EE 524 June 6, 2011
Final Exam Solutions

1. Consider a transmitter operating in free space at 170 MHz. Suppose that the transmitter power is 1 W, and uses an antenna with a gain of 2. Determine the maximum distance that a receiver can pick the signals of the transmitter. Assume that the receiver antenna gain is also 2, the receiver noise figure is 10 and the communication bandwidth is 25 kHz. The receiver requires 13 dB SNR for reliable detection.

   SOLUTION : The receiver noise power is

   $$P_n = kTBF = -174 + 44 + 10 = -120 \text{ dBm}$$

   and the required power (receiver sensitivity) is

   $$S_r = -120 + 13 = -107 \text{ dBm}.$$

   In free space, the received signal power is given by Friis' transmission formula as

   $$P_r = P_t + G_t + G_r - 20\log_{10}\left(\frac{4\pi R}{\lambda}\right).$$

   Equating this value to the sensitivity, we get

   $$P_t + G_t + G_r - 20\log_{10}\left(\frac{4\pi R}{\lambda}\right) = -107$$

   which, with $G_t = G_r = 3$ dB, and $P_t = 30$ dBm, gives

   $$30 + 3 + 3 - 20\log_{10}\left(\frac{4\pi R}{(3\times 10^8)/(170\times 10^6)}\right) = -107$$

   or

   $$R = 1983.6\,\text{km}.$$

2. Determine the loss due to a fog with a visibility of 100 m over a path of 10 km at a temperature of 5 °C, for.

   (a) $f = 10\,\text{GHz}$, and
   (b) $f = 500\,\text{GHz}$.

   SOLUTION : The water density in a fog is related to optical visibility by the formula

   $$W = 0.0156V^{-1.43}\,\text{g/m}^3.$$

   For the given values, this corresponds to a fog of $W = 0.42\,\text{g/m}^3$. The temperature parameter $\theta = \frac{300}{278} = 1.079$ at $T = 5\,^\circ\text{C}$.

(a) For $f = 10\,\mathrm{GHz}$ we have

$$\begin{aligned} K_l &= 6.0826 \times 10^{-4} f^{1.8963} \theta^{\left(7.8087 - 0.01565 f - 3.073 \times 10^{-4} f^2\right)} \\ &= 8.56 \times 10^{-2} \ \left(\mathrm{dB/\,km}\right) / \left(\mathrm{g/\,m^3}\right). \end{aligned}$$

The fog attenuation over a path of 10 km is then

$$A_f = K_l W R = 8.56 \times 10^{-2} \times 0.42 \times 10 = 0.36 \text{ dB}.$$

(b) For $f = 500\,\mathrm{GHz}$ we have

$$\begin{aligned} K_l &= 0.07536 f^{0.9350} \theta^{\left(-0.7281 - 0.0018 f - 1.542 \times 10^{-6} f^2\right)} \\ &= 21.58 \ \left(\mathrm{dB/\,km}\right) / \left(\mathrm{g/\,m^3}\right) \end{aligned}$$

and the attenuation over a path of 10 km is

$$A_f = K_l W R = 21.58 \times 0.42 \times 10 = 90.64 \text{ dB}.$$

This result shows that fog attenuation is an important problem in Terra Hertz bands.

3. The ground wave electric field strength due to a transmitter operating at 1.7 MHz is measured to be 0.5 mV/ m (rms) at a distance of 10 km from the transmitter. The transmitting antenna used is a short vertical antenna with an efficiency of 50%. Assuming that the conductivity of the ground is 5×10^{-5} S/ cm, and its relative permittivity is 10, determine the transmitted power. You may assume that both antennas are on the ground.

SOLUTION : The formula $80/f_{\mathrm{MHz}}^{1/3} = 47.06\,\mathrm{km}$ shows that flat earth ground wave model is valid for this problem. We have180

$$\begin{aligned} \kappa &= \kappa' - j\frac{\sigma}{\omega\epsilon_0} = 10 - j\frac{5 \times 10^{-3}}{2\pi \times 1.7 \times 10^6 \times 8.854187817 \times 10^{-12}} = 10 - 52.8679517j \\ \Omega &= -jk_0 R \frac{(\kappa - 1)}{2\kappa^2} = 3.22855144 - 0.672175184\,j \\ p &= |\Omega| = 3.298; \quad b = -\angle\Omega = 11.76\,^\circ \\ F &= \left|1 - j\sqrt{\pi\Omega}e^{-\Omega}\operatorname{erfc}\left(j\sqrt{\Omega}\right)\right| = 0.235 \\ F &= \left[\frac{2 + 0.3p}{2 + p + 0.6p^2} - \sqrt{\frac{p}{2}}\exp\left(-5p/8\right)\sin b\right]_{p=3.298, b=11.76*\pi/180} = 0.219 \end{aligned}$$

The directivity of a short dipole is 3 and with an efficiency of 50% the gain becomes $G_t = 3/2$ or 1.76 dB. The field strength at a distance $R = 10\,\mathrm{km}$ is given by

$$E_{rms} = \frac{\sqrt{30 P_t G_t}}{R} F \rightarrow P_t = \frac{1}{30 \times 3/2}\left(\frac{0.5 \times 10^{-3} \times 10000}{0.235}\right)^2 = 10.1\,\mathrm{W}.$$

4. Consider the path shown in Fig. 1. The path parameters are, $h_t = 20\,\mathrm{m}$, $d_1 = 3\,\mathrm{km}$, $d_2 = 6\,\mathrm{km}$, $h_o = 15\,\mathrm{m}$. What should be the height of the receiver antenna, h_r, to secure line of sight communication at $f = 1\,\mathrm{MHz}$?

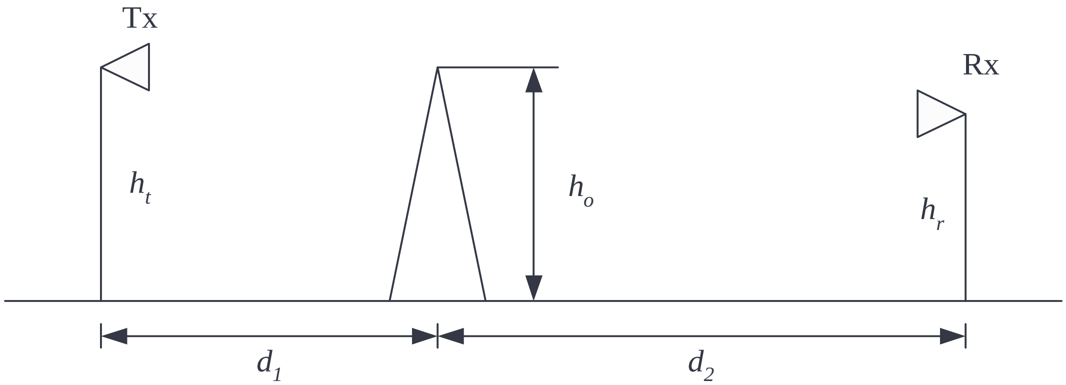

Figure 1: Geometry for Question 4.

SOLUTION : The wavelength at 1 MHz is 300 m. The radius of the first Fresnel zone is given by the formula

$$h_1 = \sqrt{\frac{d_1 d_2 \lambda_0}{d_1 + d_2}} = \sqrt{\frac{3000 * 6000 * 300}{9000}} = 774.6\,\mathrm{m}.$$

Using the geometry (neglecting the angle between line of sight and vertical) we find that the minimum height for the receiving antenna to secure LOS is $h_r = 78.5\,\mathrm{m}$.

$$\frac{x}{20} = \frac{x+3}{15+24.5} = \frac{x+9}{h_r} \rightarrow x = 7.796 \times 10^{-2}\,\mathrm{m}; \quad h_r = 2328.8\,\mathrm{m}.$$

If we don't neglect the angle between line of sight and vertical, we may proceed as follows: The tip of the obstacle must be on the first Fresnel ellipse. The distance from the Tx antenna to the obstacle is

$$x_t = \sqrt{d_1^2 + (h_t - h_o)^2}$$

and the distance from the Rx antenna to the obstacle is

$$x_r = \sqrt{d_2^2 + (h_r - h_o)^2}.$$

The equation for the first Fresnel ellipse gives

$$\begin{aligned} x_t + x_r &= \sqrt{(d_1 + d_2)^2 + (h_r - h_t)^2} + \frac{\lambda_0}{2} \\ \sqrt{d_1^2 + (h_t - h_o)^2} + \sqrt{d_2^2 + (h_r - h_o)^2} &= \sqrt{(d_1 + d_2)^2 + (h_r - h_t)^2} + \frac{\lambda_0}{2}. \end{aligned}$$

Numerically we have

$$\sqrt{3000^2 + 5^2} + \sqrt{6000^2 + (h_r - 15)^2} = \sqrt{9000^2 + (h_r - 20)^2} + 150$$

and solving we find

$$h_r = 2423.9\,\mathrm{m}.$$

5. Assume that the MUF for a radio path of distance 2000 km using reflections from a virtual height of 200 km ionospheric link is 30.6 MHz. Determine the critical frequency at the reflection point.

SOLUTION : For the given link parameters the incidence angle is

$$\begin{aligned}
\sin\psi_i &= \frac{\sin\frac{d}{2a_e}}{\sqrt{\left(\frac{h'}{a_e}\right)^2 + 4\left(1+\frac{h'}{a_e}\right)\sin^2\frac{d}{4a_e}}} = 0.968 \\
\psi_i &= \sin^{-1} 0.968 = 1.317\,\mathrm{rad} = 75.46\,^\circ
\end{aligned}$$

Then

$$f_c = \frac{30.6}{\sec 1.317} = 7.68\,\mathrm{MHz}.$$

6. A GSM system is operating at 1800 MHz in a metropolitan region. The transmit power is 10 W, and we may assume that all receivers and transmitters are polarization and impedance matched. For a base station antenna of gain 10 dBi located at 25 m above the ground, and a receive antenna gain of 2 dBi located at 2 m above the ground, determine the percentage coverage inside a circular region of radius 5 km for $x_0 = -100$ dBm threshold, using Cost-231 Hata model. Assume that the received signal is log-normally distributed with a variance $\sigma = 10$ dB.

SOLUTION : The Okumura–Hata model gives the median loss as

$$L_{50}\,(\mathrm{dB}) = F + B\log_{10} R - E + G$$

where

$$\begin{aligned}
F &= 46.3 - 13.82\log_{10} h_b + 33.9\log_{10} f_c = 137.3, \\
E &= 3.2\left[\log_{10}(11.75h_m)\right]^2 - 4.97 = 1.05, \\
B &= 44.9 - 6.55\log_{10} h_b = 35.74, \\
G &= 3.
\end{aligned}$$

Combining these results we find

$$L_{50}\,(\mathrm{dB}) = 138.35 + 35.74\log_{10} R$$

At a distance of 5 km, we get

$$L_{50}\,(5\,\mathrm{km}) = 138.35 + 35.74\log_{10} 5 = 163.3\ \mathrm{dB}.$$

The received power will then be

$$P_r = P_t + G_t + G_r - L_{50} = -111.3\ \mathrm{dBm}.$$

We can write the received signal as

$$x_{50}\,(r)\ \ \mathrm{dB} = A\,(R_0) - 10n\log_{10}\frac{r}{R_0}$$

where

$$A\left(R_0\right) = -111.3 \text{ dBm}; \qquad R_0 = 5\,\text{km}; \qquad n = 3.574.$$

Using the formulas we get

$$\begin{aligned}
P_{x_0}\left(R\right) &= \frac{1}{2} - \frac{1}{2}\operatorname{erf}\left(\frac{x_0 - x_{50}}{\sqrt{2}\sigma}\right) = 0.129 \\
\alpha &= \frac{\left(x_0 - A\left(R_0\right)\right)}{\sqrt{2}\sigma} = 0.799 \\
\beta &= \frac{\left(10n\log_{10} e\right)}{\sqrt{2}\sigma} = 1.098 \\
A_c &= P_{x_0}\left(R\right) + \frac{1}{2}e^{-\frac{2\alpha\beta-1}{\beta^2}}\left[1 - \operatorname{erf}\left(\frac{1-\alpha\beta}{\beta}\right)\right] = 0.363.
\end{aligned}$$

That is only 36% of the locations is expected to have a signal level exceeding the threshold.