda^2}{(4\pi)^3 d^4}$$

   which is the radar equation since we have assumed that the distance of both transmitter and receiver to the moon as same. Using the numerical values we get

$$\begin{aligned} P_r &= \frac{10 \times 10 \times 10 \times 2.7 \times 10^{12} \times (1.5)^2}{(4\pi)^3 (384000000)^4} = 1.41 \times 10^{-22}\,\text{W} \\ &= -188.5\ \text{dBm} \end{aligned}$$

   where $\lambda = 1.5\,\text{m}$ is used.

2. Determine the complex propagation constant for concrete at a frequency of 900 MHz. The relative dielectric constant of concrete is $\epsilon_r = 5.5$, $\mu_r = 1$, and $\sigma = 150\,\text{mS/m}$. How would you classify (good conductor, good dielectric, low-loss dielectric) this material at the above frequency? Determine the loss through a concrete wall of thickness 20 cm.

   SOLUTION: The complex dielectric constant is

$$\kappa = \epsilon_r - j\frac{\sigma}{\omega\epsilon_0} = 5.5 - j2.99.$$

   The complex propagation constant is

$$\gamma = \alpha + j\beta = jk = j\omega\sqrt{\kappa\epsilon_0\mu_0} = 11.6 + j45.7\ \text{m}^{-1}\ .$$

Since $\sigma/\omega\epsilon_0 \approx \epsilon_r$ we can say that the material is a lossy dielectric. The loss factor is α and we have

$$L = e^{-\alpha l}$$

where l is the thickness. This gives

$$L = 9.83 \times 10^{-2} = -20.2 \text{ dB}$$

3. Determine the distance d at which a vertically polarized surface wave is attenuated by 40 dB at a frequency of 20 MHz above medium dry ground. Assume that the transmitting antenna is on the ground.

   SOLUTION: Using the parameters of medium dry ground, we find

$$\kappa = \epsilon_r - j\frac{\sigma}{\omega\epsilon_0} = 15 - j0.90$$

   Using

$$\begin{aligned} \Omega &\approx -jk_0R\frac{(\kappa-1)}{2\kappa^2} = \left(7.24\times10^{-4} - 1.30\times10^{-2}j\right) \\ &= 0.013R\,\exp\left(-j1.515\,161\,49\right) \end{aligned}$$

   We see that the phase constant is $b \approx 90\,^\circ$ and the numerical distance is $p = 0.013R$. Assuming flat earth and using Fig.4.2 (or the formula) we see that for $F = -40$ the numerical distance must be 50. Then we get

$$0.013R = 50 \Rightarrow R = 3.8\,\text{km}.$$

   Notice that $80/f_{\text{MHz}}^{1/3} = 29.5\,\text{km}$ and the flat earth assumption is justified.

4. Calculate the knife edge loss at 900 MHz and 1800 MHz for an obstacle located midway between a transmitter and receiver separated by 3 km. The top of the obstacle protrudes 25 m above the direct path between the transmitter and receiver.

   SOLUTION: The wavelengths at 900 MHz and 1800 MHz are 1/3 m and 1/6 m, respectively. The radius of the first Fresnel ellipse is given by

$$h_1 = \frac{1}{2}\sqrt{R\lambda_0} = \begin{cases} 15.8\,\text{m} & @\ 900\,\text{MHz} \\ 11.2\,\text{m} & @\ 1800\,\text{MHz} \end{cases}$$

   The corresponding Fresnel parameters are

$$v = \frac{\sqrt{2}h}{h_1} = \begin{cases} 2.24 & @\ 900\,\text{MHz} \\ 3.16 & @\ 1800\,\text{MHz} \end{cases}$$

   Using formula 2.2.10 we get

$$F_d = -6.9 - 20\log_{10}\left(\sqrt{1+(v-0.1)^2} + v - 0.1\right) = \begin{cases} -19.97\text{ dB} & @\ 900\,\text{MHz} \\ -22.86\text{ dB} & @\ 1800\,\text{MHz} \end{cases}.$$

5. A GSM system is operating at 900 MHz in an urban area. The transmit power is 100 W, and we may assume that all receivers and transmitters are polarization and impedance matched. For a base station antenna of gain 10 dBi located at 15 m above the ground, and a receive antenna gain of 0 dBi located at 1.5 m above the ground, determine the percentage coverage inside a circular region of radius 10 km for $x_0 = -110$ dBm threshold, using Okumura-Hata model. Assume that the received signal is log-normally distributed with a variance $\sigma = 10$ dB.

   NOTE: You can use the approximation

$$\begin{aligned} \operatorname{erf}(x) &\approx \sqrt{1-\exp\left(-x^2\frac{4/\pi+ax^2}{1+ax^2}\right)}, \quad \text{for } x \geq 0 \\ a &= 0.14 \end{aligned}$$

   to calculate values of the error function.

   SOLUTION: The Okumura-Hata model gives the median path loss for urban area as

$$L_{50} = A + B\log_{10} R - E$$

   where

$$\begin{aligned} A &= 69.55 + 26.16\log_{10} f_c - 13.82\log_{10} h_b = 130.6 \\ B &= 44.9 - 6.55\log_{10} h_b = 37.2 \\ E &= 3.2\left[\log_{10}(11.75h_m)\right]^2 - 4.97 = -9.2\times 10^{-4} \end{aligned}$$

   and E is for large cities and $f_c \geq 300$ MHz. Note that the formula for E for medium or small cities gives

$$E = (1.1\log_{10} f_c - 0.7)\, h_m - (1.56\log_{10} f_c - 0.8) = 0.016.$$

   In both cases the value of E is very small compared to other quantities in the formula, hence use of either value does not change the results much. Using the value for E or large cities we get

$$L_{50} = 167.8 \text{ dB}.$$

   The median received power will then be

$$P_r = P_t + G_t + G_r - L_{50} = -107.8 \text{ dBm}.$$

   We can write the received signal as

$$x_{50}(r) \text{ dB} = A(R_0) - 10n\log_{10}\frac{r}{R_0}$$

   where

$$A(R_0) = -107.8 \text{ dBm}; \quad R_0 = 10\,\text{km}; \quad n = 3.72.$$

Using the formulas we get

$$
\begin{aligned}
P_{x_0}(R) &= \frac{1}{2} - \frac{1}{2}\operatorname{erf}\left(\frac{x_0 - x_{50}}{\sqrt{2}\sigma}\right) = 0.587 \\
\alpha &= \frac{(x_0 - A(R_0))}{\sqrt{2}\sigma} = -0.156 \\
\beta &= \frac{(10n \log_{10} e)}{\sqrt{2}\sigma} = 1.142 \\
A_c &= P_{x_0}(R) + \frac{1}{2} e^{-\frac{2\alpha\beta - 1}{\beta^2}} \left[1 - \operatorname{erf}\left(\frac{1 - \alpha\beta}{\beta}\right)\right] = 0.792 \,.
\end{aligned}
$$

That is 79.2% of the locations is expected to have a signal level exceeding the threshold.