Last revised May 2, 2011

CHAPTER 7

Ionospheric Propagation

The ionosphere is the name given to the upper layer of the atmosphere that constitutes of ionized gases mainly due to solar radiation. To understand the radio wave propagation in the ionosphere, we need a general understanding of the physical properties of the ionosphere. We will first give a brief summary of the structure of the ionosphere and its characteristics.

7.1. The Structure of the Ionosphere

The main constituents of the upper atmosphere are the same gases found in the atmosphere. However, their concentrations differ with height. Heavier gases are found in the lower parts, while the upper parts are richer in lighter gases, mainly oxygen and nitrogen. The upper atmosphere is stratified owing to the differences in masses of gases. In the lower parts of the atmosphere, meteorological phenomena mix the air well and prevent the atmosphere to become stratified despite the difference in masses of the constituent gases.

Due to the stratification (along with other reasons) the upper atmosphere is divided into several layers. Just above the troposphere is the *stratosphere* which is layered in temperature with warmer layers above and cooler layers below. The border of the troposphere and stratosphere, the *tropopause*, is marked by where this inversion begins. The reason for temperature inversion in the stratosphere is the ozone (O_3) which absorbs high energy UVB and UVC light. In the upper stratosphere the O_2 and O_3 molecules are broken down to atomic oxygen by the prevailing bombardment of the UV light. The mid stratosphere has less UV light passing through it. The O and O_2 are able to combine to produce the majority of natural ozone. In this process heat is released and this is the reason of temperature inversion in the stratosphere. The stratosphere typically extends between altitudes of 10 km and 50 km.

Above the stratosphere is the *mesosphere*. The boundary between the stratosphere and mesosphere is called the *stratopause*. In the mesosphere temperature decreases with increasing height. The upper boundary of the mesosphere is the *mesopause* with temperatures below 130 K. The exact boundaries of the mesosphere vary with latitude and with season, but the lower boundary of the mesosphere is usually located at heights of about 50 km and the upper boundary at heights near $80 - 100$ km.

The layer above the mesosphere is called the *thermosphere*. The thermosphere begins at about $80 - 100$ km above the earth. This is the layer in which the residual atmospheric gases sort into strata according to molecular mass. Thermospheric temperatures increase with altitude due to absorption of highly energetic solar radiation by the small amount of residual oxygen still present. Temperatures are highly dependent on solar activity, and can rise up to 1500 °C. Solar radiation ionizes the atmospheric particles in this layer, enabling radio waves to bounce off and be received beyond the horizon. The upper boundary of thermosphere is called the *thermopause*. The exact altitude of thermopause varies by the energy inputs at the location, time of day, solar flux, season, etc. and can be anywhere between $500 - 1000$ km.

The *exosphere* is the uppermost layer of the atmosphere. The main gases within the exosphere are the lightest gases, mainly hydrogen, with some helium, carbon dioxide, and atomic oxygen. The exosphere is the last layer before outer space. Since there is no clear boundary between outer space and the exosphere, the exosphere is sometimes considered a part of outer space.

The *ionosphere* is a part of the upper atmosphere, comprising portions of the mesosphere, thermosphere and exosphere, distinguished because it is ionized by solar radiation. Figure 7.1 shows the layers of the atmosphere and the temperature and electron density variations with height.

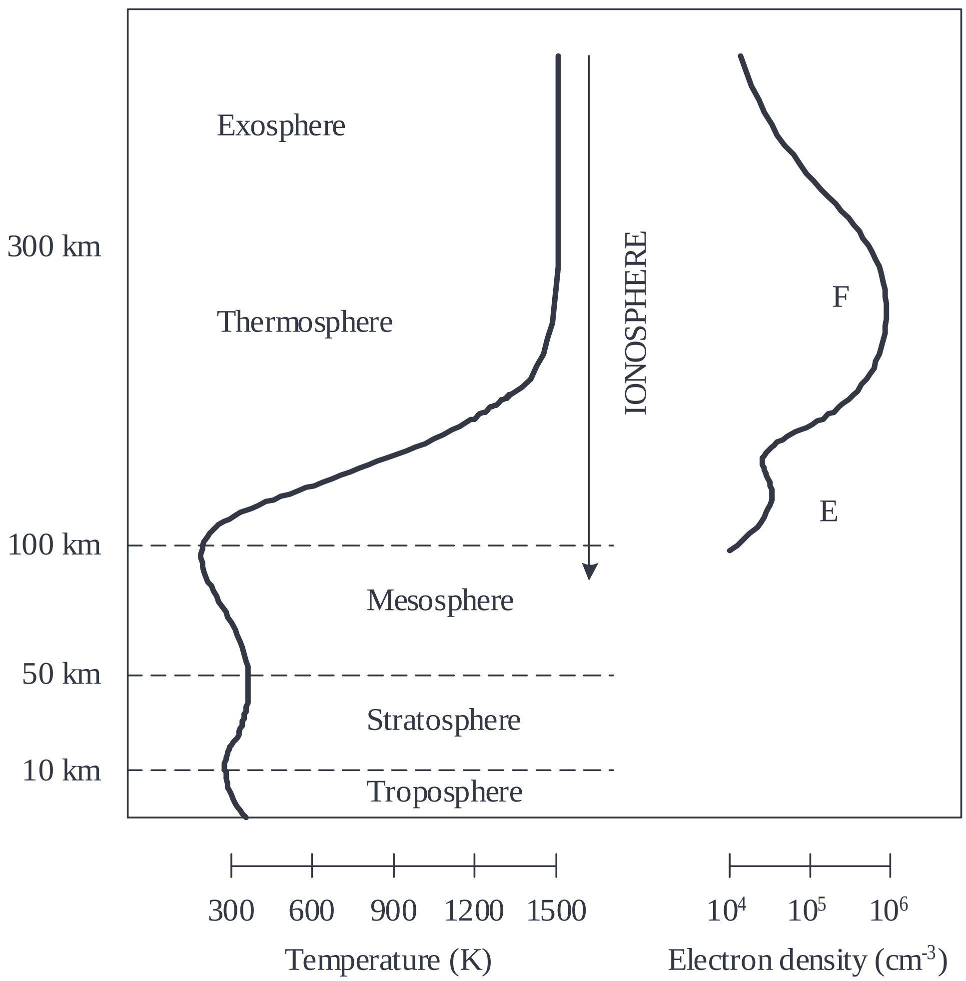

FIGURE 7.1. Properties of the atmosphere and ionosphere.

In the upper atmosphere, the solar radiation causes the molecules to break into atoms as they absorb a quantum of energy. The energy of a photon that breaks the O_2 molecule corresponds to a wavelength of $0.24\,\mu$m, and the energy of a photon required to break down the N_2 molecule into its atoms corresponds to a wavelength of $0.127\,\mu$m. These wavelengths are in the ultraviolet and X-ray bands. Thus, the ionosphere absorbs these waves and protects the life on earth from their hazardous effects.

7.1.1. Ionization. Experimental studies show that the dissociation of O_2 begins at an altitude of about 90 km, while dissociation of N_2 begins at an altitude of about 220 km. At 130 km above the earth, the atmosphere still retains 25% of its O_2, and the molecular oxygen practically disappears at heights above 160 km. At 300 km and above molecular nitrogen also disappears. Between 30 to 60 km, solar radiation produces an ozone region.

The atoms of the gases in the atmosphere consist of positively charged nuclei and negatively charged electron shells around them. Sometimes an electron is broken loose from the outer shell and a positively charged molecule or atom, called *ion*, is left behind. The process is called *ionization*. The electrons are attracted to the nucleus and energy is required to break them loose. The energy required to remove an electron from its shell is called the *ionization energy*. The ionization energies of the atmospheric gases in the molecular and atomic form are studied in laboratory conditions and Table 7.1 lists some of them. It must be noted that the energy required to break O_2 molecule into atoms is not the ionization energy, but the bond energy. The bond energy of O_2 molecule is 498 kJ/mol, while that of N_2 is 945 kJ/mol.

TABLE 7.1. Ionization energies of some of the atmospheric gases.

Gas	**Ionization energy** (eV)	**Ionization frequency** (Hz)	**Ionization wavelength** (Å)
O	13.62	3.29×10^{15}	910
O_2	12.18	2.95×10^{15}	1018
N	14.53	3.51×10^{15}	853
N_2	15.58	3.77×10^{15}	796

If we denote the ionization energy as W, we can say that when a gas is exposed to radiation with energy of photons greater than W, ionization will occur. This is *ionization by absorption of radiation*. Since the energy of photons is given by $h\nu$, the frequency of the radiation must be greater than a certain frequency, called the *ionization frequency*, to ionize a given gas. The wavelength of the ionizing radiation is called the *ionization wavelength*. When the frequency of radiation is above the ionization frequency, the extra energy is transferred to the kinetic energy of the electron knocked-out of its orbit. If such electrons (or other molecules) have energies above the ionization energy of another gas, they can ionize that gas upon collision. This is *ionization by collision*. These two processes take place continuously in the atmosphere.

7.1.2. Recombination. Ionization is not a one way process. The ionized particles *recombine* with free electrons when they happen to move close enough to one another through random thermal motion to be attracted to each other by electrostatic forces. When this happens, energy is released in the form of a photon. If the photon released by this process has enough energy, it will ionize other molecules, otherwise it will penetrate to lower atmosphere. Obviously, the photons that can reach lower atmosphere will mostly have lower energies, although some high energy photon may escape collisions and reach the lower atmosphere. This is how the ionosphere protects life on earth by absorbing ionizing radiation.

Assume that there is a single electron and a single positive ion in one cubic meter of space. Let α_e be the probability that they will recombine, which is called the *recombination coefficient*. It means that the expected time for the electron and positive ion to collide is $1/\alpha_e$. If there are two electrons and two ions, the probability of recombination increases four-fold, since there are four different possibilities for recombination. By the same reasoning, if there are N electrons and N ions in one cubic meter of space, the probability of recombination will increase by N^2. The number N is actually the number of electrons per unit volume, i.e., the *electron density*.

The recombination and ionization processes work against each other. At some point, the rate of ionization will be equal to the rate of recombination. This is a dynamic balance determined by the equation

$$\frac{dN}{dt} = 0 = I_s - \alpha_e N^2 \ \ \mathrm{m}^{-3}\,\mathrm{s}^{-1} \tag{7.1.1}$$

where I_s denotes the ionization rate, i.e., the number of electrons produced per unit volume per unit time by ionization.

The rate of ionization depends on the solar energy. If the solar energy suddenly changes, the equilibrium condition is changed and a new equilibrium will be achieved after some time. This may happen for example after sunset. Let at time $t = 0$ the electron density be N_0. If $I_s = 0$ at t^+ the equilibrium condition becomes

$$\frac{dN}{dt} = -\alpha_e N^2 \tag{7.1.2}$$

which can be solved for $N(t)$ as

$$\int_{N_0}^{N} \frac{dN}{N^2} = -\alpha_e \int_0^t dt \tag{7.1.3}$$

or equivalently

$$N(t) = \frac{N_0}{1 + \alpha_e N_0 t}. \tag{7.1.4}$$

7.1.3. Ionization in the Real Ionosphere. So far we have discussed the basic mechanisms of ionization in the ionosphere and stated the basic principles. In the real ionosphere, several other factors play an important role. Among these are the presence of non-ionized particles, changes in the temperature, density, and pressure with height, stratification of the atmosphere, and other sources of ionization. All of these factors make it quite difficult to theoretically determine the electron density in the ionosphere. More importantly, the ionization density depends heavily on the solar activities, time of day, season of the year and so on. Therefore, the ionization density in the ionosphere is determined through experiments and observations.

There are several layers in the ionosphere in which the ionization density reaches a peak or remains roughly constant. There are three well defined layers designated D, E, and F in order of increasing height. In the day time the D layer extends between 60 km and 90 km above the surface of the Earth and vanishes at night. The E layer extends between heights of 90 km and 130 km and exists both during the day and night. However, at certain times thin clouds of intense ionization occur in the upper E layer and lower F layer, called sporadic E layer, designated E_s. The F layer layer extends from about 180 km to more than 500 km.

During the day time the F layer splits into two layers designated $F1$ (extending between 180 km and 240 km) and $F2$ (extending between 230 km and 400 km). At night, the $F1$ layer disappears.

7.2. Radio Wave Propagation in the Ionosphere

The high electron density in the ionosphere forms a relatively high conductivity region. The electromagnetic waves of lower frequencies are reflected back from the ionosphere making very long distance communication possible. Such waves are called ionospheric waves or *sky waves*. IEEE standard 211 reserves the word ionospheric waves to the waves internal to ionosphere, [**40**], however, its use is quite common. To understand the electromagnetic wave propagation in the ionosphere, we will assume that it is a homogeneous ionized gas.

7.2.1. Dielectric Constant of Homogeneous Ionized Gas. In an ionized gas there are two types of charged particles: the positively charged ions, and the free electrons. These particles move under the influence of the electric field component of an electromagnetic wave. However, positively charged ions are much heavier than the electrons and their motion can be neglected. The motion of the electrons are governed by the equation

$$m_e \frac{d\mathbf{v}}{dt} = -e\mathcal{E}(t) \tag{7.2.1}$$

where m_e, e, and $\mathbf{v}$ denote the mass, charge, and velocity of an electron, respectively, and $\mathcal{E}(t)$ is the time varying electric field vector. For a sinusoidal field we can write

$$j\omega m_e \mathbf{V} = -e\mathbf{E}. \tag{7.2.2}$$

The induced current in a homogeneous ionized gas having an electron density of N per cubic meter will then be

$$\mathbf{J} = -eN\mathbf{V} = \frac{e^2 N}{j\omega m_e}\mathbf{E}. \tag{7.2.3}$$

From Maxwell's equation we have

$$\nabla \times \mathbf{H} = j\omega\epsilon_0\mathbf{E} + \mathbf{J} = j\omega\epsilon_0 \left(1 - \frac{e^2 N}{\omega^2 \epsilon_0 m_e}\right)\mathbf{E} \tag{7.2.4}$$

which gives the dielectric constant as

$$\kappa = 1 - \frac{e^2 N}{\omega^2 \epsilon_0 m_e}. \tag{7.2.5}$$

The *plasma frequency* is defined as $\omega_p = e\sqrt{N/\epsilon_0 m_e}$. Using this definition we can write

$$\kappa = 1 - \frac{\omega_p^2}{\omega^2}. \tag{7.2.6}$$

It must be noted that when $\omega > \omega_p$, κ is less than unity, when $\omega = \omega_p$, κ is zero, and when $\omega < \omega_p$, κ is negative. The propagation constant of a plane wave is $k = \omega\sqrt{\mu_0\kappa\epsilon_0} = \sqrt{\kappa}k_0$. This means that when $\omega < \omega_p$, k will be purely imaginary and the plane waves will become evanescent and decay exponentially as they propagate.

In this derivation, we have assumed that there is no collision between the electrons. This assumption is valid only if the density is very small, as in the case of upper atmosphere. At lower parts of the atmosphere, the density is much higher and electrons suffer a large number of collisions with other electrons, ions, and neutral particles. To account for collisions, we must write (7.2.1) as

$$m_e \frac{d\mathbf{v}}{dt} + \nu m_e \mathbf{v} = -e\mathcal{E}(t) \tag{7.2.7}$$

where ν is the number of collisions an electron experiences per unit time. With this correction, the dielectric constant becomes

$$\kappa = 1 - \frac{\omega_p^2}{\omega(\omega - j\nu)} = 1 - \frac{\omega_p^2}{\omega^2 + \nu^2} - j\frac{\omega_p^2}{(\omega^2 + \nu^2)}\frac{\nu}{\omega}. \tag{7.2.8}$$

The real part of κ is the relative permittivity, and the imaginary part is the ratio of conductivity to $\omega\epsilon_0$. Note that, the collisions make κ complex. The nonzero imaginary part of κ implies energy loss, which means that some of the energy of the wave is transferred to thermal energy.

If $\omega^2 \gg \nu^2$, we may write

$$\kappa' = 1 - \frac{1}{\omega^2 + \nu^2}\omega_p^2 \approx 1 - \frac{e^2 N}{\epsilon_0 m_e \omega^2}, \tag{7.2.9a}$$

$$\sigma = \frac{e^2 N}{m_e \omega^2}\nu \tag{7.2.9b}$$

Substituting the numerical values of $e = 1.602 \times 10^{-19}$ C, and $m_e = 9.11 \times 10^{-31}$ kg we find

$$\kappa' \approx 1 - 81\frac{N}{f^2} \quad \text{F/m}, \tag{7.2.10a}$$

$$\sigma \approx 2.82 \times 10^{-8}\frac{N\nu}{\omega^2} \quad \text{S/m}. \tag{7.2.10b}$$

If we ignore the collisions we may write the refractive index of ionized gas as

$$n = \sqrt{1 - 81\frac{N}{f^2}}. \tag{7.2.11}$$

7.2.2. Reflection of Waves from the Ionosphere. Since the electron density increases with height, the refractive index decreases. Just as in the case of tropospheric refraction, the waves will bend downward as they propagate in the ionosphere. If a plane wave is normally incident on the ionosphere, it will be reflected back from the height where the electron density is high enough to make $\kappa = 0$. For the case of oblique incidence with an angle of incidence ψ_i as defined in Fig. 7.2, the plane wave will be reflected back from a height at which $\sqrt{\kappa} = \sin\psi_i$ if such a condition is satisfied.

For a given value of ψ_i, the requirement that $\kappa = \sin^2\psi_i$ implies that higher electron densities are required as the frequency is increased since $\kappa = 1 - 81N/f^2$. Conversely, if the maximum electron density is given, there will be a maximum frequency above which the waves will not return back. For a given frequency f, the critical value of electron density, N_{critical}, can be found as

$$N_{\text{critical}} = \frac{f^2\cos^2\psi_i}{81}. \tag{7.2.12}$$

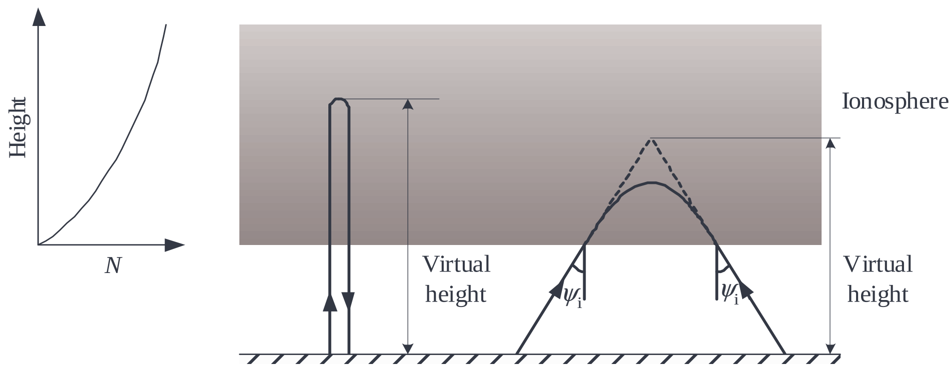

FIGURE 7.2. Reflection of plane waves from the ionosphere.

In practice, the maximum electron density, $N_{\max}$, of the ionosphere at a given time is determined by the conditions in the ionosphere. For normal incidence, this implies frequencies above

$$f_c = 9\sqrt{N_{\max}} \tag{7.2.13}$$

will not be reflected back. This frequency is called the *critical frequency*. For oblique incidence, we find that frequencies above

$$f = 9\sqrt{N_{\max}} \sec\psi_i = f_c \sec\psi_i \tag{7.2.14}$$

will not be reflected back. This frequency is called the *maximum usable frequency*, (MUF).

For a typical value of $N_{\max} = 5 \times 10^{11}/\,\mathrm{m}^3$, the critical frequency is 6.4 MHz. If $\psi_i = 30\,^\circ$, the MUF will be 7.4 MHz. If we increase ψ_i to $60\,^\circ$, the MUF will be 12. 8 MHz. The MUF generally does not exceed 40 MHz. During low solar activity periods, the upper frequency limit falls down to 25 to 30 MHz.

The height at which the lines in the direction of incident and returned rays meet is called the *virtual height* of the ionosphere as shown in Fig. 7.2. For all electromagnetic calculations (including time delay), we can replace the ionosphere by a PEC layer at this height. The virtual height of the $F1$ layer ranges from 200 to 250 km, while for $F2$ layer ranges between 250 and 400 km. At night, the two layers combine and the F layer has a virtual height of about 300 km. The virtual height of E layer is about 110 km.

The main idea in ionospheric communication is to reflect the signals from the ionosphere to arrive at far away distances on the earth's surface. This makes beyond the horizon communication possible. Actually, the wave can be reflected from the earth's surface and ionosphere more than once as shown in Fig. 7.3. The signal transmitted by the transmitter at Tx will be received at points A and B, but not in between. The distance from the transmitter to the point where the signal reflected from the ionosphere arrives is called the *skip distance*. Obviously, the skip distance depends both on the initial angle of the rays and the virtual height of the ionosphere.

The horizon angle α (angle between transmitter radiation direction and the local horizon) of the transmitter and the incidence angle ψ_i are related by

$$\frac{\cos\alpha}{a_e + h'} = \frac{\sin\psi_i}{a_e}. \tag{7.2.15}$$

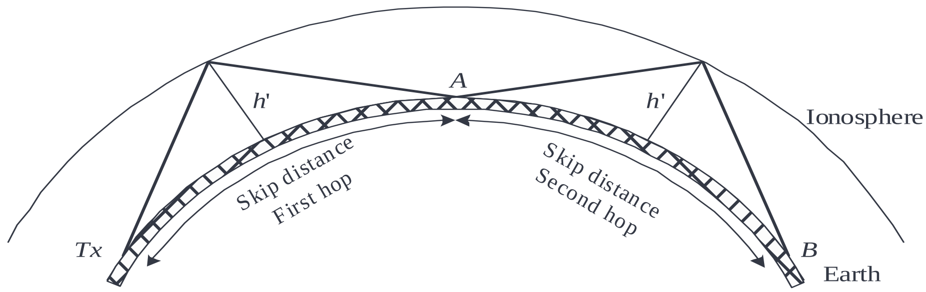

FIGURE 7.3. Multiple hop communication link.

The skip distance is then given by

$$d_{\text{skip}} = 2a_e \left(\frac{\pi}{2} - \alpha - \psi_i \right). \tag{7.2.16}$$

The skip distance will be a maximum for $\alpha = 0$, and the skip distance can be written approximately as

$$d_{\max} = 2\sqrt{2a_e h'}. \tag{7.2.17}$$

The corresponding maximum value of ψ_i is given by

$$\psi_i = \sin^{-1} \frac{a_e}{a_e + h'} \approx \frac{\pi}{2} - \sqrt{\frac{2h'}{a_e + h'}}. \tag{7.2.18}$$

This value is about 74° and using this value in (7.2.14) gives the maximum usable frequency for maximum ψ_i as

$$f_{\max} = 3.6 f_c. \tag{7.2.19}$$

For an electron density of $N = 10^{12}/\,\text{m}^3$ $f_c = 9\,\text{MHz}$ and $f_{\max} = 32.4\,\text{MHz}$. Using the virtual height of the F layer, which is 300 km, the maximum skip distance is calculated as 4500 km. For the E layer with a virtual height of 110 km, the maximum skip distance is 2700 km. If the desired range is less than the maximum skip distance, the transmitter beam must be elevated above the horizon, resulting in a lower value for MUF. If the desired range is greater than the maximum skip distance, a multi-hop link must be used, in which the waves are reflected more than once between the ionosphere and the surface of the earth, as shown in Fig. 7.3. It must be noted that the properties of the ionosphere at the reflection point must be used for MUF calculations.

7.2.3. Ionospheric Measurements. There are many methods to measure the electron density as a function of height in the ionosphere. Among these are direct methods such as direct measurements carried out by space probes, satellites, and stratospheric balloons. An indirect yet very reliable method to observe the ionosphere is the use of ground based ionospheric stations, called *ionosondes*. An ionosonde, or chirpsounder, is a special radar for the examination of the ionosphere. It has a high frequency (HF) transmitter that can be tuned over HF frequency range, typically 0.5–23 MHz or 1–40 MHz, though normally sweeps are confined to approximately 1.6 – 12 MHz. An antenna with a pattern maximum pointed vertically upwards and is efficient over the whole frequency range is used. A receiver which tracks the frequency of the transmitter, receives the signals reflected from the ionosphere, and the results are used to plot the virtual height as a function of frequency. Such a plot is called an *ionogram*. A typical ionogram is shown in Fig. 7.4. There are two important characteristics of an ionogram. Firstly, the virtual heights increase steeply as the critical frequency is reached. Secondly, there are double reflections from the $F1$ and $F2$ layers. This is due to the fact that ordinary and extraordinary waves have different critical frequencies. This topic will be discussed later.

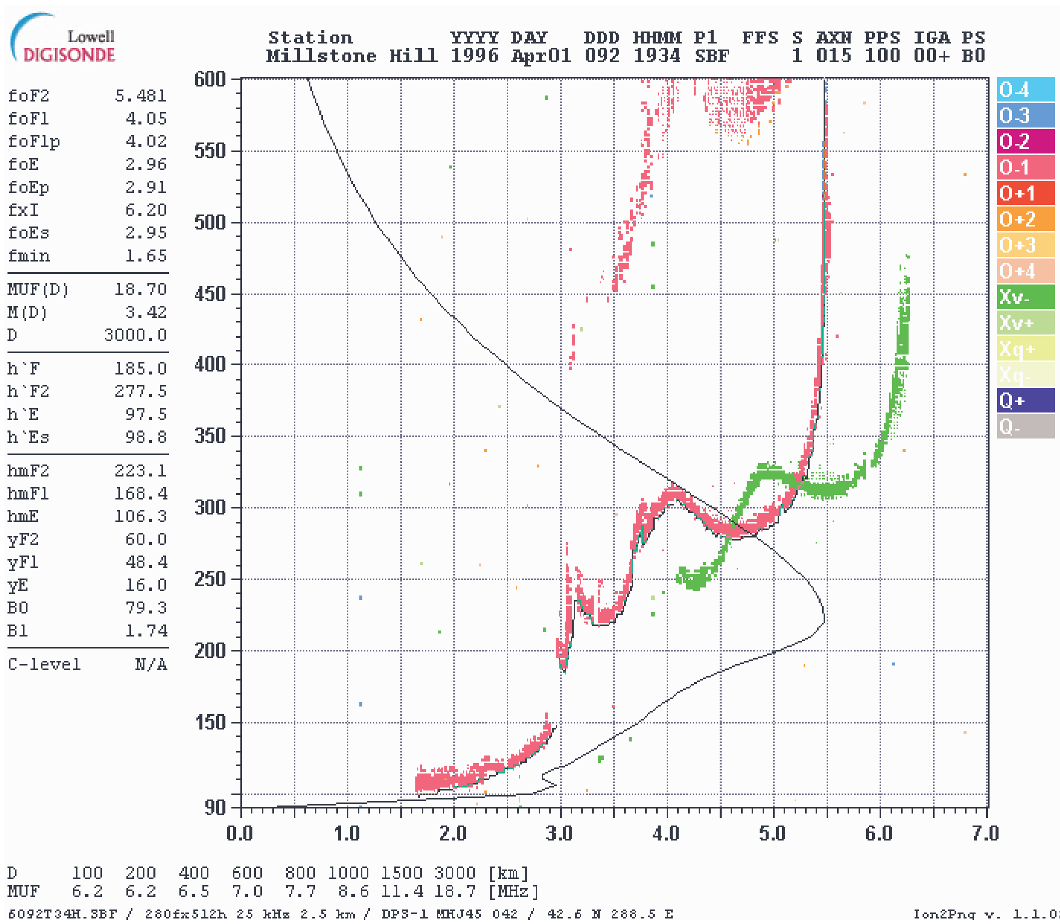

FIGURE 7.4. A sample ionogram, [**2**].

An oblique incidence sounding stations use a transmitter and receiver located at the end points of a propagation path. The transmitter and the receiver must be synchronized for such a measurement. This is achieved by transmitting synchronization pulses. However, the difficulty of synchronization and the fixed location of the stations are major drawbacks of such a system. Instead, oblique incidence backscatter sounding stations are much more useful. In these systems, the transmitter and receiver are located at the same site. The transmit antenna can be directed at different angles. The waves are scattered at the point M where they hit the earth's surface and some of the energy is propagated back towards the transmitter as shown in Fig. 7.5. In the case of oblique incidence sounding, the ionogram will exhibit a double reflection at high frequency end as shown schematically in Fig. 7.6. The geometry causing double reflection is shown in the same figure.

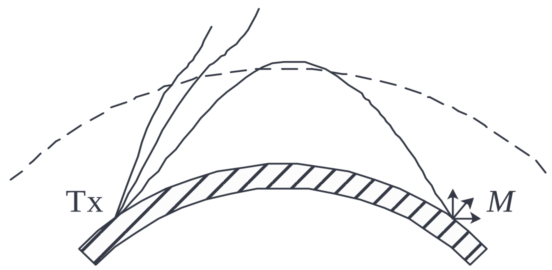

FIGURE 7.5. Ray path in an oblique incidence backscatter sounding.

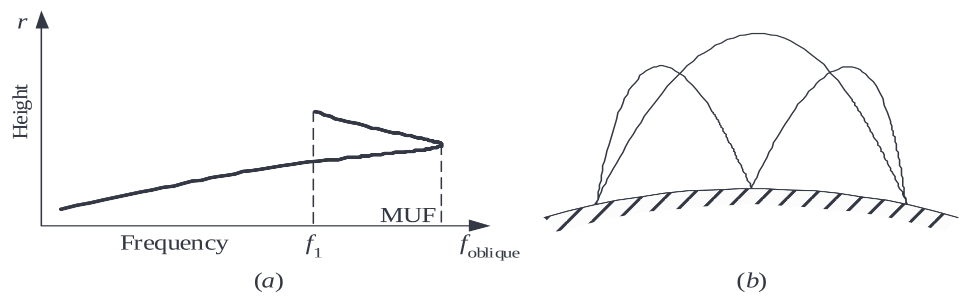

FIGURE 7.6. Oblique incidence ionogram showing reflections from different heights.

In order to establish an ionospheric propagation link between two stations on earth, one needs to know the MUF for that path. The ionosonde data can be used to obtain the required data. Practically, daily MUF charts are prepared for different locations and propagation distances and these charts are used to determine the frequency of operation. Such a daily MUF plot is shown in Fig. 7.7 for a winter month.

7.2.4. Maximum Usable Frequency and Optimum Frequency. The critical frequency and MUF are different concepts as we have discussed above. The critical frequency is the maximum frequency that can be reflected by a layer for vertical incidence, while MUF is the maximum frequency that can be reflected by a layer for a given incidence angle. As (7.2.14) indicates, the MUF differs from the critical frequency by a factor of $\sec\psi_i$. From Fig. 7.7, we can see that 30 MHz would be satisfactory for transmission over a 2000 km path during midday. During other times, one may need to change the frequency to establish communication.

The MUF may show variations about the monthly average of up to 15%. Furthermore, it is desirable to restrict the number of frequencies required to a reasonable number. Therefore, the *optimum* frequency is selected somewhere between about 50% and 90% of the predicted MUF for a given communication link.

As the frequency of a transmission is reduced, multiple hops will be needed for a given communication path. This will increase the losses. Also, the losses due to the D layer

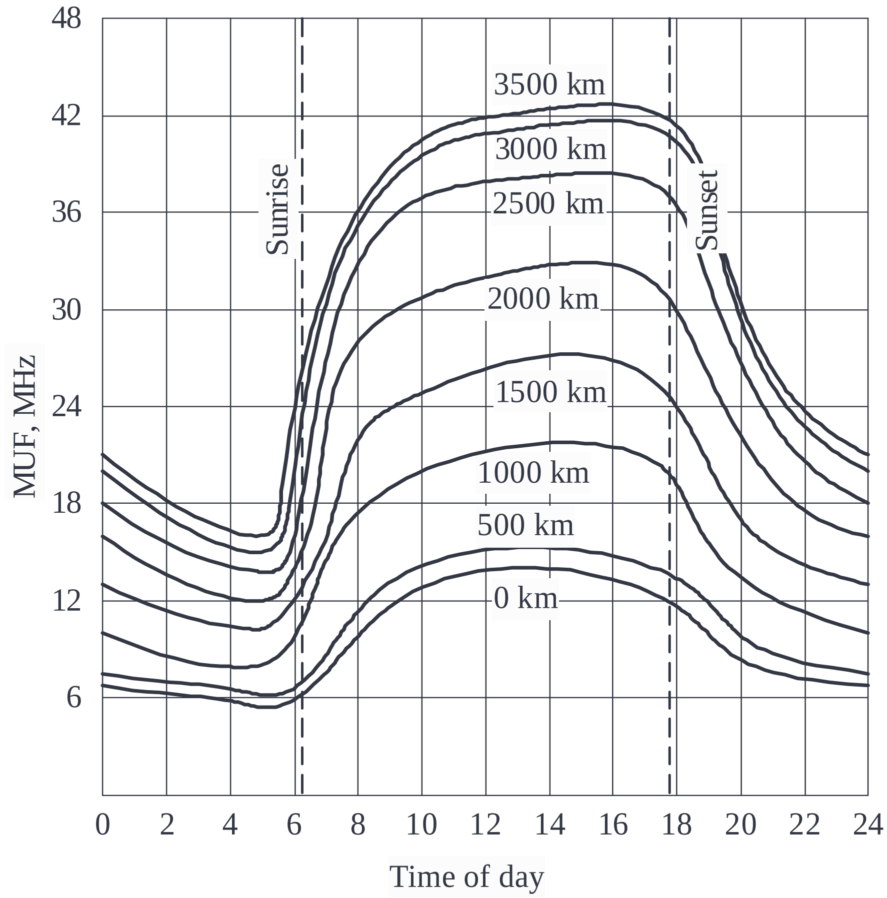

FIGURE 7.7. A typical MUF chart for propagation paths of different lengths.

increase as the frequency is decreased. These two effects mean that there is a frequency below which radio communications between two stations will be lost due to reduced SNR. The *lowest usable frequency* (LUF) is defined as that frequency below which the signal falls below the minimum strength required for satisfactory reception.

The LUF is dependent upon both the transmitter and receiver. Their antennas, receivers, transmitter powers, the level of noise in the vicinity, and so forth all affect the LUF. The type of modulation used is also important since different types of modulations require different SNR for reliable communication. A practical but rough estimate of LUF is given as $0.25\times$MUF with an upper limit of 12 MHz.

The ITU-R Recommendation P.373-8, [**51**], gives the following definitions:

Operational MUF: The highest frequency that would permit acceptable performance of a radio circuit by signal propagation via the ionosphere between given terminals at a given time under specified working conditions, (antennas, power, emission type, required SNR, and so forth).

Basic MUF: The highest frequency by which a radio wave can propagate between given terminals, on a specified occasion, by ionospheric refraction alone.

Optimum working frequency (OWF): The lower decile of the daily values of operational MUF at a given time over a specified period, usually a month. That is, it is the frequency that is exceeded by the operational MUF during 90% of the specified period.

Highest probable frequency (HPF): Highest probable frequency (HPF): the upper decile of the daily values of operational MUF at a given time over a specified period, usually a month. That is, it is the frequency that is exceeded by the operational MUF during 10% of the specified period.

Lowest usable frequency (LUF): The lowest frequency that would permit acceptable performance of a radio circuit by signal propagation via the ionosphere between given terminals at a given time under specified working conditions.

ITU-R Recommendation 1239-2, [**52**], gives methods for the prediction of long term ionospheric characteristics needed for radio-circuit design, service planning and frequency band selection.

7.2.5. Attenuation of Waves in the Ionosphere. The complex dielectric constant of the ionosphere is given in (7.2.8). The propagation constant can be written as $\beta + j\alpha$ where

$$\alpha = \mathrm{Re}\left\{j\omega\sqrt{\mu_0\kappa}\right\} = \mathrm{Re}\left\{\sqrt{(j\omega\mu_0)(\sigma + j\omega\epsilon)}\right\} \tag{7.2.20a}$$

$$= \omega\sqrt{\frac{\mu_0\epsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}-1\right)}, \tag{7.2.20b}$$

$$\beta = \mathrm{Im}\left\{j\omega\sqrt{\mu_0\kappa}\right\} = \mathrm{Im}\left\{\sqrt{(j\omega\mu_0)(\sigma + j\omega\epsilon)}\right\} \tag{7.2.20c}$$

$$= \omega\sqrt{\frac{\mu_0\epsilon}{2}\left(\sqrt{\left(1+\frac{\sigma^2}{\omega^2\epsilon^2}\right)}+1\right)}. \tag{7.2.20d}$$

Using (7.2.8) in these equations gives

$$\alpha = \omega\sqrt{\frac{\mu_0\epsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}-1\right)} \tag{7.2.21a}$$

$$= \omega\sqrt{\mu_0\epsilon_0}\sqrt{\frac{1}{2}\left(\sqrt{(\kappa')^2+\left[(1-\kappa')\frac{\nu}{\omega}\right]^2}-\kappa'\right)}. \tag{7.2.21b}$$

It must be noted that as $\nu \to 0$, α also goes to zero. Thus, the attenuation is higher at the lower parts of the ionosphere (D layer) since there are more collisions in the denser layers of the ionosphere. If $\sigma/\omega\epsilon \ll 1$, we can write

$$\alpha \approx \frac{\sigma}{2}\sqrt{\frac{\mu_0}{\epsilon}} = \frac{1}{2}\frac{\omega_p^2\nu\epsilon_0}{(\omega^2+\nu^2)}\sqrt{\frac{\mu_0}{\epsilon}} = \frac{60\pi e^2 N\nu}{\sqrt{\kappa'}m_e(\omega^2+\nu^2)} \tag{7.2.22}$$

which shows that the attenuation is lower at higher frequencies. Therefore, it is desirable to use as high a frequency as possible without exceeding the MUF. The relative permittivity is typically very close to unity and is zero at the reflection point. Therefore, as a first order approximation we may set $\kappa' = 1$. In the HF band $(3 - 30\,\mathrm{MHz})$ we also have $\omega^2 \gg \nu^2$ and we can write

$$\alpha \approx \frac{60\pi e^2 N\nu}{m_e\,(2\pi f)^2} = 1.35 \times 10^{-7}\frac{N\nu}{f^2}. \tag{7.2.23}$$

Ionospheric measurements show that the maximum electron density (at noon) in $F2$ layer is 10^{12} per m^3, and for E layer, 10^{11} per m^3. The collision frequency is about 10^3 per second for the $F2$ layer and about 10^6 per second for the E layer. The product $N\nu$ is 10^{17} in E layer and 10^{15} in $F2$ layer. This means that the absorption of HF waves in the E layer is hundred times larger in than that in the $F2$ layer. Actually, the D layer just below the E layer has much higher electron density and collision frequency. In this region however, the assumption $\omega^2 \gg \nu^2$ is no longer valid. Although the absorption does not decay with f^2 in D layer, it still decreases as the frequency is increased.

7.2.6. Effect of Earth's Magnetic Field. When an electromagnetic wave passes through the ionosphere, the ions and electrons in the ionosphere are influenced by the field. The earth's magnetic field also affects the motion of the ions and electrons in the ionosphere. This forces the charged particles to move in spiral paths. This makes the ionosphere an anisotropic medium, that is, a medium which has different properties in different directions.

The force on a free electron moving with a velocity $\mathbf{v}$ in a permanent magnetic field $\mathbf{B_0}$ is given by

$$\mathbf{F} = -e\mathbf{v} \times \mathbf{B_0} \tag{7.2.24}$$

which forces the electron to rotate on a circular orbit. This force must be countered by the centrifugal force which gives

$$m_e\frac{v^2}{a} = evB_0 \tag{7.2.25}$$

where a is the radius of the circular orbit of the electron. Solving for a from this equation, we find

$$a = \frac{m_e v}{eB_0}. \tag{7.2.26}$$

The rotation period of the electron is then

$$T = \frac{2\pi a}{v} = \frac{2\pi m_e}{eB_0} \tag{7.2.27}$$

from which we can calculate the angular frequency as

$$\omega_c = \frac{2\pi}{T} = \frac{eB_0}{m_e} \tag{7.2.28}$$

which is called the *cyclotron frequency.*

Let us choose a coordinate frame in which the earth's magnetic field is $\mathbf{B_0} = B_0\mathbf{a}_z$. If an ac field $\mathbf{E}e^{j\omega t}$, $\mathbf{H}e^{j\omega t}$ is also present, the force acting on an electron will be (in phasor notation)

$$\mathbf{F} = -e\,(\mathbf{E} + \mathbf{V} \times \mathbf{B_0} + \mathbf{V} \times \mathbf{B})\,. \tag{7.2.29}$$

We also have

$$B = \mu_0 H = \mu_0 Y_0 E = \frac{E}{c} \tag{7.2.30}$$

where c is the speed of light. Thus, the force due to ac magnetic field is smaller than the force due to ac electric field by a factor of V/c, and can be neglected. Including the damping force due to collisions, we can write the equation of motion as

$$(j\omega + \nu)\, m_e \mathbf{V} = -e\left(\mathbf{E} + \mathbf{V} \times \mathbf{B}_0\right). \tag{7.2.31}$$

The motion of the electrons will cause an ac current. assuming N electrons per unit volume, we can write the current density as

$$\mathbf{J} = -eN\mathbf{V}. \tag{7.2.32}$$

Combining (7.2.31) and (7.2.32) we get

$$(j\omega + \nu)\,\mathbf{J} + \omega_c \mathbf{J} \times \mathbf{a}_z = \frac{e^2 N}{m_e}\mathbf{E} = \epsilon_0 \omega_p^2 \mathbf{E} \tag{7.2.33}$$

where we have used the definitions of plasma and cyclotron frequencies. In matrix form

$$\begin{bmatrix} j\omega + \nu & \omega_c & 0 \\ -\omega_c & j\omega + \nu & 0 \\ 0 & 0 & j\omega + \nu \end{bmatrix} \begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \epsilon_0 \omega_p^2 \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \tag{7.2.34}$$

which can be inverted to give

$$\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} = \frac{\epsilon_0 \omega_p^2}{\omega_c^2 + (\nu + j\omega)^2} \begin{bmatrix} j\omega + \nu & -\omega_c & 0 \\ \omega_c & j\omega + \nu & 0 \\ 0 & 0 & \frac{\omega_c^2}{j\omega+\nu} + j\omega + \nu \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}. \tag{7.2.35}$$

This equation defines the conductivity of the ionosphere as a tensor. Using tensor notation we can write

$$\mathbf{J} = \bar{\boldsymbol{\sigma}} \cdot \mathbf{E} \tag{7.2.36}$$

where

$$\bar{\boldsymbol{\sigma}} = \frac{\epsilon_0 \omega_p^2}{\omega_c^2 + (\nu + j\omega)^2} \begin{bmatrix} (j\omega + \nu)\left(\mathbf{a}_x \mathbf{a}_x + \mathbf{a}_y \mathbf{a}_y\right) \\ +\omega_c\left(\mathbf{a}_y \mathbf{a}_x - \mathbf{a}_x \mathbf{a}_y\right) + \frac{\omega_c^2 - (\omega - j\nu)^2}{j\omega+\nu} \mathbf{a}_z \mathbf{a}_z \end{bmatrix}. \tag{7.2.37}$$

From Maxwell's equation

$$\nabla \times \mathbf{H} = j\omega\epsilon_0 \mathbf{E} + \bar{\boldsymbol{\sigma}} \cdot \mathbf{E} \tag{7.2.38a}$$

$$= j\omega\epsilon_0 \left(\bar{\mathbf{I}} + \frac{\bar{\boldsymbol{\sigma}}}{j\omega\epsilon_0}\right) \cdot \mathbf{E} \tag{7.2.38b}$$

we see that the dielectric constant is also a tensor given by

$$\bar{\boldsymbol{\kappa}} = \bar{\mathbf{I}} + \frac{\bar{\boldsymbol{\sigma}}}{j\omega\epsilon_0} = \begin{bmatrix} \kappa_1 & -j\kappa_2 & 0 \\ j\kappa_2 & \kappa_1 & 0 \\ 0 & 0 & \kappa_3 \end{bmatrix} \tag{7.2.39}$$

where

$$\kappa_1 = 1 - \frac{(1 - j\nu/\omega)\,\omega_p^2}{\omega^2\,(1 - j\nu/\omega)^2 - \omega_c^2}, \tag{7.2.40a}$$

$$\kappa_2 = \frac{\omega_p^2\,(\omega_c/\omega)}{\omega^2\,(1 - j\nu/\omega)^2 - \omega_c^2}, \tag{7.2.40b}$$

$$\kappa_3 = 1 - \frac{\omega_p^2/\omega^2}{(1 - j\nu/\omega)}. \tag{7.2.40c}$$

If we assume a uniform plane wave solution in the ionosphere, the E and H fields will be in the form $\mathbf{E}e^{-j\mathbf{k}\cdot\mathbf{r}}$ and $\mathbf{H}e^{-j\mathbf{k}\cdot\mathbf{r}}$ where $\mathbf{k}$ is the propagation vector. The Maxwell's curl equations will give

$$-j\mathbf{k} \times \mathbf{E} = -j\omega\mu_0\mathbf{H}, \tag{7.2.41a}$$

$$-j\mathbf{k} \times \mathbf{H} = j\omega\epsilon_0\bar{\boldsymbol{\kappa}} \cdot \mathbf{E}. \tag{7.2.41b}$$

Taking the cross product of the first equation and substituting in the second gives

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) = \mathbf{k}\mathbf{k} \cdot \mathbf{E} - k^2\mathbf{E} = -\omega^2\epsilon_0\mu_0\bar{\boldsymbol{\kappa}} \cdot \mathbf{E} \tag{7.2.42}$$

which can be written as

$$\left(\mathbf{k}\mathbf{k} + k_0^2\bar{\boldsymbol{\kappa}} - k^2\bar{\mathbf{I}}\right) \cdot\mathbf{E} = 0. \tag{7.2.43}$$

Writing this equation in matrix form gives

$$\begin{bmatrix} k_x^2 + k_0^2\kappa_1 - k^2 & k_xk_y - jk_0^2\kappa_2 & k_zk_x \\ k_xk_y + jk_0^2\kappa_2 & k_y^2 + k_0^2\kappa_1 - k^2 & k_yk_z \\ k_zk_x & k_yk_z & k_z^2 + k_0^2\kappa_3 - k^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0. \tag{7.2.44}$$

A nontrivial solution exists if and only if the determinant of the coefficient matrix vanishes. Instead of trying to obtain general solutions, we will consider two special cases.

7.2.6.1. *Case 1: Propagation perpendicular to* $\mathbf{B_0}$. Let us choose $\mathbf{k} = k_x\mathbf{a_x} = k\mathbf{a}_x$. The matrix equation in (7.2.44) becomes

$$\begin{bmatrix} k_0^2\kappa_1 & -jk_0^2\kappa_2 & 0 \\ jk_0^2\kappa_2 & k_0^2\kappa_1 - k_x^2 & 0 \\ 0 & 0 & k_0^2\kappa_3 - k_x^2 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0. \tag{7.2.45}$$

If $E_z \neq 0$, we must have

$$\frac{k_1}{k_0} = \sqrt{\kappa_3}. \tag{7.2.46}$$

If the determinant of the 2×2 leading principle minor of the determinant is nonzero, we must have $E_x = E_y = 0$ which means $H_x = H_z = 0$. We then find

$$-\frac{E_z}{H_y} = \frac{1}{\sqrt{\kappa_3}}Z_0. \tag{7.2.47}$$

The wave in this case is not affected by the permanent magnetic field. Indeed, since $\mathbf{E}$ is in the same direction with $\mathbf{B_0}$, the electrons move in the direction of the magnetic field and are not deflected by $\mathbf{B_0}$.

If

$$\begin{vmatrix} k_0^2\kappa_1 & -jk_0^2\kappa_2 \\ jk_0^2\kappa_2 & k_0^2\kappa_1 - k_x^2 \end{vmatrix} = k_0^2\left(k_0^2\left(\kappa_1-\kappa_2\right)\left(\kappa_1+\kappa_2\right) - \kappa_1 k_x^2\right) = 0 \tag{7.2.48}$$

we can have a solution for which E_x and E_y are nonzero. This condition can be written as

$$\frac{k_2}{k_0} = \sqrt{\frac{\left(\kappa_1-\kappa_2\right)\left(\kappa_1+\kappa_2\right)}{\kappa_1}}. \tag{7.2.49}$$

Since (7.2.46) and (7.2.49) cannot be satisfied simultaneously (except for $\omega = 0$) we must have $E_z = 0$, which implies that $H_x = H_y = 0$. Then

$$\frac{E_y}{H_z} = \frac{k_0}{k_2} Z_0, \tag{7.2.50}$$

$$\frac{E_x}{E_y} = \frac{j\kappa_2}{\kappa_1}. \tag{7.2.51}$$

These results show that there are two distinct cases for propagation in the direction perpendicular to the earth's magnetic field. If $\mathbf{E}$ is parallel to $\mathbf{B_0}$, the wave is not affected by $\mathbf{B_0}$ and has the same phase velocity as if $\mathbf{B_0}$ were not present. This wave is called the *ordinary wave*. If $\mathbf{E}$ is perpendicular to $\mathbf{B_0}$, the phase velocity will be affected by $\mathbf{B_0}$. This wave is called the *extraordinary wave*. In general, a wave will have components both parallel and perpendicular to $\mathbf{B_0}$. Such a wave will split into ordinary and extraordinary waves which will travel different paths with different phase velocities and time delays. The extraordinary ray suffers greater absorption and has a slightly higher critical frequency.

7.2.6.2. *Case 2: Propagation parallel to* $\mathbf{B_0}$. Let us now assume that $\mathbf{k} = k_z\mathbf{a_z} = k\mathbf{a}_z$. The matrix equation in (7.2.44) for this case is

$$\begin{bmatrix} k_0^2\kappa_1 - k^2 & -jk_0^2\kappa_2 & 0 \\ jk_0^2\kappa_2 & k_0^2\kappa_1 - k^2 & 0 \\ 0 & 0 & k_0^2\kappa_3 \end{bmatrix}\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0. \tag{7.2.52}$$

Obviously, we must have $E_z = 0$, making $H_x = H_y = 0$. For a nontrivial solution we must have

$$\begin{vmatrix} k_0^2\kappa_1 - k^2 & -jk_0^2\kappa_2 \\ jk_0^2\kappa_2 & k_0^2\kappa_1 - k^2 \end{vmatrix} = \left(k_0^2\kappa_1 - k^2\right)^2 - \left(k_0^2\kappa_2\right)^2 = 0. \tag{7.2.53}$$

There are two possibilities to satisfy this equation:

$$\frac{k_3^2}{k_0^2} = \kappa_1 + \kappa_2 \rightarrow \frac{E_x}{E_y} = -j, \tag{7.2.54a}$$

$$\frac{k_4^2}{k_0^2} = \kappa_1 - \kappa_2 \rightarrow \frac{E_x}{E_y} = j. \tag{7.2.54b}$$

In both cases the field is circularly polarized but in opposite senses. The phase velocities of the two waves are different.

Any linearly polarized wave can be considered as a superposition of two circularly polarized waves rotating in opposite directions. With the above notation, we can write

$$\mathbf{E}_3 = E\left(\mathbf{a}_x - j\mathbf{a}_y\right)e^{-jk_3 z}, \tag{7.2.55a}$$

$$\mathbf{E}_4 = E\left(\mathbf{a}_x + j\mathbf{a}_y\right)e^{-jk_4 z}. \tag{7.2.55b}$$

Note that $\mathbf{E}_T = \mathbf{E}_3 + \mathbf{E}_4$ is a wave linearly polarized in the x direction at $z = 0$. When such a wave propagates in the direction of the earth's magnetic field in the ionosphere, the two circularly polarized components will propagate with different phase velocities. After a distance, say l, in the ionosphere the electric field is

$$\mathbf{E}_T\left(l\right) = E\left(\mathbf{a}_x - j\mathbf{a}_y\right)e^{-jk_3 l} + E\left(\mathbf{a}_x + j\mathbf{a}_y\right)e^{-jk_4 l} \tag{7.2.56}$$

which can be written as

$$\mathbf{E}_T\left(l\right) = 2Ee^{-j(k_3+k_4)l/2}\left(\mathbf{a}_x \cos\frac{\left(k_3 - k_4\right)l}{2} - \mathbf{a}_y \sin\frac{\left(k_3 - k_4\right)l}{2}\right). \tag{7.2.57}$$

Notice that the field is still linearly polarized, but in a different direction. The direction of polarization is ϕ relative to the x axis where

$$\phi = \frac{\left(k_4 - k_3\right)l}{2}. \tag{7.2.58}$$

That is, the polarization direction rotates at a constant rate in the ionosphere. This phenomenon is known as *Faraday rotation*.

7.3. Regular and Irregular Variations of the Ionosphere

Conditions in the ionosphere depend heavily on the solar radiation. This causes fairly regular variations throughout the day and with the season of the year. The F layer which has a height of about 300 km during the night, splits into two separate layers called $F1$ (lower) and $F2$ (upper). The E layer exists only during the day. At night its critical frequency falls below 1 MHz. Its virtual height remains constant at $110 - 120$ km and does not show any seasonal or yearly variations.

Since there are irregular variations over a day, the monthly averages of the critical frequencies and the virtual heights of the normal ionospheric layers are plotted for different seasons. Such plots are shown in Fig.s 7.8 and 7.9. The sunspot cycle affects the ionosphere and the ionospheric properties show variations in synchronism with the 11 year sunspot cycle. The critical frequencies are much higher during sunspot maxima. Typically, critical frequencies for both ordinary and extraordinary waves are plotted in such figures and are designated by a letter o for ordinary and x for extraordinary waves. For example $foF1$ would designate the critical frequency of the ordinary wave for the $F1$ layer, and fxE would designate the critical frequency of the extraordinary wave for the E layer. However, only critical frequencies for the ordinary waves are shown in Fig.s 7.8 and 7.9. The ionosphere varies also with seasons. In winter, the sun is always lower over the horizon than in summer. This affects the critical frequencies of the D, E and $F1$-layers, which are higher in summer since the solar energy per unit area is higher in summer than in winter. But this is the opposite for the $foF2$ at mid-latitudes that shows its greatest variation in winter. This difference is known as the "mid-latitude seasonal anomaly" and is basically due to the seasonal changes in the relative concentrations of atoms and molecules.

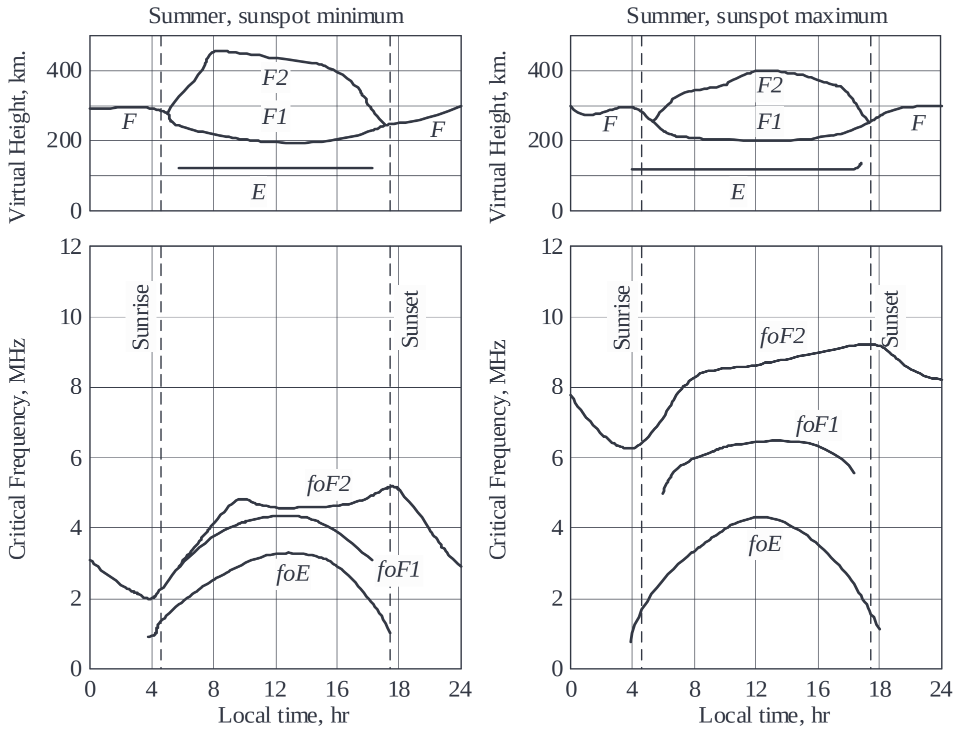

FIGURE 7.8. Monthly average of diurnal variations of critical frequency and virtual height of regular ionospheric layers for summer, (after [**3**])

In addition to the regular variations of the ionosphere, there are irregular and often unpredictable variations. One of these irregular variations is known as the *sudden ionospheric disturbance* (SID or Dellinger effect) which is caused by sudden bright eruptions on the sun. This will cause total radio fade out which may last from a few minutes to a few hours. The solar flares cause an increase in the ionization of the D layer which absorbs the higher HF frequencies. Only the very low frequencies can be reflected and actually, the sky wave strength of these signals increase during an SID. This effect always happens during day.

Sometimes a solar flare causes a gradual change in the ionization densities. In such cases the disturbance generally lasts longer, up to several hours. The absorption of the waves is not as complete as in the case of an SID and radio communication may be continued at higher frequencies.

A third type of irregularity is known as *ionospheric storms* which are caused by high energy electrons released by the sun. This makes the ionosphere turbulent and the normal stratification of the ionosphere is disturbed. As a result, the propagation of electromagnetic waves becomes very erratic. Generally, the communication can be maintained by lowering the frequency. Ionospheric storms generally occur at 27 day intervals which is the period of the rotation of sun about itself. This indicates that certain areas on the sun are more active. Ionospheric storms may cause damage to power distribution lines and may result in blackouts.

Another type of irregularity occurs only in polar regions during a sunspot maximum and is known as *polar cap absorption* (PCA). This phenomenon is associated with release of

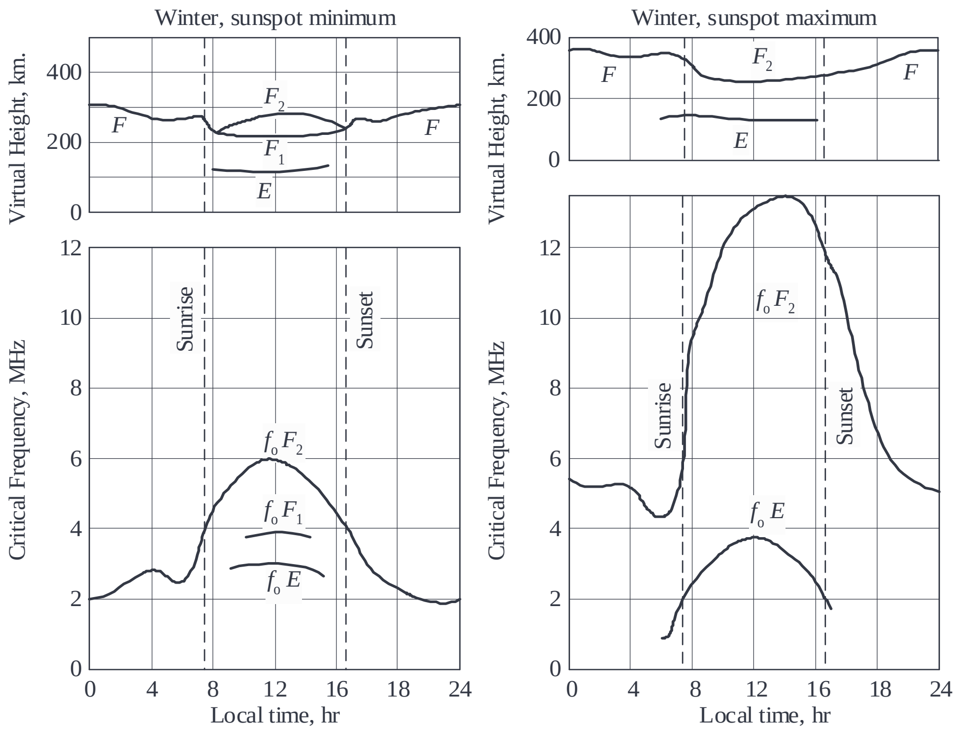

FIGURE 7.9. Monthly average of diurnal variations of critical frequency and virtual height of regular ionospheric layers for winter, (after [**3**])

high-energy protons from solar flares. These particles can hit the earth within 15 minutes to 2 hours after the solar flare. The protons spiral around and down the magnetic field lines of the earth and penetrate into the atmosphere near the magnetic poles increasing the ionization of the D and E layers. PCA's typically last anywhere from about an hour to several days, with an average of around 24 to 36 hours.

7.4. HF Ionospheric Link Calculations

The design and calculation of HF ionospheric links is basically a two step procedure. The first step is to determine the link structure and choose the optimum working frequency, and the second step is to estimate the field strength.

For the first step, ionospheric maps are used. These maps give the MUF for various predetermined distances and are designated as MUF-3000 or MUF-4000, depending on the distance. The MUF-0 is actually a map of critical frequencies. Of course, the maps for $F2$ and E layer are different, and the maps vary with time of day. It must be noted that the MUF at the reflection point (or points if more than one hop is used) must be considered. For multiple hop links, the MUF at each reflection point must be calculated and the smallest of these values must be used since the reflection conditions are decided by the one with the lowest MUF. The optimum working frequency is chosen as 10% lower than the MUF to ensure continuous working despite unforeseen fluctuations in ionosphere conditions. The Institute for Telecommunication Sciences (a U.S government agency NTIA/ITS) provides software *free of* *charge, as is* to be used for any purpose at the web site http://elbert.its.bldrdoc.gov/hf.html. The software HFWIN32 for Windows-NT/2000/XP can be used for HF circuit calculations.

Although the ionosphere generally behaves very regularly, it is a random medium. The field strength has regular and random variations. Therefore, as in the case of tropospheric propagation, median values of the field strength is estimated. Although there are several methods for median field strength prediction, we will consider a simple method described in [**19**]. In this method, the received field is a combination of the waves that arrive at the receiver after different numbers of reflections. The total field is then given by

$$E_{rms} = \sqrt{\sum_{i=1}^{m} E_{rms(i)}^2} \tag{7.4.1}$$

where m is the number of rays (not the number of hops). The 1st ray is the intended ray and the rays are sorted in increasing number of reflections. Therefore, it is proposed that m should be less than three.

The median value of attenuation function for the each ray is given by

$$F_m = \frac{1}{2}\frac{1+R}{2}R^{n-1}\exp\left[-\sum_{j=1}^{n}\Gamma_j\right] \tag{7.4.2}$$

where R is the modulus of the reflection coefficient for the earth's surface, n is the number of reflections from the ionosphere, and Γ_j is the absorption coefficient at the jth reflection point.

The coefficient 1/2 in (7.4.2) represents a 6 dB attenuation. 3 dB of this factor is due to the receiving antenna being linearly polarized while the wave becomes elliptically polarized in the ionosphere. The remaining 3 dB is due to the losses caused by the wave splitting into ordinary and extraordinary rays.

The term $(1+R)/2$ accounts for the effect of ground reflected wave at the receiving end. In practice, the beam of the receiving antenna cannot be directed exactly in the direction of the incoming wave, since the height of the reflecting layer is not exactly known and/or may change due to variations in the ionospheric conditions.

The next term in (7.4.2) accounts for the losses due to reflections from the ground in a multi-hop link.

The fourth term takes the ionospheric absorption into account. The absorption coefficient at the jth reflection point is the sum of the absorption coefficients due to different layers of the ionosphere given by

$$\Gamma_j = \Gamma_D + \Gamma_E + \Gamma_{F1} + \Gamma_{F2}. \tag{7.4.3}$$

These coefficients can be calculated using (7.2.21) as $\Gamma_x = -\alpha_x l_x$ where α_x and l_x are the attenuation constant and path length of the ray in respective layers. If the reflection occurs from lower layers, the contributions from higher layers must be omitted. For layers below the reflection layer, approximations in (7.2.22) and (7.2.23) can be used. For the reflection layer, the assumption $\kappa' = 1$ is not valid since at the reflection height we need to have $\kappa' = 0$. The latter case is called *deviative* absorption while the former case is called *non-deviative* absorption.

The ITU-R Recommendation P.533-10, [**53**] gives a detailed procedure for the computation of median field strength. The procedure uses empirical formulations from the fit to measured data. A computer program (REC533) associated with the prediction procedures described in ITU-R Recommendation P.533-10 is available from that part of the ITU-R website dealing with Radiocommunication Study Group 3. This procedure is also included in HFWIN32.

7.5. Examples

EXAMPLE 27. *Determine the velocity v that an electron should have in order to ionize molecular oxygen by collision.*

SOLUTION 27. *Equating the kinetic energy of the electron to the ionization energy we get*

$$\frac{1}{2}m_e v^2 = 12.18\text{ eV} = 1.95 \times 10^{-18}\text{ J} \tag{7.5.1}$$

from which we find $v = 2.07 \times 10^6\text{ m/s}$. *This velocity is only* 0.69% *of the speed of light and therefore relativistic correction is not necessary.*

EXAMPLE 28. *Assume that the MUF for a radio link of* 2000 km *is* 30.6 MHz *and the virtual height is* 200 km. *Calculate the critical frequency of the reflecting region.*

SOLUTION 28. *For distance d and a virtual height h', the incidence angle can be solved by using (7.2.15) and (7.2.16) as*

$$\sin\psi_i = \frac{\sin\frac{d}{2a_e}}{\sqrt{\left(\frac{h'}{a_e}\right)^2 + 4\left(1+\frac{h'}{a_e}\right)\sin^2\frac{d}{4a_e}}} \tag{7.5.2}$$

which gives $\psi_i = 1.317\text{ rad}$. *Using (7.2.14) we find*

$$f = \frac{30.6}{\sec 1.317} = 7.68\text{ MHz}. \tag{7.5.3}$$

EXAMPLE 29. *Assuming that the D layer has an electron density of* $N = 4 \times 10^8\ \text{m}^{-3}$, *determine the frequency at which the index of refraction becomes* 0.5. *Ignore collisions of the electrons.*

SOLUTION 29. *Using (7.2.11) we can write*

$$0.5 = \sqrt{1 - 81\frac{4 \times 10^8}{f^2}} \tag{7.5.4}$$

which yields $f = 0.208\text{ MHz}$.

EXAMPLE 30. *Determine the minimum electron densities corresponding to critical frequencies of* 2.5 MHz *and* 8.5 MHz.

SOLUTION 30. *The critical frequency and electron density are related by (7.2.13). Solving the electron density gives* $N = f_c^2/81$. *Thus* $N = 7.72 \times 10^{10}\text{ m}^{-3}$ *for* $f_c = 2.5\text{ MHz}$ *and* $N = 8.92 \times 10^{11}\text{ m}^{-3}$ *for* $f_c = 8.5\text{ MHz}$.

EXAMPLE 31. *Assume that an HF communication link between two points on earth at a distance of* 2500 km *is to be established using reflections from* $F1$ *layer with a virtual height of* 200 km*. The critical frequency is* 5 MHz*. Determine the maximum usable frequency.*

SOLUTION 31. *For this geometry the incidence angle is* $\psi_i = 1.341\,\mathrm{rad}$*. Then the maximum usable frequency is*

$$f = f_c \sec \psi_i = 21.95\,\mathrm{MHz} \tag{7.5.5}$$

EXAMPLE 32. *Assume that the maximum skip distance for* $f = 30\,\mathrm{MHz}$ *is* 3400 km*. Determine the virtual height and the critical frequency of the reflecting layer.*

SOLUTION 32. *From (7.2.17) we can determine the virtual height as*

$$h' = \frac{d_{\max}^2}{8a_e} = 170\,\mathrm{km}. \tag{7.5.6}$$

The value of α *for maximum skip incidence is zero. Then from (7.2.18) the incidence angle will be*

$$\psi_i = \sin^{-1} \frac{a_e}{a_e + h'} = 1.372\,\mathrm{rad} \tag{7.5.7}$$

and from (7.2.14) the critical frequency is found as

$$f_c = \frac{f}{\sec \psi_i} = 5.92\,\mathrm{MHz}. \tag{7.5.8}$$

EXAMPLE 33. *Determine the link geometry and basic parameters of an HF communication link with a great circle nominal distance of* 6800 km*.*

SOLUTION 33.

SOLUTION 34. *The virtual height of the reflecting layer to have* 6800 km *skip distance is* $h' = d^2/(8a_e) = 680\,\mathrm{km}$*. However, this is above the* $F2$ *layer. Therefore the required service must be based on a double hop with* 3400 km *each. Using a nominal virtual height value of* 300 km*, we find* $\psi_i = 1.299\,\mathrm{rad} = 74.46\,^\circ$*. This gives* $\alpha = 4.08\,^\circ$*, i.e., the transmitting and receiving antennas must have their direction of maximum radiation (horizon angle) above the horizon by this amount. The electron concentration of the* F *layer is about* $5\times10^{11}\,\mathrm{m}^{-3}$ *under daytime conditions. The critical frequency is then* $f_c = 9\sqrt{N} = 6.36\,\mathrm{MHz}$ *and the maximum usable frequency is* $f_{MUF} = 6.36 \sec \psi_i = 23.75\,\mathrm{MHz}$*. The operational frequency can be chosen about* 21.3 MHz*. This corresponds to international* 13 m *band (*21.450 *to* 21.850 MHz*). But the seasonal and diurnal variations will cause interruptions in the communication. Generally, this band has somewhat shaky day reception, and very little night reception. The link may be used only by adjusting the frequency several times during the day. During the night, multiple hops should be considered to secure communication.*

EXAMPLE 34.

SOLUTION 35. *To direct the main beam in the desired direction, one can use an antenna at a height* h_a *above the ground. The antenna and its image forms a two antenna array with a separation of* $2h_a$*. Typically vertically polarized antennas are used thus the antenna and its image are in phase. The array pattern will then be given by*

$$f(\theta) = A \cos(kh_a \sin\theta) \tag{7.5.9}$$

where θ is measured from the local horizon. For this pattern to have a maximum in the direction α we must have

$$kh_a \sin\alpha = n\pi \tag{7.5.10}$$

which gives for $h_a = 99\,\mathrm{m}$ (for $n = 1$).

Note that this solution is overly simplified. Actually, the variation of MUF during the day at the location of reflection should be considered, which is affected by the time of day, time of year, and solar activity.

EXAMPLE 35. *A short-wave broadcasting service is to be established over a distance of $6000\,\mathrm{km}$ in three hops. Assume that the reflection takes place at a height of $250\,\mathrm{km}$ and that the electron density is $9 \times 10^{12}\,\mathrm{m}^{-3}$. Determine the maximum usable frequency, the operational frequency and the angle of incidence.*

SOLUTION 36. *The solution procedure is similar to the previous example. We find $\psi_i = 72.80\,^\circ$, $\alpha = 10.46\,^\circ$, $f_{MUF} = 27\,\mathrm{MHz}$, $f_{OWF} = 24.3\,\mathrm{MHz}$ ($11\,\mathrm{m}$ band).*

EXAMPLE 36. *A plane wave of frequency $15\,\mathrm{MHz}$ propagates through a distance of $200\,\mathrm{km}$ in the ionosphere with an average electron density of $5 \times 10^{10}\,\mathrm{m}^{-3}$. Assuming that the collisions are negligible, find the amount of Faraday rotation produced. The earth's magnetic field is $30\,\mu\mathrm{T}$.*

SOLUTION 37. *The cyclotron frequency is calculated from (7.2.28) as $\omega_c = eB_0/m_e = 7.91 \times 10^6\,\mathrm{rad/s}$. The plasma frequency is calculated as $\omega_p = e\sqrt{N/\epsilon_0 m_e} = 1.26 \times 10^7\,\mathrm{rad/s}$. If we neglect the collisions, (7.2.40) reduces to*

$$\kappa_1 = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2} = 0.982\ , \tag{7.5.11a}$$

$$\kappa_2 = \omega_p^2 \frac{\omega_c}{\omega\left(\omega^2 - \omega_c^2\right)} = 1.51 \times 10^{-3}. \tag{7.5.11b}$$

Thus, from (7.2.54) we find

$$k_3 = \sqrt{0.983\,51}\frac{2\pi}{\lambda}; \qquad k_4 = \sqrt{0.980\,49}\frac{2\pi}{\lambda}. \tag{7.5.12}$$

The wavelength is $20\,\mathrm{m}$ and the amount of Faraday rotation can be calculated as

$$\phi = \frac{(k_3 - k_4)\,l}{2} = 4.79\,\mathrm{rad} = 274\,^\circ.$$

EXAMPLE 37. *A plane wave is propagating in the direction of earth's magnetic field in the ionosphere with a cyclotron frequency of $f_c = 1.2\,\mathrm{MHz}$. The frequency of the wave is $6\,\mathrm{MHz}$ and the average electron density is $10^{11}\,\mathrm{m}^{-3}$. Assuming a collision frequency of $\nu = 10^3$ determine the attenuation constants for the left and right circularly polarized waves.*

SOLUTION 38. *Using (7.2.40) we first find*

$$\kappa_1 = 1 - \frac{(1 - j\nu/\omega)\,\omega_p^2}{\omega^2 (1 - j\nu/\omega)^2 - \omega_c^2} = 0.658 - 1.22 \times 10^{-5} j, \tag{7.5.13a}$$

$$\kappa_2 = \frac{\omega_p^2 (\omega_c/\omega)}{\omega^2 (1 - j\nu/\omega)^2 - \omega_c^2} = 8.210 \times 10^{-2} + 5.55 \times 10^{-6} j, \tag{7.5.13b}$$

$$\kappa_3 = 1 - \frac{\omega_p^2/\omega^2}{(1 - j\nu/\omega)} = 0.678 - 1.03 \times 10^{-5} j. \tag{7.5.13c}$$

and from (7.2.54) we get

$$k_3 = \sqrt{(\kappa_1 + \kappa_2)}\frac{2\pi}{50} = 0.108 - 4.857 \times 10^{-7} j, \tag{7.5.14}$$

$$k_4 = \sqrt{(\kappa_1 - \kappa_2)}\frac{2\pi}{50} = 9.536 \times 10^{-2} - 1.470 \times 10^{-6} j. \tag{7.5.15}$$

The propagation constant of the left hand polarized wave is k_3 *and that of the right hand polarized wave is* k_4. *Thus, the attenuation constant for the left hand polarized wave is* $4.857 \times 10^{-7} Np/\mathrm{m}$ *or* $4.22 \times 10^{-3} dB/\mathrm{km}$, *while the attenuation constant for the right hand polarized wave is* $1.470 \times 10^{-6} Np/\mathrm{m}$ *or* $1.28 \times 10^{-2} dB/\mathrm{km}$.