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CHAPTER 6

Effect of the Troposphere

The troposphere is the lowest portion of earth's atmosphere extending from the surface up to a height of approximately 17 km in the middle latitudes. It is deeper at the equator and shallower near the poles. It contains approximately 75% of the atmosphere's mass and 99% of its water vapor and aerosols. Most of the phenomena we associate with day-to-day weather occur in the troposphere.

The most important property of the troposphere as far as radio wave propagation is concerned, is that its temperature decreases with height. The average vertical temperature gradient of the troposphere is 6 °C/km. The troposphere is almost completely transparent to sun rays. Therefore, sun rays cannot heat the troposphere while passing through it. The solar energy is absorbed by the earth's surface, which in turn radiates thermal energy heating the troposphere in an upward direction, which causes the negative temperature gradient. The temperature difference between the layers of the troposphere results in convective air motion, which is the main reason of meteorological phenomena.

The average pressure at the earth's surface is 1014 mbar. The pressure also decreases with height and the typical value of the pressure gradient in the troposphere is 9 Pa/m (90 mbar/km). At an altitude of 5 km, the pressure is nearly halved, at 11 km it is about 225 mbar.

The troposphere is relatively rich in water vapor content which is caused by the evaporation of water from the surface of the oceans, seas and water reservoirs. The water vapor content also decreases with height. At an altitude of 1.5 km, the water vapor content is about one half and at the upper boundary of the troposphere it drops to only a few thousandths of its value at the earth's surface.

The three parameters discussed above, namely temperature, pressure, and water vapor content determines the refractivity of the troposphere. As we have discussed previously in chapter 3, this typically causes the radio waves to bend downward as they propagate in the troposphere. However, this is not always the case.

6.1. Forms of Atmospheric Refraction

The atmospheric refractivity is related to the refractive index by the formula

$$n = 1 + N \times 10^6 \tag{6.1.1}$$

where the refractivity N is given by, [**20**],

$$N = \frac{77.6}{T}\left(P + 4810\frac{e}{T}\right). \tag{6.1.2}$$

Here, P is the atmospheric pressure in hPa, e is the water vapor pressure in hPa, and T is the absolute temperature in K. The variation of the refractivity with height is typically modeled by an exponential function (reference atmosphere). However, the exponential refractivity profile is harder to handle and a constant refractivity gradient (standard atmosphere) is commonly used in the literature, which gives quite good results in the troposphere. In the standard atmosphere model, the value of the refractivity gradient is

$$\frac{dN}{dh} = -4 \times 10^{-2}\,\mathrm{m}^{-1}. \tag{6.1.3}$$

The curvature of the rays as they propagate in a layered atmosphere can be expressed as (see eq. (3.3.11))

$$R = \frac{10^6}{-dN/dh}\,\mathrm{m}. \tag{6.1.4}$$

In terms of this quantity, we can define an effective earth radius as

$$a_e = \frac{1}{1 - a/R} \tag{6.1.5}$$

where a is the physical radius of the earth.

Weather conditions in the troposphere may lead to a refractive index distribution with height which is materially different from its average state. Such changes in the refractivity gradient cause different forms of refraction. The possible forms of refraction are listed in Table 6.1.

If the refractivity gradient is positive, the rays bend upwards and the refraction is referred to as negative. If the refractivity does not change with height, the rays will propagate along lines and this case is called zero refraction. Occurrence of negative or zero refraction is infrequent but such possibilities must be considered.

Positive refraction is the name given to the case when the refractivity decreases with height, and this is the most typical case. The rays bend downwards. This case is divided into five cases. If the bending of the rays is smaller than the standard case, it is called sub-refraction. When the bending of the rays are such that their radius of curvature is the same as the earth's radius, it is called critical refraction. Augmented refraction is the case between critical and standard refraction. The ray bending is more than the standard case. Super refraction occurs when the ray bending is more pronounced than the critical refraction.

The critical refraction occurs when $R = a$. Substituting the numerical values in (6.1.4) we find

$$\frac{dN}{dh} = -\frac{10^6}{6375000} = -0.157\,\mathrm{m}^{-1}\,. \tag{6.1.6}$$

The effective earth radius becomes infinitely large, that is, the equivalent earth becomes a plane. Under critical refraction conditions, a horizontal ray will propagate at constant height.

With super refraction, the radius of curvature of the ray is more than the earth's radius. Rays leaving the transmitter at small elevation angles will undergo a total internal reflection and return back to the earth at some distance from the transmitter. The wave will then be reflected by the earth's surface and will undergo a total internal reflection and return back to the earth again. The waves become trapped in this way and may undergo several reflections giving rise to abnormally large communication distances beyond line of sight. This phenomenon is known as *ducting*. The occurrence of ducting is quite frequent and deserves further investigation.

TABLE 6.1. Forms of atmospheric refraction.

Form of refraction	$dN/dh\,(\mathrm{m}^{-1})$	$\mathbf{R}\,(\mathrm{km})$	$a_e\,(\mathrm{km})$	$k = a_e/a$
Negative	> 0	< 0	< 6375	< 1
Zero	0	∞	6375	1
Positive				
Sub-refraction	0 to -0.04	∞ to 25000	6375 to 8500	1 to 4/3
Standard	-0.04	25000	8500	4/3
Augmented	-0.04 to -0.157	25000 to 6375	8500 to ∞	4/3 to ∞
Critical	-0.157	6375	∞	∞
Super-refraction	< -0.157	< 6375	< 0	< 0

6.2. Super-Refraction and Ducting

The refractivity gradient can be obtained by differentiating (6.1.2) with respect to height:

$$\frac{dN}{dh} = 77.6\left(\frac{1}{T}\frac{dP}{dh} - \left(\frac{9620e}{T^3} + \frac{P}{T^2}\right)\frac{dT}{dh} + \frac{4810}{T^2}\frac{de}{dh}\right). \tag{6.2.1}$$

The pressure always decreases with height and its gradient depends on the weather conditions only very slightly. Therefore, the first term is always negative and is nearly a constant. The temperature and humidity distributions depend strongly on the weather conditions and large variations in their gradients are possible. Under normal conditions dT/dh and de/dh are both negative. Their values may change drastically, and may even become positive under certain conditions. Temperature inversion is the condition when dT/dh is positive. The humidity gradient may become positive inside clouds and moisture pockets. Thus, super-refraction conditions are mainly determined by temperature and humidity gradients. Among these two factors, temperature inversions is the decisive one.

When the refraction conditions differ from the standard atmosphere, significant changes in propagation occur. Such conditions are known as *anomalous*, or *nonstandard* propagation. In dealing with nonstandard refraction phenomena, a *modified refractivity* which takes the earth's curvature into account is defined as

$$M = N + \frac{h}{a} \times 10^6. \tag{6.2.2}$$

Its use lies in the fact that critical refraction occurs when dM/dh is zero negative gradients of M indicates super-refraction. A plot of modified refractivity versus altitude is visually useful for identifying nonstandard propagation conditions, especially ducting.

6.2.1. Evaporation Ducts. The standard refractive conditions are seldom encountered in the sea environment. The most common case is the evaporation duct. The air in contact with the sea surface is saturated with water vapor. The meteorological conditions determine the ambient value of the humidity and this value is reached within several meters above the surface. The rapid decrease in the humidity level is accompanied by a rapid decrease in the refractivity that results in the formation of a low lying surface duct. Such a duct may cause extended communication ranges considerably greater than free space range. Normally, both the transmitter and the receiver must be within the duct layer to benefit from the extended communication range. For example, if a transmitter is located below the height of the evaporation duct, it may be able to communicate over the horizon with a receiver that is also located within this duct. However, if the receiver is located instead above the height of the duct, it may have difficulty receiving the transmitted signal, even at ranges within the normal radio horizon.

Evaporation ducts are almost always present, yet their heights are highly variable in space and time. The height of the duct is a measure of its strength (not the height below which the antennas must be located to have extended propagation) and depends on location, season, time of day, and wind speed. Evaporation duct heights are typically in the range of 6 to 30 m.

A knowledge about the presence and parameters of an evaporation duct is important, especially in order to estimate the detection range of marine radars. There is no direct method for measuring the duct height. Instead, theoretical models based on the meteorological data are used, [**5**]. One such model is given in [**45**].

The lowest frequency that can be propagated by a duct is a function of the duct height. The lowest frequency that can be propagated within an evaporation duct is said to be 3 GHz. Table 6.2 gives a rough guide for the lowest frequency that can be trapped by a given duct height based on [**4**]. However, as the frequency is increased, the path loss is also increased due to high water vapor content.

TABLE 6.2. Minimum duct height required for propagation, [**4**].

Duct Height	**Minimum frequency that can be propagated**
25 m	3 GHz
14 m	7 GHz
10 m	10 GHz
6 m	18 GHz

The presence of an evaporation duct increases the range in the diffraction region. In the interference region, the main effect is the increased ray curvature which causes the interference maxima to occur at lower heights. Near the horizon range, the presence of duct decreases the signal level, as compared to no duct case.

6.2.2. Surface Based Duct. Surface based ducts occur when the upper air is exceptionally warm and dry as compared to lower air. A body of warm and dry air may be blown over a cooler air, or ground which cools the lower air and results in a temperature inversion. This may occur in the spring time when large masses of air are blown from the warmer regions of earth over the ground still covered by snow. More commonly, the land is heated faster than the sea in day time and the warm, dry air is blown over sea. Furthermore, the water content of the lower air is increased by evaporation and drops of water lifted from the crests of the waves causing a humidity gradient.

Another cause of surface based ducts is the fast cooling of land surfaces on clear nights, especially in the summer time. The earth's surface loses heat and the surface temperature drops, but there is little or no change in the temperature of the upper atmosphere, resulting in a temperature inversion.

The common property of these two type of duct is that the inversion starts from the surface, hence the base of the duct touches the surface. Typically, the height of surface based ducts is below a few hundred meters. The increased height of these ducts (as compared to evaporation ducts) implies that lower frequencies can be trapped. Long range propagation can occur at frequencies exceeding 100 MHz, and the propagation is relatively insensitive to frequency.

6.2.3. Elevated Ducts. The base of an elevated duct is above the earth's surface. Elevated ducts commonly occur in the trade wind region. The high pressure regions present in the trade wind region causes the high altitude air to sink slowly to meet the low altitude maritime air. This leads to warmer drier air above a cooler moist air, thus forming a duct. The base of the trade wind elevated ducts may range from hundreds of meters to thousands of meters. Their thicknesses may vary from a few meters to several hundred meters. Thicker ducts can support propagation of frequencies as low as 100 MHz. Trade wind ducts can occur about 40% of the time and give rise to strong persistent propagation throughout most of the year over at least one third of the ocean, [**46**].

In some cases, the temperature inversion layer causes a stratus cloud layer to form. In such cases, the elevated duct layer can be visually identified. In some other cases, the temperature inversion can be identified by a haze layer formed below it.

Figure 6.1 shows the plots of modified refractivity versus altitude for different propagation cases.

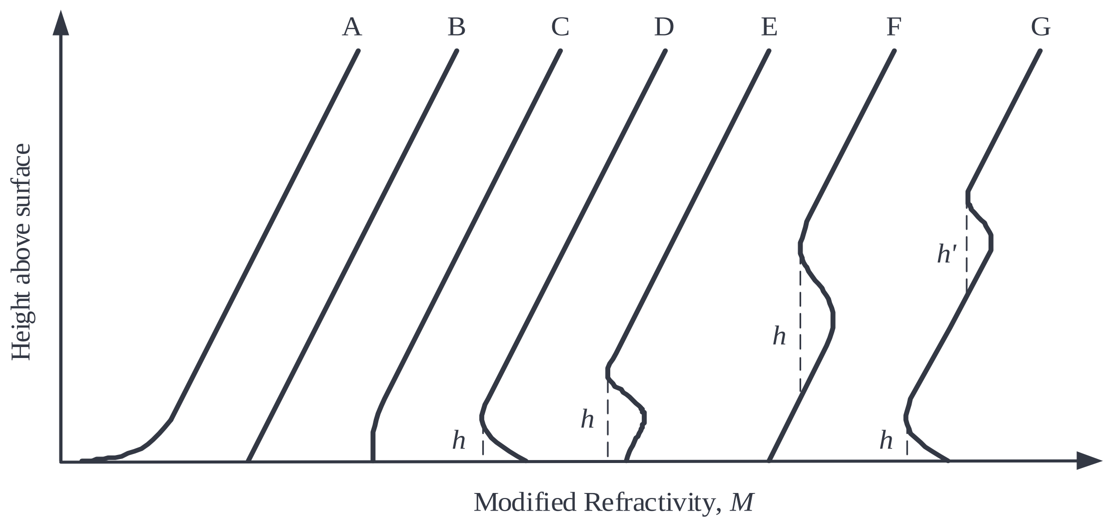

FIGURE 6.1. Idealized modified index profiles: (A) Substandard surface layer; (B) profile for standard refraction; (C) superrefractive surface layer; (D) superrefractive surface layer with a surface duct; (E) elevated superrefractive layer with surface duct; (F) elevated superrefractive layer with elevated duct; (G) surface and elevated superrefractive layers with both surface and elevated ducts. In all cases the duct extends along the dashed lines.

6.3. Fluctuations in the Troposphere

The models discussed so far are time invariant models. Actually, there are lots of things that are changing with time in a propagation environment. For ground waves and ground reflected waves, the surface or reflection point may change its characteristics due to wind blown motion of vegetation in rural areas, and due to wave motion over sea surfaces. Motion of vehicles and people may effect the propagation path in an urban environment. One can think of many other factors that cause variations in received signal strength. In many cases the 2-D model that we have considered so far is inadequate. Waves may be reflected and/or diffracted from other objects such as mountains and buildings. This will give rise to a field due to interference of three or more waves. The phase and amplitude variations in each path will cause variations in the field strength. One other cause of signal variation is the changes in the state of troposphere due to slow changes in the weather conditions. At longer wavelengths and for line of sight paths, these changes are relatively slow and the signal strength will remain almost constant. Therefore the models previously discussed give quite accurate results. However, as the wavelength is decreased, small changes in the environment (comparable to wavelength) will change the phase of the waves arriving at the receiver through different paths resulting in relatively large changes in the signal strength. The variations in the signal level is called fading and will be discussed later in Chapter **??**.

6.4. Tropospheric Scattering

The diffraction region equations discussed in Chapters 4 and 5 shows that the attenuation factor decreases rapidly with distance, especially at shorter wavelengths. However, it has been observed that the fields of transmitters in the diffraction region are much larger than predicted by the diffraction theory. The main characteristic of the fields observed over long distances is that they are subject to fading, that is, random variations in signal strength caused by changes in the propagation medium.

The troposphere is actually a random medium. The refractivity models that we have considered so far only give the mean value of the refractivity. While the refractivity basically follows this model in the mean, its actual value changes quite randomly about the mean. The twinkling of stars and far away lights, the wavering appearances of objects seen over warmer regions (such as cities, earth's surface heated by the sun), the motion of smoke leaving chimneys are all facts that indicate the randomness of the air in the troposphere. The discontinuities in the refractive index causes the waves to be scattered while they propagate through the troposphere.

Let us assume that the irregularities in the refractive index form small scatterers in the troposphere as shown in Fig. 6.2. This gives us a mechanism to understand the propagation of short wave lengths beyond the horizon due to tropospheric scattering. Consider a small volume dV at point Q that can be considered as a scatterer. Let us define σ as the scattering cross section per unit volume in the troposphere. It must be noted that σ is the bi-static cross section and will change with the scattering angle θ. Also, scattering cross section has the unit of area which means that scattering cross section per unit volume will have the unit $\mathrm{m}^2/\mathrm{m}^3$ (or m^{-1}). Then $\sigma(\theta)\,dV$ will give us the scattering cross section of the volume dV. By definition, the power re-radiated by the volume dV will be the product of the incident power density, say S_Q by the scattering cross section $\sigma(\theta)\,dV$.

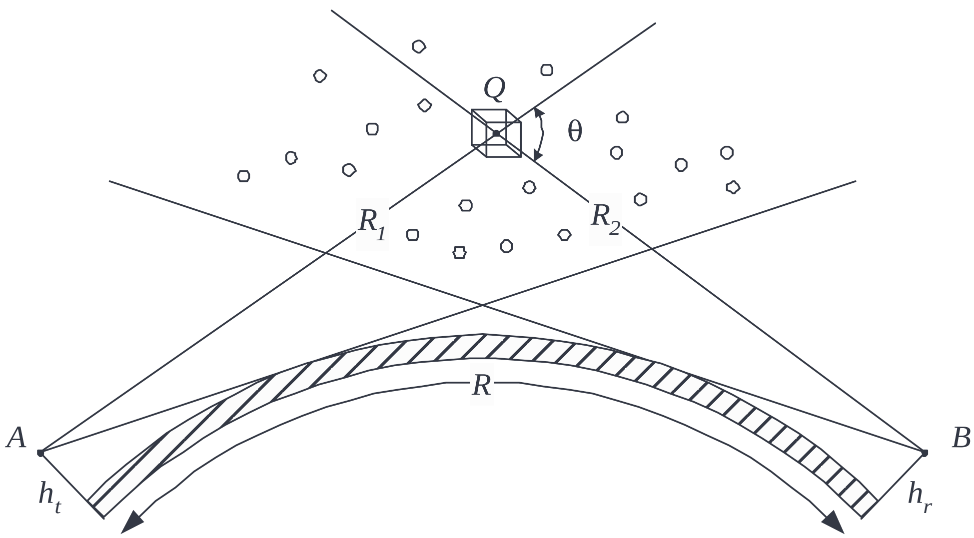

FIGURE 6.2. Tropospheric scatter radio link.

If we denote the transmitter power by P_t and the transmitter antenna gain by G_t the power incident power density at Q will be

$$S_Q = \frac{P_t G_t}{4\pi R_1^2}\,\mathrm{W/m^2} \tag{6.4.1}$$

and the power re-radiated by the volume dV in the direction of receiver will be

$$dS_Q = \frac{P_t G_t}{4\pi R_1^2}\sigma(\theta)\,dV\ \ \mathrm{W}. \tag{6.4.2}$$

The power density at the location of the receiver due to the volume dV will be

$$dS_B = \frac{dS_Q}{4\pi R_2^2}\,\mathrm{W/m^2}. \tag{6.4.3}$$

The power extracted by the receiving antenna will be

$$dP_r = dS_B A_r\,\mathrm{W} \tag{6.4.4}$$

where A_r is the effective aperture of the receiver antenna. Using the fact that effective aperture is related to gain by

$$A_r = \frac{\lambda_0^2}{4\pi}G_r\ \ \mathrm{m^2} \tag{6.4.5}$$

and combining the above results we get

$$dP_r = \frac{P_t G_t G_r \lambda_0^2}{64\pi^3 R_1^2 R_2^2}\sigma(\theta)\,dV\ \ \mathrm{W}. \tag{6.4.6}$$

The total power due to all scatterers in the troposphere is obtained by integrating this result over all scatterers, or equivalently over the relevant volume of troposphere as

$$P_r = \frac{P_t G_t G_r \lambda_0^2}{64\pi^3}\int_V \frac{\sigma(\theta)}{R_1^2 R_2^2}dV\ \ \mathrm{W}. \tag{6.4.7}$$

The total power that would be received if the propagation were in free space is

$$P_{r,fs} = \frac{P_t G_t G_r \lambda_0^2}{(4\pi R)^2} \text{ W}. \tag{6.4.8}$$

The attenuation factor is the square root of the ratio of P_r to $P_{r,fs}$. Thus, we can write

$$F = \frac{R}{2\sqrt{\pi}}\sqrt{\int_V \frac{\sigma(\theta)}{R_1^2 R_2^2} dV}. \tag{6.4.9}$$

To calculate the propagation factor, we need to determine the volume V and the distribution of the bi-static radar cross section $\sigma(\theta)$ over this volume. The volume V is that volume of the troposphere seen by both antennas. When the receiving and transmitting antennas are highly directive, R_1 and R_2 may be assumed to be constant and equal to $R/2$ within the scattering volume. Furthermore, if the volume is very small, $\sigma(\theta)$ may be assumed to be constant over the volume. Then we get

$$F = \frac{2}{R\sqrt{\pi}}\sqrt{V\sigma(\theta)}. \tag{6.4.10}$$

Although the approximations made in deriving this result seem to be very crude, they are valid in tropospheric scattering links and the final expression turns out to be a good approximation. In fact, the fading of the signal in tropo-scatterer links is too deep and the average value of the signal changes with time. The value of the attenuation factor is expected to give a long time average which may change by several tens of dB. Therefore, such crude approximations are acceptable in tropospheric scatterer link calculations.

When the antenna gains are small, the volume V is bounded by the tangent rays from below and the upper boundary of the troposphere which is approximately 17 km in the middle latitudes. The length of the arc seen by the main beam of an antenna with a beam width of 20° at a range of 50 km is approximately 17.5 km. Therefore, 20° beam width may be considered as the upper limit to assume that the antennas are high gain.

6.4.1. Determination of $\sigma(\theta)$. The troposphere is a *turbulent* medium in which the local air flow changes with time and position randomly both in direction and magnitude. Therefore, the permittivity may be written as

$$\kappa'(\mathbf{r}, t) = \kappa'_0 + \Delta\kappa'(\mathbf{r}, t) \tag{6.4.11}$$

where $\mathbf{r}$ is the position vector, t is the time, κ'_0 is the average permittivity, and $\Delta\kappa'(\mathbf{r}, t)$ is a zero mean random process that describes the random variations due to turbulent medium. Assuming that $\Delta\kappa'(\mathbf{r}, t)$ is stationary (in space), the spatial correlation function

$$C(\rho) = \frac{\overline{\Delta\kappa'(\mathbf{r}_1, t)\Delta\kappa'(\mathbf{r}_2, t)}}{\overline{(\Delta\kappa')^2}} = \lim_{T\to\infty} \frac{1}{2T\overline{(\Delta\kappa')^2}} \int_{-T}^{T} \Delta\kappa'(\mathbf{r}_1, t)\,\Delta\kappa'(\mathbf{r}_2, t)\,dt \tag{6.4.12}$$

gives the correlation between the fluctuations in permittivity at point $\mathbf{r}_1$ and $\mathbf{r}_2$ spaced ρ apart, where the bar over terms indicates a time average. The term $\overline{(\Delta\kappa')^2}$ is the mean square variations in the amplitude of κ' which is independent of position and time due to assumed stationarity. If we also assume that $\Delta\kappa'(\mathbf{r},t)$ is *ergodic*, we may replace the time averages by ensemble averages. The correlation function is then

$$C(\rho) = \frac{1}{V\overline{(\Delta\kappa')^2}} \int_V \Delta\kappa'(\mathbf{r}_1)\,\Delta\kappa'(\mathbf{r}_1+\boldsymbol{\rho})\,dV \tag{6.4.13}$$

where all values are evaluated at the same time. The assumed spatial stationarity implies that the correlation function depends only on the distance and not on the direction. Therefore, $C(\rho)$ is a spherically symmetric function, with a peak at $\rho = 0$ which is equal to unity, and decreasing monotonically as ρ increases. The average value of $C(\rho)$ given by

$$l = \int_0^\infty C(\rho)\,d\rho\,\mathrm{m} \tag{6.4.14}$$

has the unit of length and is a measure of the average size of the *eddies* in the turbulence.

The scattering cross section per unit volume of the troposphere can be obtained by a statistical analysis as, [**18**],

$$\sigma(\theta) = \frac{\overline{(\Delta\kappa')^2}k_0^4\cos^2\theta}{4\pi}\int_0^\infty C(\rho)\,\frac{2\sin K\rho}{K\rho}\rho^2 d\rho \tag{6.4.15}$$

Therefore we need to determine the spatial correlation function $C(\rho)$.

The Fourier transform of the spatial correlation function is the spatial energy spectrum given by

$$F(K) = \int_{-\infty}^\infty C(\rho)\,e^{-jK\rho}d\rho = 2\int_0^\infty C(\rho)\cos K\rho\,d\rho. \tag{6.4.16}$$

The transform variable $K = 2\pi/L$ can be considered as the mechanical wave number, with L being a mechanical wavelength (not to be confused with the wavelength of the electromagnetic waves). That is, $F(K)\,dK$ is the energy due to eddies of lengths between K and $K + dK$.

Differentiating both sides of (6.4.16) with respect to the variable K and working the differentiation under the integral, we can write

$$\frac{1}{K}\frac{dF(K)}{dK} = -2\int_0^\infty C(\rho)\,\frac{\sin K\rho}{K\rho}\,\rho^2 d\rho. \tag{6.4.17}$$

Using this result in (6.4.15) gives

$$\sigma(\theta) = -\frac{\overline{(\Delta\kappa')^2}k_0^4}{4\pi}\left(\frac{1}{K}\frac{dF(K)}{dK}\right)\cos^2\theta. \tag{6.4.18}$$

This final expression relates the scattering cross section to the spatial energy spectrum, $F(K)$.

Notice that (6.4.18) expresses $\sigma(\theta)$ as a function of the mechanical wavelength K. Indeed, the radio waves will be scattered by all sizes of eddies in the turbulent atmosphere, and the total scattered power will be given by the sum of them. However, discontinuities of certain size have much larger contribution. To explain this effect, we will simplify the problem and assume a layered troposphere as shown in Fig. 6.3.

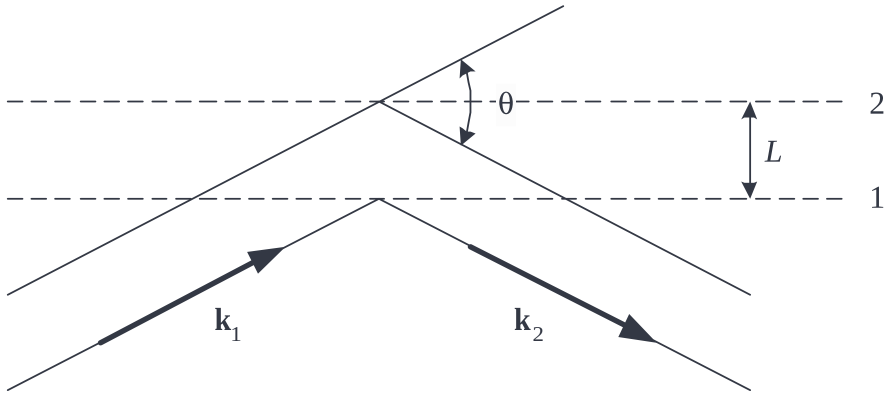

FIGURE 6.3. Reflections from a layered troposphere.

The phase difference between the waves reflected from the boundary 1 and 2 can be written as

$$\Delta\phi = 2k_0 L \sin\frac{\theta}{2} = \left|\mathbf{k}_{0,1} - \mathbf{k}_{0,2}\right| L. \tag{6.4.19}$$

Obviously, the scattered power will be a maximum in the θ direction if $\Delta\phi = 2\pi$, or

$$2k_0 \sin\frac{\theta}{2} = \frac{2\pi}{L} = K. \tag{6.4.20}$$

In other words, for a given frequency of the radio waves, the discontinuities (eddies) that contribute most to the power scattered in the direction of the receiver will have a size determined by (6.4.20). Actually, (6.4.20) defines a very narrow range of sizes. The scattered power will be a maximum if (6.4.20) is exactly satisfied, and will rapidly decrease for discontinuities having different sizes. Therefore, for a given frequency and propagation path length (i.e., for a given scattering angle θ) the scattering will be determined by eddies of size L. Figure 6.4 shows the spatial spectrum of functions, $F(K)$, as a function of the mechanical wavelength along with the scattered power for a wave of wave number k_0. The function $F(K)$ is assumed to have the Kolmogorov spectrum, [**47**]. Due to the narrowband characteristic of the scattered power for a given frequency, the value of σ is determined by only the value of $F(K)$ for the value of K determined by (6.4.20).

In the turbulent region, the spatial spectrum function can be written as $F(K) = AK^{-a}$. Using this in (6.4.18) we get

$$\sigma(\theta) = \frac{Aa\pi\overline{(\Delta\kappa')^2}\cos^2\theta}{4}\frac{\lambda_0^{a-2}}{(4\pi)^a}\left(\sin\frac{\theta}{2}\right)^{-(a+2)}. \tag{6.4.21}$$

The value of a is 11/16 for turbulent atmosphere. Then, one would expect that the higher frequencies are more favorable for tropospheric scattering links. However, the measurements indicate that the spatial spectrum function may differ from the theoretical Kolmogorov spectrum. There are different theories for deriving and measuring the spatial spectrum function, but this topic is beyond our scope.

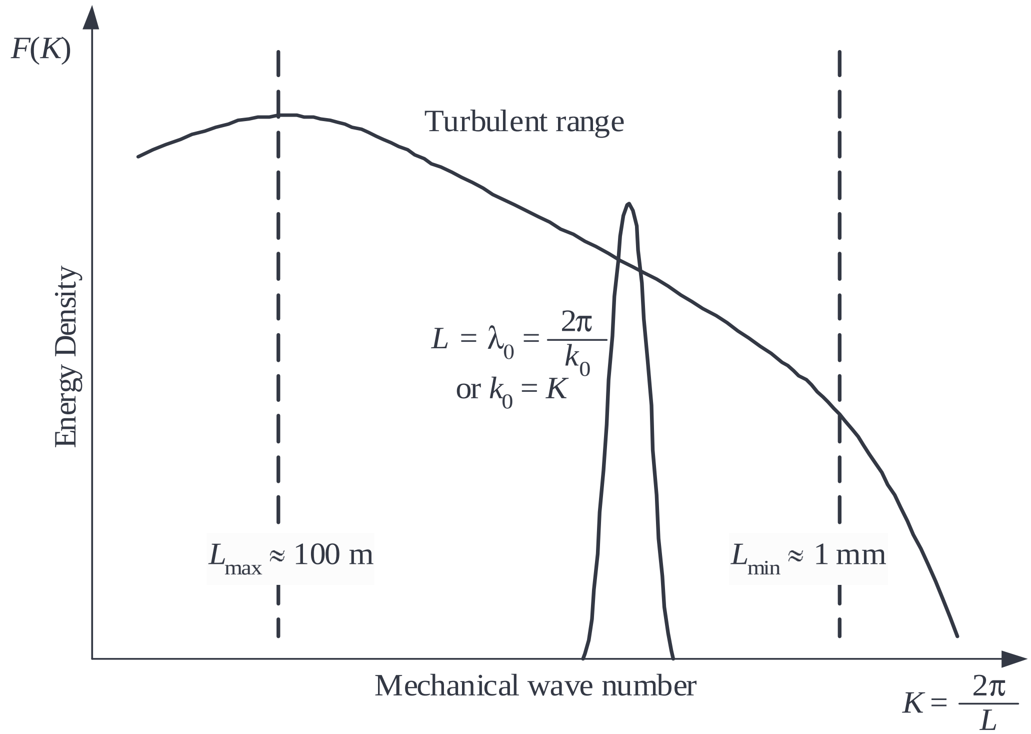

FIGURE 6.4. Spatial spectrum of fluctuations, $F(K)$, and the scattering condition.

Collin, [**5**], referring to Tatarski, [**48**], gives the formula

$$-\left(\frac{\overline{(\Delta\kappa')^2}}{K}\frac{dF(K)}{dK}\right) = 32\pi^3\left(3.3\times10^{-2}C_n^2\right)K^{-11/3}\,\mathrm{m}^3 \tag{6.4.22}$$

and

$$V_c = \frac{R^3\left(\theta_{t,1/2}\right)^{3/2}\left(\theta_{r,1/2}\right)^{3/2}}{\theta} \tag{6.4.23}$$

where $2\theta_{1/2}$ is the half power beam width of the antenna. The factor C_n is called the structure constant and is given as

$$C_n^2 = 4.2\times10^{-14}h^{-1/3}e^{-h/h_0}\,\mathrm{m}^{-2/3} \tag{6.4.24}$$

where $h_0 = 3200\,\mathrm{m}$ and h is the height above the earth's surface.

6.5. Transmission Loss Calculations for Tropospheric Scatter Paths

ITU-R Recommendation 617-1, [**49**],describes an algorithm to estimate the average annual median transmission loss $L(q)$ not exceeded for percentages of the time q. The *horizon angle* of a directional antenna, is defined as the angle, in a vertical plane, subtended by the lines extending from the antenna to the local horizon (tangent to earth's surface) and from the antenna in its direction of maximum radiation, as shown in Fig. 6.5. Due to the antenna height, the local horizon and the radio horizon may be different. This angle, denoted by ψ in Fig. 6.5 is given by $\psi = h/\sqrt{2a_e h}$.

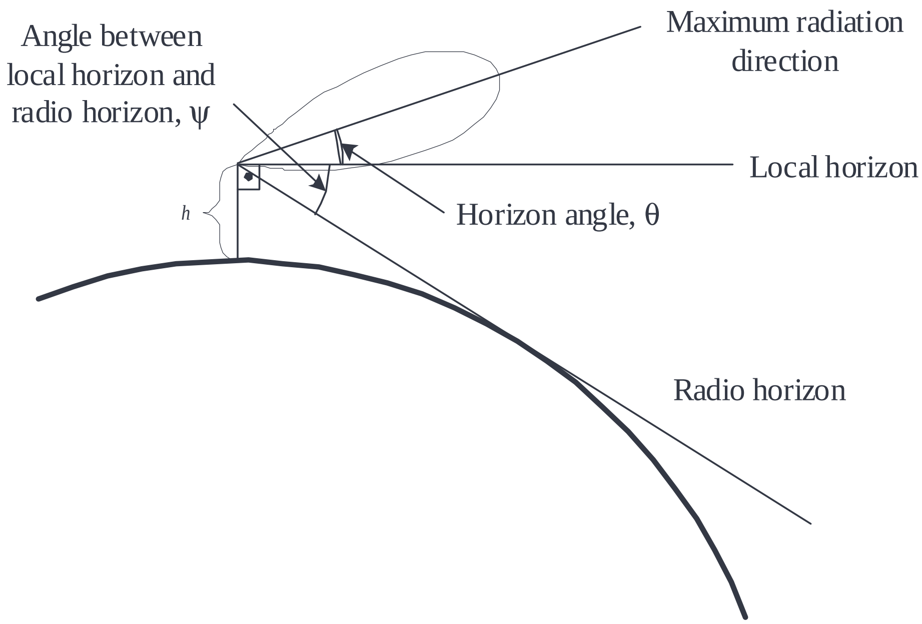

Figure 6.5. Definition of horizon angle.

We will assume that the great-circle path length d (km), frequency f (MHz), transmitting antenna gain G_t (dB), receiving antenna gain G_r (dB), horizon angle θ_t (mrad) at the transmitter, and the horizon angle θ_r (mrad) at the receiver are given. The procedure described in [**49**] to determine the average annual median transmission loss $L(q)$ for $q = 50\%$ is summarized below.

(1) Decide on the appropriate climate for the link in question from the list of nine climates. Detailed description of the climates are given in [**49**]. Table 6.3 gives the meteorological and atmospheric structure parameters.

Table 6.3. Values of meteorological and atmospheric structure parameters

Climate	**Description**	M (dB)	γ (km^{-1})
1	Equatorial	39.60	0.33
2	Continental sub-tropical	29.73	0.27
3	Maritime sub-tropical	19.30	0.32
4	Desert	38.50	0.27
5	Mediterranean	38.50	0.27
6	Continental temperate	29.73	0.27
7a	Maritime temperate, overland	33.20	0.27
7b	Maritime temperate, oversea	26.00	0.27
8	Polar	33.20	0.27

(2) Calculate the scatter angle θ (defined in Fig. 6.2) as

$$\theta = \theta_e + \theta_t + \theta_r \,\mathrm{mrad} \tag{6.5.1}$$

where $\theta_e = d \times 10^3 / a_e$ (mrad), and a_e (km) is the effective radius of the earth.

(3) Determine the transmission loss dependence L_N on the height of the common volume from

$$L_N = 20\log_{10}(5 + \gamma H) + 4.34\gamma h \text{ dB} \tag{6.5.2}$$

where $H = 10^{-3}\theta d/4$ (km), $h = 10^{-6}\theta^2 a_e/8$ (km), γ is the atmospheric structure parameter obtained from Table 6.3.

(4) Estimate the aperture-to-medium coupling loss L_c from:

$$L_c = 0.07\exp\left[0.055\,(G_t + G_r)\right] \text{ dB.} \tag{6.5.3}$$

(5) The estimate of the average annual transmission loss not exceeded for 50% of the time is given by

$$L(50) = M + 30\log_{10} f + 10\log_{10} d + 30\log_{10}\theta + L_N + L_c - G_t - G_r \text{ dB.} \tag{6.5.4}$$

This expression is an empirical formula based on data for the frequency range between 200 MHz and 4 GHz. For non-exceedance percentages q greater than 50%, $L(q)$ is given by

$$L(q) = L(50) - Y(q) \tag{6.5.5}$$

where $Y(q) = C(q)\,Y(90)$ (dB) and $C(q)$ and $Y(90)$ are defined in [**49**].

The loss calculated by the above procedure gives annual median values. In fact, transmission loss varies annually and diurnally. In temperate climates, monthly median losses tend to be higher in winter than in summer, and the difference may be $10 - 15$ dB on $150 - 250$ km overland paths but decreases as the distance increases. Diurnal variations are most pronounced in summer, with a range of 5 to 10 dB on $100 - 200$ km overland paths. The transmission loss is greatest in the afternoon, and least in early morning. Greater variations are expected for oversea paths than land paths, since the former are more likely to be affected by super-refraction and elevated layers.

In dry, hot desert climates, the variation of the monthly median losses with season of the year is reversed /as compared to temperate climates) and attenuation reaches a maximum in the summer. The annual variations of the monthly medians for medium-distance paths exceed 20 dB, while the diurnal variations are very large.

In equatorial climates, the annual and diurnal variations are generally small, [**49**].

Apart from the seasonal and diurnal variations, there is also variations in the signal strength in much shorter time scales as mentioned previously. This is called scintillation or fading and will be discussed in Chapter **??**.

6.6. Examples

EXAMPLE 24. *A tropospheric scatterer link operating at* 3 GHz *uses* $P_t = 1\,\mathrm{kW}$, *and* $G_t = G_r = 10^5$ *(50 dB) with* 78% *efficiency. The distance is* 400 km *and link uses* 1 MHz *bandwidth. Assuming an equivalent noise temperature of* $T = 600\,\mathrm{K}$, *calculate the received power, the SNR, and the attenuation factor. Take* $C_n = 10^8$.

SOLUTION 25. *We can assume that the antennas have symmetrical patterns about their bore sight since typically parabolic reflector antennas are used. The directivity of the antennas are* $D = \eta G = 78000$ *and we can estimate the half power beam width from* $4\pi / \left(2\theta_{1/2}\right)^2 = D$*, which gives* $\theta_{1/2} = 6.3 \times 10^{-3}\,\mathrm{rad}$*. The scattering angle is* $\theta = d/a_e$ *and we get* $K = 2.96$*. Using (6.4.22) we find* $\sigma(\theta) = 2.0 \times 10^{-9}$*, and* $V_c = 3.5 \times 10^{11}$*. Finally we get* $P_r = 8.3 \times 10^{-13}\,\mathrm{W}$*. The noise power is* $P_n = 8.28 \times 10^{-15}\,\mathrm{W}$ *giving* $SNR = 20$ *dB.*

The received power for free space conditions would be $P_{r,fs} = 3.96 \times 10^{-3}\,\mathrm{W}$*. Therefore the attenuation factor is* 2.78×10^{-10} *or* -96.8 *dB.*

Using (6.5.4) we would obtain $L(50) = 146.1$ *dB for climate zone* 2*. There is a large difference between the two results. The reason may be that (6.5.4) gives average median values. However, the variations in climate zone* 2 *is about* 20 *dB, so this does not explain the discrepancy. Another reason may be the addition of atmospheric losses and including the possibility of precipitation over the propagation path.*

EXAMPLE 25. *Assume that a tropospheric scatterer link operates at* 144 MHz*. The propagation path length is* 250 km*, and antenna gains are* 16 *dB. Both transmitting and receiving antennas have a gain of* 16 *dB and are looking towards the radio horizon. The path is in Mediterranean climate zone. ITU-R Rec. 617-1 gives* $Y(90) = -9$ *dB and* $C(99.9) = 2.41$*. Determine the average annual median transmission loss not exceeded for* 99.9% *of the time.*

SOLUTION 26. *By definition we have* $\theta_t = \theta_r = 0$ *and* $\theta = \theta_e = 29.4\,\mathrm{mrad}$*. From Table 6.3 we find that* $M = 38.50$ *dB, and* $\gamma = 0.27\,\mathrm{km}^{-1}$*. Using (6.5.2) we have* $H = 1.84\,\mathrm{km}$*,* $h = 0.918$ km*, and* $L_N = 15.88$ *dB. Using (6.5.3) we find* $L_c = 0.41$ *dB. Then, using (6.5.4) we get* $L(50) = 155.6$ *dB. Finally, using (6.5.5) we obtain* $L(99.9) = 155.6 + 21.7 = 177.3$ *dB. The free space path loss for this case would be* $L_{fs} = 123.6$ *dB. Including the antenna gains, we have a difference of* 64 *dB, but this cannot be defined as an attenuation factor in the usual sense. Instead we can define it as the annual median attenuation factor.*

EXAMPLE 26. *Determine the transmitter power necessary to secure a communication reliability of* 99.9% *over a propagation path* 400 km *long in the Mediterranean climate zone. The frequency of operation is* 800 MHz*, and antenna gains are* 40 *dB looking horizontally. The receiver noise figure is* 10 *and requires* 13 *dB SNR for operation and the communication bandwidth is* 500 kHz*. Ignore fading effects.*

SOLUTION 27. *The angle between the local tangent and radio horizon is very small for practical antenna heights and we can take* $\theta_t = \theta_r = 0$*. Repeating the procedure, we find*

$$\begin{aligned} \theta &= 47.1\,\mathrm{mrad}; \quad H = 4.71\,\mathrm{km}; \quad h = 2.35\,\mathrm{km}; \\ L_N &= 18.7\ dB; \quad L_c = 5.7\ dB; \quad L(50) = 146.2\ dB; \\ L(99.9) &= 168\ dB. \end{aligned}$$

The required power at the receiver is

$$P_r = 20kTBF = 4.14 \times 10^{-13}\,\mathrm{W} = -94\ dBm.$$

Thus, the transmitter power should be

$$P_t = -94 + 168 = 74\ dBm = 25\,\mathrm{kW}.$$

Notice that if the loss were constant at its median value, a transmitter power of 52.2 *dBm would be sufficient. The additional* 21.7 *dBm is to secure a communication reliability of* 99.9%.