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CHAPTER 5

Propagation over Earth (Ground Wave)

The calculation of the propagation factor over a earth is rather involved. There are several methods for the calculation of the propagation factor, some of which require intensive calculations. These methods are discussed in detail in Chapters (3) and (4). In this chapter we will discuss the regions of validity of different methods and, compare their advantages and disadvantages.

For propagation calculations, the region around a transmitting antenna is divided into three regions. In the first region, the direct and reflected waves are the main contributors to the total field. The total field is determined by their interference. Therefore, this region is therefore called the *interference region*. The third region is the region where the observation point lies below the line-of-sight horizon of the transmitter. The field in this region is due to diffraction of the waves and therefore it is called the *diffraction region*. In the region between the two, the field calculations are based on a full-wave integral and is called the *intermediate region*. The procedures for the calculation of the propagation factor is different in each region and will be detailed in the following sections.

There is no general rule based on analytical derivations for determining the validity of the computation methods as a function of distance. Therefore, the boundaries of these regions are not clearly defined. However, in the literature a generally accepted rule of thumb is to define the interference region as the region that extends from the transmitter up to $0.8R_h$, where R_h is the total horizon distance. The diffraction region starts at $1.2R_h$, and the rest is the intermediate region. This definition ignores the heights of the transmitting and the receiving antenna. To overcome this problem, some authors define the interference region as the region for which the interference equations are valid, but this description does not answer the question. A more general description should use two dimensional regions as shown in Fig. 5.1. However, the borders of the regions fail to have clear-cut definitions. Nonetheless, we will try to give certain rules for the applicability of different formulas in the following sections.

The waves that we consider in this chapter are called *ground waves* in the propagation literature. We will use the definitions of the terms as given by the IEEE Standard 211, "IEEE Standard Definitions of Terms for Radio Wave Propagation," [**37**]. The relevant definitions are exerpt from this document below:

Direct wave: A wave propagated over an unobstructed ray path from a source to a point.

Ground wave: From a source in the vicinity of the surface of the Earth, a wave that would exist in the vicinity of the surface in the absence of an ionosphere.

Note: The ground wave can be decomposed into the Norton surface wave and a space wave consisting of the vector sum of a direct wave and a ground-reflected wave.

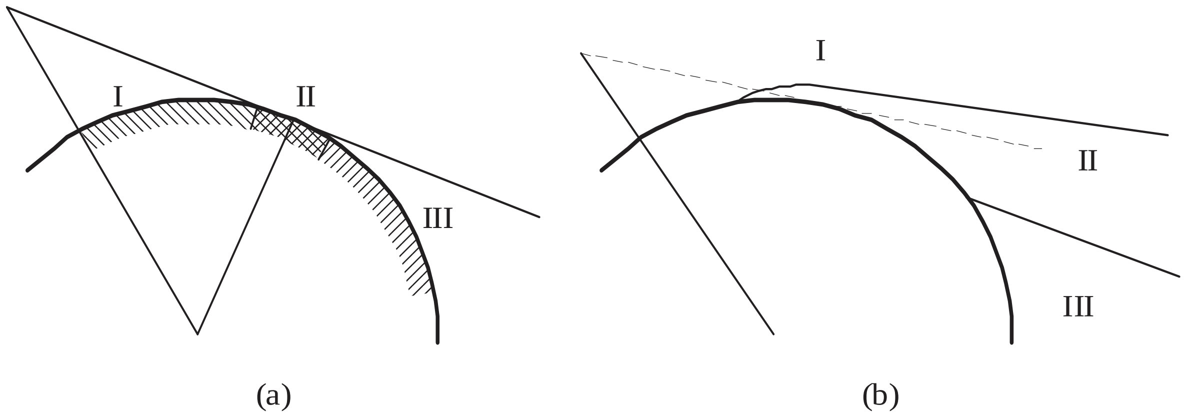

FIGURE 5.1. Definitions of the (I) interference, (II) intermediate, (III) diffraction regions. (a) range dependent definition, (b) zones.

Norton surface wave: The propagating electromagnetic wave produced by a source over or on the ground. The Norton wave consists of the total field minus the geometrical-optics field.

Reflected wave: For two media, separated by a planar interface, that part of the incident wave which is returned to the first medium. The direction of propagation of the reflected wave is given by Snell's Law.

Surface wave: A wave guided by a boundary with a surface impedance whose reactive part exceeds the resistive part. A surface wave is generally characterized as a slow wave having a magnitude that exponentially decreases with distance from the interface but may be modified by curvature.

The definition of ground wave implicitely defines a *space wave* as the vector sum of a direct wave and a ground-reflected wave, which is also used in the literature.

5.1. Interference Region

In the interference region the surface wave is negligible and prime interest is in the space wave component. Therefore, the attenuation function is given by

$$F = \left|1 + D\Gamma e^{-jk\Delta R}\right| \tag{5.1.1}$$

for either plane earth or spherical earth model excluding the antenna pattern effects. The pattern effect can be added by considering the direction of direct and reflected waves with respect to antenna bore sight. If these angles are denoted by ξ_d and ξ_r, respectively, the attenuation function becomes

$$F = \left|f(\xi_d) + f(\xi_r)\, D\Gamma e^{-jk\Delta R}\right|. \tag{5.1.2}$$

In the plane earth model $D = 1$ is used.

The first question is whether to use flat earth geometry or the spherical earth geometry. There are three parameters that we should consider to answer this question, namely, the reflection coefficient, the path length difference, and the divergence factor.

The reflection coefficient depends on the grazing angle and the electrical parameters of the ground at the reflection point. Since the grazing angle and the reflection point may be different in the two models, the reflection coefficients calculated with the two models will be different. In most practical situations however, the difference in the reflection coefficients will be negligible. The most important difference between the two models is in the calculation of the path difference ΔR. Figures 5.2 and 5.3 show the values of ψ and ΔR as a function of the range R, calculated using three different approaches. The transmitter and receiver heights are 50 m and 100 m, respectively. For these values the receiver falls below horizon at a range of about 70 km. The error in the flat earth model increases with the range as expected. This becomes especially important at higher frequencies where the wavelength is comparable to the error.

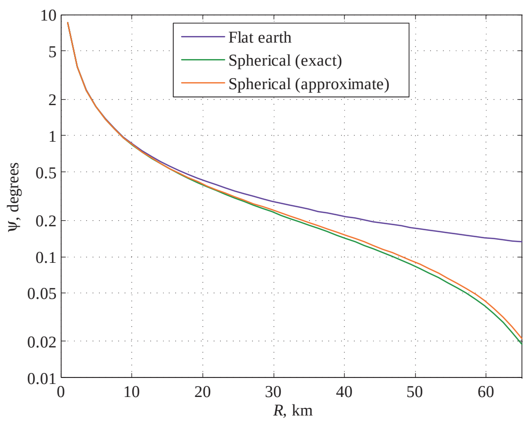

FIGURE 5.2. Values of ψ calculated with different models, for $h_t = 50$m and $h_r = 100$m.

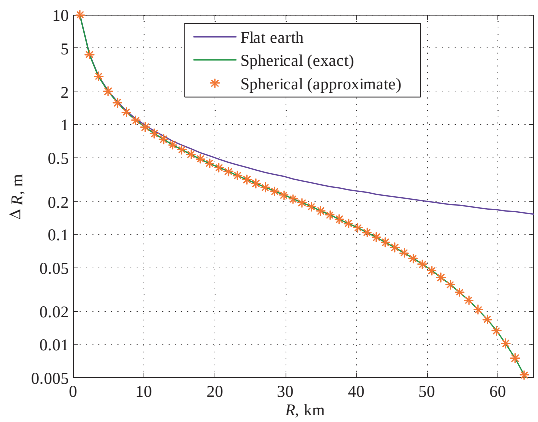

FIGURE 5.3. Values of ΔR calculated with different models, for $h_t = 50$m and $h_r = 100$m.

To determine the range of validity of the flat earth model, we have to compare the path differences calculated by the two models. The flat earth model gives

$$\Delta R_f = \frac{2h_1 h_2}{d} \tag{5.1.3}$$

while for spherical earth model we can use the effective heights and write

$$\Delta R_s = \frac{2h_1' h_2'}{d}. \tag{5.1.4}$$

where $h_i' = h_i - d_i^2/(2a_e)$. The difference between two results is then

$$\Delta R_f - \Delta R_s = \frac{2h_1 h_2}{d} - \frac{2h_1' h_2'}{d} \approx \frac{(h_1 d_2^2 + h_2 d_1^2)}{d a_e} \tag{5.1.5}$$

where we have neglected $d_1^2 d_2^2/(2a_e)^2$ term. To a first approximation, we can replace d_1 and d_2 by the flat earth values and obtain

$$\Delta R_f - \Delta R_s \approx \frac{d h_1 h_2}{a_e (h_2 + h_1)}. \tag{5.1.6}$$

The two models will yield the same result if this difference is a fraction of the wavelength, say $\lambda_0/10$. Then, we can say that the flat earth model will be valid if

$$\frac{dh_1h_2}{a_e\left(h_2+h_1\right)}<\frac{\lambda_0}{10} \tag{5.1.7}$$

or equivalenly, if

$$d<\frac{\lambda_0 a_e\left(h_2+h_1\right)}{10h_1h_2}. \tag{5.1.8}$$

It must be noted that the validity region of the flat earth model depends on the frequency. For the geometry of Fig. 5.3, and for $\lambda_0 = 10\,\mathrm{cm}$, the flat earth model will be valid for ranges below 2550 m, while for $\lambda_0 = 1\,\mathrm{m}$, the range of validity extends to 25.5 km. Of course, if the limit given by (5.1.8) exceeds the total horizon range, the interference region formulas cannot be used. So we should limit the validity region of flat earth model as

$$d<\min\left(\frac{\lambda_0 a_e\left(h_2+h_1\right)}{10h_1h_2}, \sqrt{2a_eh_1}+\sqrt{2a_eh_2}\right). \tag{5.1.9}$$

As the antenna heights get smaller, the flat earth model will be valid for increasing portions of the total horizon range as can be seen from (5.1.9). At sufficiently small antenna heights, the flat earth model is known to give accurate results up to and beyond the total horizon range, [**19**]. However, in such cases the surface wave component must also be considered.

The variation of the divergence factor with range is shown in Fig. 5.4 for $h_t = 50\,\mathrm{m}$ and $h_r = 100\,\mathrm{m}$. The divergence factor decreases to very small values as the horizon is reached, which is not correct. It is recommended to use D from (3.2.12i) if $\Delta R \geq \lambda_0/4$ and take $D = 1$ if $\Delta R < \lambda_0/4$, [**16**].

There is also the problem of solving the spherical earth geometry. One method is to use the exact formula given in (3.2.4), the other is the method outlined in section 3.2 by the equations (3.2.12). The error in (3.2.12) is not known, but generally believed to yield accurate results in all practical situations. The exact formula can easily be solved by iteration, i.e., if we start with an initial value ψ_0 and define

$$\psi_{k+1}=g\left(\psi_k\right), \tag{5.1.10}$$

then $\lim_{k\to\infty}\psi_k$ exists and is the unique solution of equation (3.2.4), [**38**]. However, care must be exercised in calculations, since the convergence may be very slow, especially for larger values of ψ. On the other hand the approximate formulas for spherical earth give very accurate results up to the horizon. The grazing angles and path length differences calculated by the exact and approximate methods for spherical earth are shown in Fig. 5.2 and 5.3. With the modern computers, calculations required for spherical earth model is not much of a task, and it is generally better to use the spherical earth models. However, flat earth model is still useful at least to quickly analyze and get an understanding for certain problems.

The total field is always determined by the sum of space and surface wave components. In the interference region, the surface wave component is generally negligible, and the total field is approximated by the space wave only. As the distance is increased towards the total horizon, the space wave component decreases and the surface wave cannot be neglected. Therefore, the interference formulas are not valid all the way to the total horizon range. No

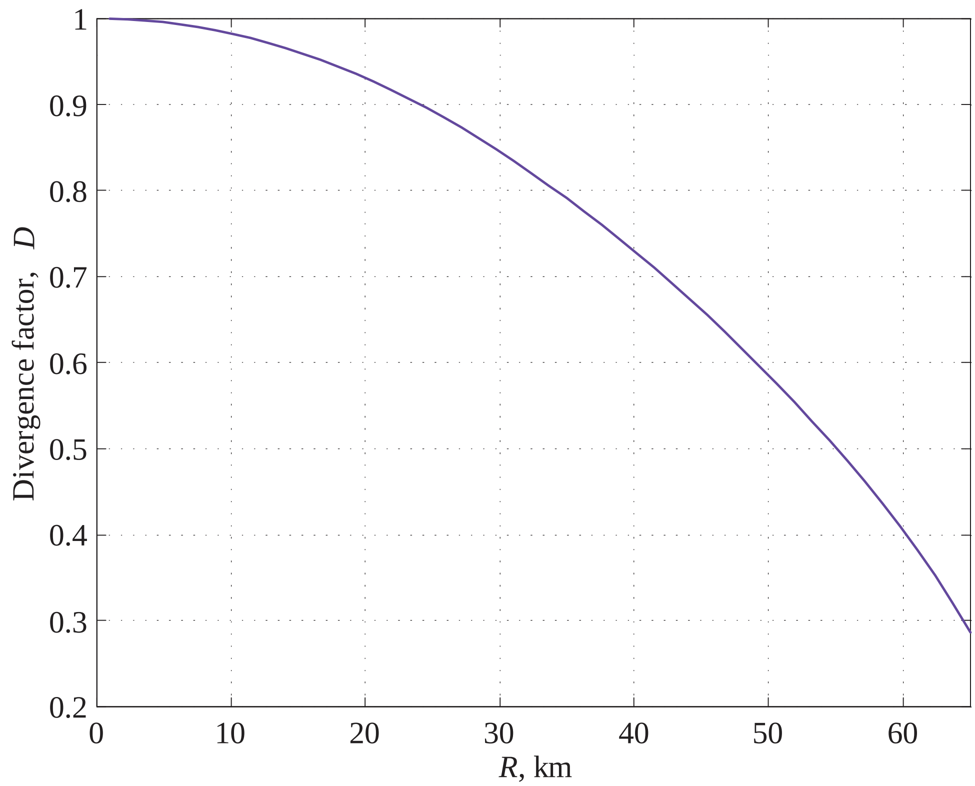

FIGURE 5.4. Variation of D with range, for $h_t = 50\,\mathrm{m}$ and $h_r = 100\,\mathrm{m}$.

exact analytic expression is known for the limit of validity of interference formulas, but it is believed that it will always be valid for $\Delta R \geq \lambda_0/4$.

If the antennas are very close to the surface, only surface wave propagation should be considered and it is accepted that, when the distance between the transmitter and the receiver exceeds $80/f_{\mathrm{MHz}}^{1/3}$ km or when either of the transmitter or receiver antenna height exceeds $610/f_{\mathrm{MHz}}^{2/3}$ m, the effect of earths sphericity must be taken into account in surface wave calculations, [**39**].

5.1.1. Solution of Approximate Spherical Formulas. The approximate formulas for the solution of spherical geometry are arranged in a way to determine the so called correction factors $J(S,T)$ and $K(S,T)$ and the related terms S, T, S_1, and S_2 in terms of the geometric parameters. However, in some problems like generating coverage diagrams, we need to determine the geometric parameters from a knowledge of these parameters. Although not obvious from the equations, only two of these parameters are independent, and all parameters can be determined from a knowledge of any two of them. On the other hand, we have three independent geometric parameters. This seems like a contradiction but, scaling the antenna heights by a factor and the distance by the square root of the same factor does not change the solution.

The equations that relate S, T, S_1, and S_2 are

$$S = \frac{S_1 T + S_2}{1 + T}, \tag{5.1.11}$$

$$T = \frac{S_1 \left(1 - S_2^2\right)}{S_2 \left(1 - S_1^2\right)}. \tag{5.1.12}$$

If any two of S_1, S_2, S, and T are given, the other two can be solved from these equations. Still, the equations are nonlinear and generally require the solution of a cubic equation. The parameters S and T are directly related to the physical parameters h_1, h_2, and d, and are relatively easy to calculate. Therefore, plots giving the correction factors J, K, and the divergence factor D as functions of S and T are generated. The use of these graphs makes it easier to solve the spherical geometry. Figures 5.16, 5.17, and 5.18 given in the appendix can be used for this purpose.

In constructing coverage diagrams, one is required to solve the distance at which the attenuation factor has a certain value. Under certain assumptions, this means the determination of distance between transmitter and receiver with known heights for a given phase difference between the direct and the reflected paths. Multiplying (3.2.12g) by R_h and recognizing $d/R_h = S$, we can write

$$\frac{R_h \Delta R}{2 h_r h_t} = \frac{J\left(S,T\right)}{S} = \frac{\left(1 - S_1^2\right)\left(1 - S_2^2\right)}{S}. \tag{5.1.13}$$

We define the ratio $J\left(S,T\right)/S$ as a new function $Q\left(S,T\right)$ of the variables S and T. Figure of the appendix gives $Q\left(S,T\right)$ as a function of S and T and can be used for this type of problems.

5.2. Diffraction Region

At points sufficiently far beyond the total horizon range, the diffraction field can be calculated by only a single term in the residue series given by (4.2.7) which is repeated here for convenience

$$W\left(x,q\right) = 2\left(\frac{\pi x}{j}\right)^{1/2} \sum_{n=1}^{\infty} \frac{G_n\left(y_t\right) G_n\left(y_r\right) \exp\left(-jxz_n\right)}{\left(z_n - q^2\right)}. \tag{5.2.1}$$

The convergence of the residue series is basically determined by the exponential term. The poles z_n always lie in the fourth quadrant of the complex plane and z_n have a negative imaginary part and a positive real part. If we write $z_n = \alpha_n - j\beta_n$, with $\alpha_n, \beta_n > 0$, we observe that the terms in the series are proportional to $\exp\left(-x\beta_n\right)$. Thus, for large x, the terms decay rapidly. For large values of q, we have $z_1 = \zeta_1 = 1.17 - j2.02$ and we can write

$$F = \left|W\right| = V\left(x\right) U\left(y_t\right) U\left(y_r\right), \tag{5.2.2}$$

$$V\left(x\right) = 2\sqrt{\pi x}\exp\left(-2.02x\right) \tag{5.2.3}$$

$$U\left(y\right) = \left|\frac{u\left(z_1 - y\right)}{qu\left(z_1\right)}\right| = \left|\frac{u\left(z_1 - y\right)}{u'\left(z_1\right)}\right| \tag{5.2.4}$$

which is fairly easy to calculate. At the total horizon range and for a slight distance beyond it, single term is not sufficient to obtain a correct value of F. There is no simple rule that determines the distance beyond which the single mode representation is valid but for standard conditions the total horizon range needs to be increased by only a small fraction of itself to make the single mode representation valid. One idea is to compare the magnitudes of the second and first term. Again for large values of q, we have $z_2 = 2.054 - j3.540$. Thus the ratio of the second term to the first term can be found to be approximately given by $\exp(-1.52x)$. If $x \geq 10/3$, or equivalently $R > (k_0 a_e/2)^{-1/3}\, 10a_e/3$, the second term will be less than 0.7% of the first term, and the use of single term expansion is justified. If more error is acceptable, we can use the single term expansion at smaller ranges.

The height-gain function $U(y)$ (in dB) is shown in Fig. 5.5 as a function of the normalized height y in the region $1 < y < 100$. The value of z_1 is taken as the first zero of $u(z)$ which is valid for large values of q. With large q assumption, we have $U(y) \approx y$ for $y \leq 1$.

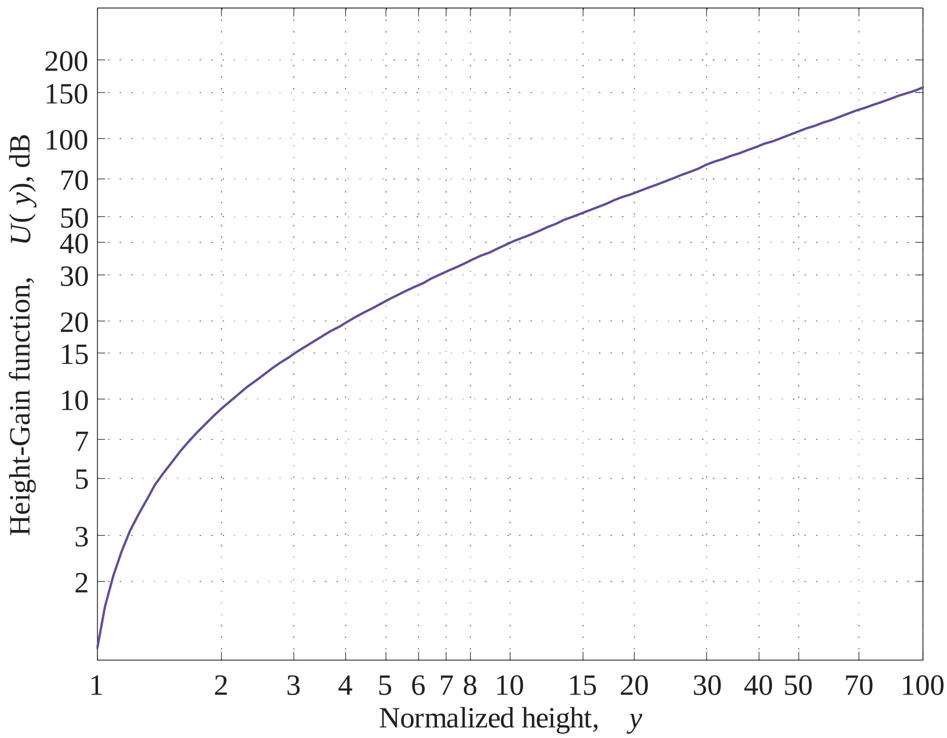

FIGURE 5.5. Height-gain function $20\log_{10}|U(y)|$ as a function of normalized height, for large q, $a_e = 8500\,\text{km}$.

From the definition of $V(x)$, we have

$$20\log_{10} V(x) = 20\log_{10}(2) + 10\log_{10}(\pi) + 10\log_{10}(x) + 20\left(\frac{-2.02x}{\ln 10}\right), \tag{5.2.5}$$

$$= 10.9921 - 17.5455x + 10\log_{10}(x). \tag{5.2.6}$$

For ranges of interest, $x > 1$ and $\log_{10}(x) \ll x$ which means that we can approximate $20\log_{10} V(x)$ by a linear approximation. A very good fit to this function for the ranges of interest is

$$20\log_{10} V(x) \approx -17x + 14 \tag{5.2.7}$$

with an error less than 4 dB. Figure 5.6 shows $20\log_{10} V(x)$ and the linear fit as a function of x.

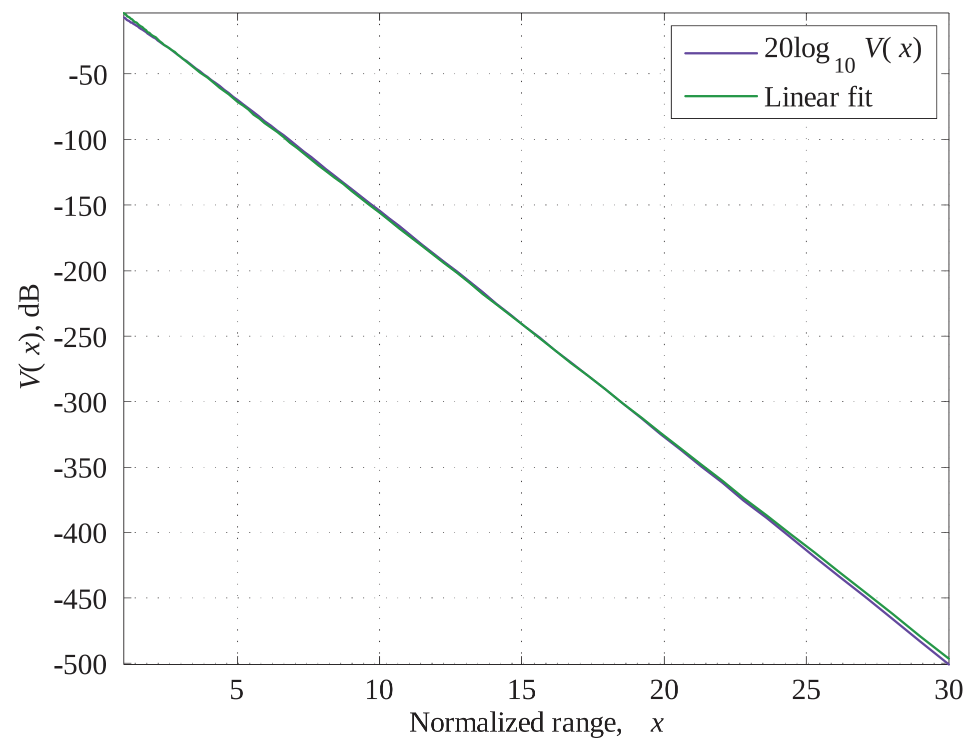

FIGURE 5.6. $20\log_{10}(x)$ as a function of x, for $1 \leq x \leq 30$, $a_e = 8500\,\mathrm{km}$.

5.3. Intermediate Region

We have seen that ray methods that are used in the interference region generally break down before the total horizon range is reached. In a similar manner, the single term expression is valid not at the total horizon range but slightly beyond it. This leaves a region known as the intermediate region near the horizon for which the two methods are not applicable. Of course, using the residue series with more terms is a possibility, but it is generally quite complicated. One method is to calculate the fields in the two regions with appropriate formulas and then fill the intermediate region by a smooth interpolation. For this purpose, typically two points are used in the interference region, one at $\Delta R = \lambda_0/2$ where the first maximum occurs and $F = 1 + D\,|\Gamma|$, the other at the first point of quadrature, $\Delta R = \lambda_0/4$ and $F = \sqrt{1 + D^2\,|\Gamma|^2}$. In the diffraction region, two or three points are used and an appropriate interpolation method is used. A shape preserving spline interpolation is a good choice for this purpose.

The fields in the interference region are basically due to surface waves and the flat earth surface wave attenuation function can be used to predict the field in this region. As mentioned previously, the flat earth surface wave model is valid up to $80/f_{\mathrm{MHz}}^{1/3}$ km. Beyond this range earth sphericity must be taken into account. The small curvature expansion in powers of $\delta^3 = -1/(2q^3)$ given in (4.2.21) is useful in this region, however, its use is restricted also for $q \geq 0.1$, [**40**].

The surface wave formulas do not take antenna heights into account. In principle, these formulas are valid only for antennas on the ground, but it is claimed that the flat earth surface wave model can be used upto height of $610/f_{\mathrm{MHz}}^{2/3}$ m. However, as we shall see in the study cases below, the results will differ from residue series calculations by an appreciable amount at fractions of this height.

5.4. Sample Calculations

In this section we will consider propagation at three different frequencies for the purpose of comparing different formulations and their validities.

5.4.1. Case I. Suppose that a transmitter operates at 1 MHz with vertical polarization over "wet ground" with $\kappa' = 30$ and $\sigma = 10^{-2}$ S/m. We could expect flat earth model to be valid up to $80/f_{\mathrm{MHz}}^{1/3} = 80$ km range provided that the antennas are below $610/f_{\mathrm{MHz}}^{2/3} = 610$ m. The value of q can be calculated as

$$|q| = \left| -j \left(\frac{k_0 a_e}{2} \right)^{1/3} \frac{\sqrt{\kappa - 1}}{\kappa} \right| = 3.3 \tag{5.4.1}$$

which is greater than 0.1 and the small curvature assumption is expected to give good results. The attenuation factors calculated with different methods for $h_t = h_r = 1$ m are shown in Fig. 5.7. The total horizon distance for this case is 8.2, and therefore the interference models completely fail in the region of computations. The surface wave attenuation function with flat earth assumption and small curvature correction are very close to each other. Only a small difference is observed for ranges exceeding 35 km. In this region small curvature correction coincides with residue series. Around the horizon range, the surface wave approximation fails but the residue series shows the beginning of the interference region. The residue series were calculated using 50 terms, although only a few terms in the residue series were sufficient for larger values of R. The single term expression is not considered since q is not large enough to justify the assumptions made, i.e., the location of the zero is not accurate.

Figure 5.8 shows the attenuation functions for $h_t = h_r = 100$ m. For this case the total horizon distance is 82 km. Although the antenna heights are below 610 m, there is appreciable difference between the residue series expansion is due to increased heights.

5.4.2. Case II. Next we will consider a 10 MHz vertically polarized transmitter operating over a ground with parameters $\kappa' = 15$ and $\sigma = 3 \times 10^{-3}$ S/m. Figure 5.9 shows the attenuation functions for $h_t = h_r = 1$ m, calculated using the flat earth model, the small curvature approximation, and the residue series. We could expect flat earth model to be valid up to $80/f_{\mathrm{MHz}}^{1/3} = 37.1$ km range. It can be seen that small curvature approximation departs from the flat earth formula at about 15 km range, the difference being less than 3 dB up to about 40 km. Between $15 - 70$ km small curvature approximation and residue series give similar results. Beyond 70 km, small curvature approximation falls of rapidly and differs from the residue series result. The value of $|q|$ for this case is 23.4.

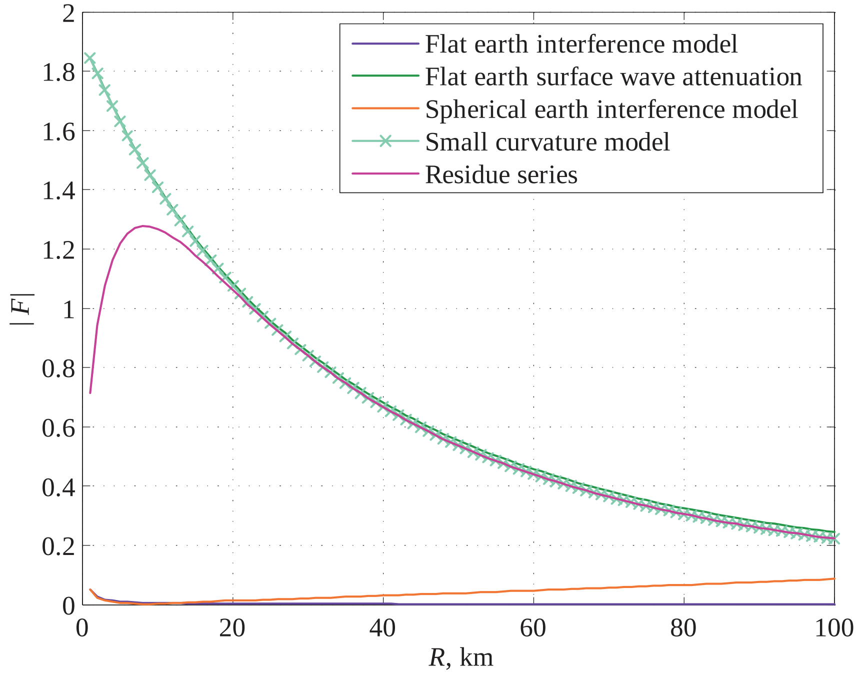

FIGURE 5.7. Attenuation factors for Case I calculated with different methods at 1 MHz, $h_t = h_r = 1\,\mathrm{m}$.

5.4.3. Case III. In the third case the frequency of operation is 1 GHz and antennas are at $h_t = h_r = 50\,\mathrm{m}$. Propagation is over "wet ground" with $\kappa' = 30$ and $\sigma = 10^{-2}\,\mathrm{S/m}$, and vertical polarization is considered. Fig. 5.10 shows the results. The total horizon range for this case is 58.3 km. The interference region extends up to about 30 km. Eq. (5.1.9) shows that the flat earth interference formula would be valid up to 10.2 km range. The difference between flat earth and spherical earth models becomes obvious after about 15 km. The residue series was calculated using 20 terms and gives accurate results even in the interference region. The spherical interference model gives a zero field at the total horizon range. The jump in this curve is at the range where $\Delta R = \lambda_0/4$ and the divergence factor is taken as 1 for larger ranges. This point also indicates the border of the interference region. Beyond 60 km, single term is sufficient in the residue series. The intermediate region is between $40 - 60$ km and residue series appears to be quite accurate in this region. However, the smoothness of the curve in this region implies that a simple interpolation would also be quite accurate.

5.5. Mixed Path Propagation

If the ground parameters change along the propagation path between the transmitter and receiver, we have a mixed path propagation. We are still assuming a smooth earth in the sence that the irregularities of the ground is much smaller than the effective wavelength $\lambda_v = \lambda_0/\sin\psi$, in the direction perpendicular to the surface. We will also assume that the path can be described by a series of of finite segments, each with different ground

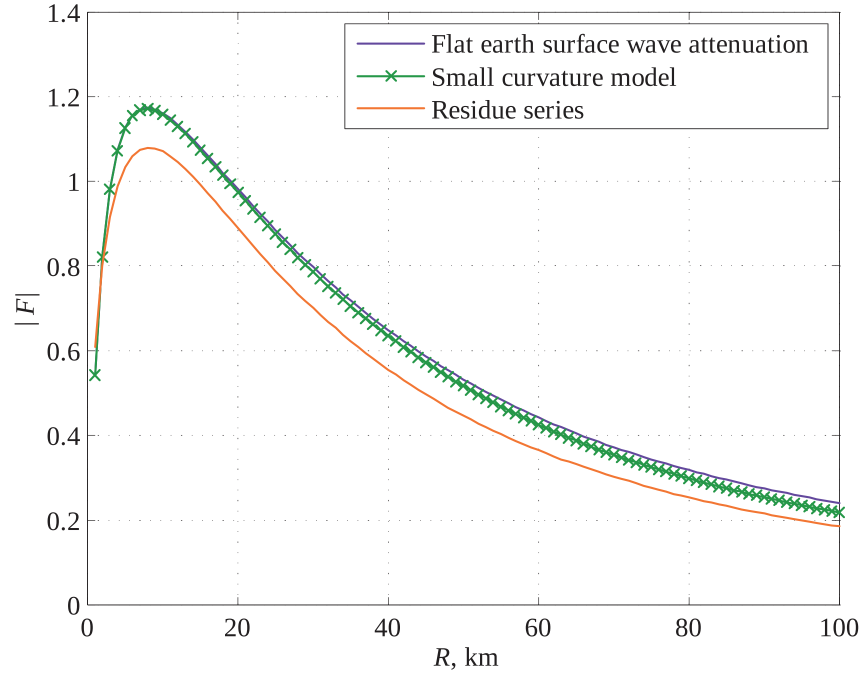

FIGURE 5.8. Attenuation factors for Case I calculated with different methods at 1 MHz, $h_t = h_r = 100\,\mathrm{m}$.

parameters. This problem typically models the situation where the path between the source and the observer passes from land to sea or from sea to land. Depending on antenna heights the space wave and/or surface wave components may be the dominant propagation mode. However, in this section we will consider onlt the surface wave propagation.

The first analytical solution to this problem was proposed by Millington, [**41**]. He argued that up to the discontinuity from the transmitting antenna, the transmission behaved as if the path were homogeneous with parameters of the first region, and afterwards the propagation is as it should if the path were homogeneous with parameters of the second region. This creates a discontinuity in the vertical component of the field at the point where ground parameters change and thus he adds a correction term. For transmission in the reverse direction, he employes the same argument, but this gives rise to different fields in the two directions, which violates the reciprocity principle. He then takes the geometric mean of the fields in two directions to satisfy reciprocity. Although the derivation is heuristic, his result was later shown to be valid, [**34**] for practical situations.

The algorithm proposed by Millington can be best explained by an example. We will consider the three segment model shown in Fig. 5.11. The first step is to calculate the path loss from the transmitter to the receiver (assuming antennas are on the ground) as

$$F_{tr} = F_1\left(d_1\right) - F_2\left(d_1\right) + F_2\left(d_1 + d_2\right) - F_3\left(d_1 + d_2\right) + F_3\left(d_1 + d_2 + d_3\right) \tag{5.5.1}$$

where $F_1\left(d_1\right)$ is the attenuation factor in dB over distance d_1 using the parameters of the first region, $F_2\left(d_1\right)$ is the attenuation factor in dB over distance d_1 using parameters of the second region, $F_2\left(d_1 + d_2\right)$ is the attenuation factor in dB over distance $d_1 + d_2$ using

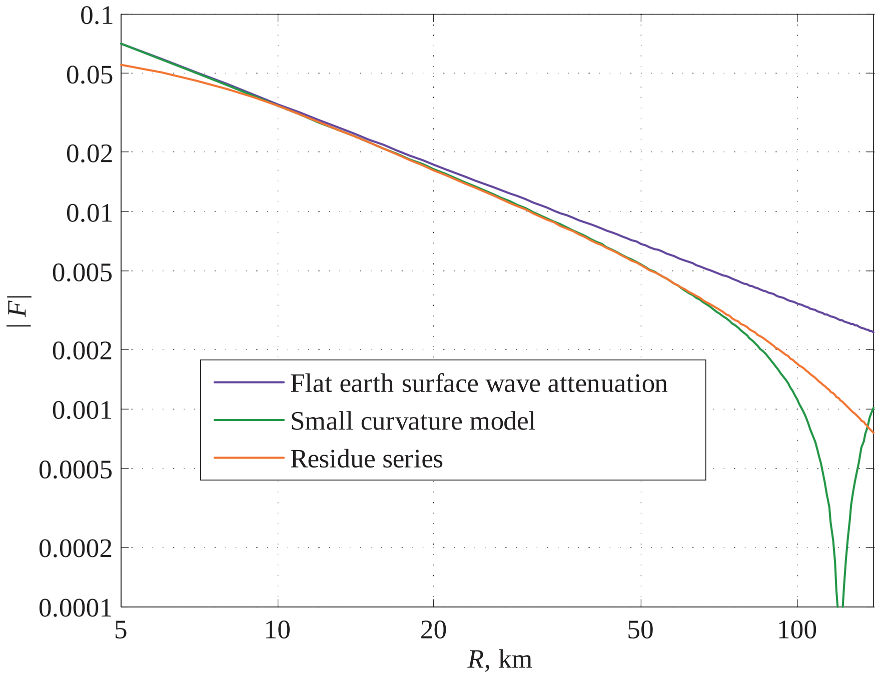

FIGURE 5.9. Attenuation factors for Case II calculated with different methods at 10 MHz, $h_t = h_r = 1\,\mathrm{m}$.

parameters of the second region, $F_3(d_1 + d_2)$ is the attenuation factor in dB over distance $d_1 + d_2$ using parameters of the third region, and $F_3(d_1 + d_2 + d_3)$ is the attenuation factor in dB over distance $d_1 + d_2 + d_3$ using parameters of the third region. Thus the subscript of L indicates the region whose parameters should be used. Notice that the negative terms take away the path loss due to its region from the following term. For example, if d_3 is made zero, the last two terms would cancel, leaving the two segment solution. Thus, the negative terms are the corrections for continuity.

The second step is to make the calculation in the reverse direction, i.e.,

$$F_{rt} = F_3(d_3) - F_2(d_3) + F_2(d_3 + d_2) - F_1(d_3 + d_2) + F_1(d_3 + d_2 + d_1). \qquad (5.5.2)$$

This time, $F_3(d_3)$ is the attenuation factor in dB over distance d_3 using the parameters of the third region, $F_2(d_3)$ is the attenuation factor in dB over distance d_3 using parameters of the second region, $F_2(d_3 + d_2)$ is the attenuation factor in dB over distance $d_3 + d_2$ using parameters of the second region, $F_1(d_3 + d_2)$ is the attenuation factor in dB over distance $d_3 + d_2$ using parameters of the first region, and $F_1(d_3 + d_2 + d_1)$ is the attenuation factor in dB over distance $d_2 + d_2 + d_1$ using parameters of the first region.

The third step is to take the geometric mean of the two results. In dB we have

$$F_0 = \frac{F_{tr} + F_{rt}}{2}. \qquad (5.5.3)$$

This step automatically satisfies reciprocity.

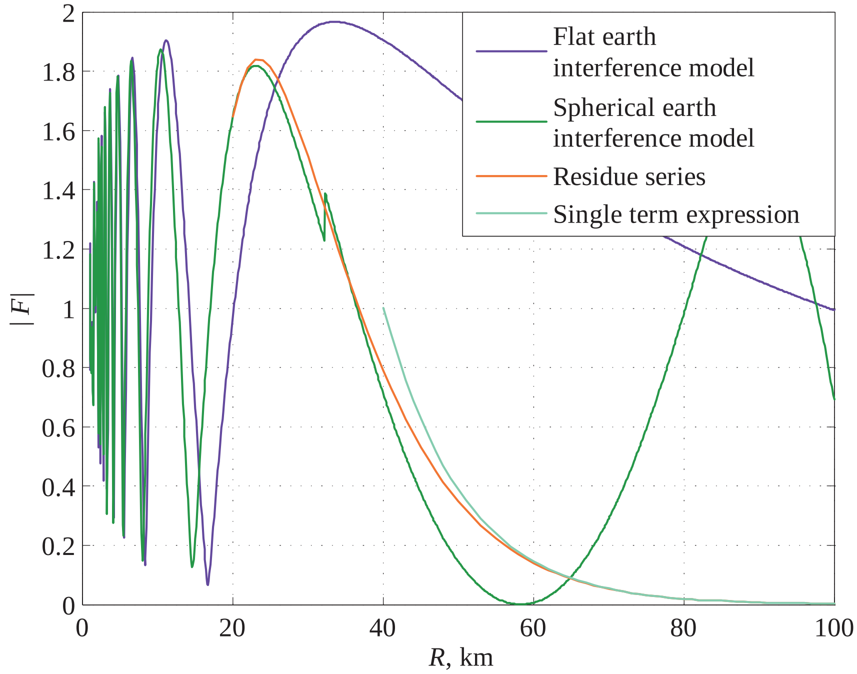

FIGURE 5.10. Attenuation factors for Case III calculated with different methods at 1 GHz, $h_t = h_r = 50\,\mathrm{m}$.

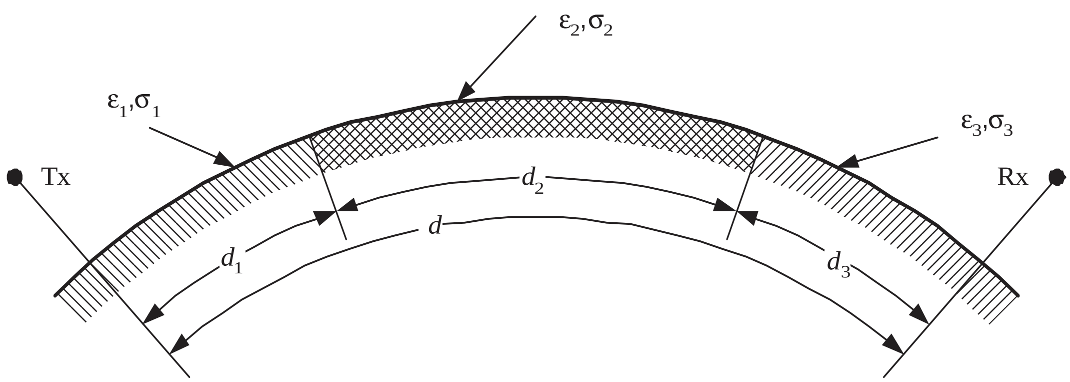

FIGURE 5.11. Geometry for three segment spherical earth mixed path.

The final step is to correct for antenna heigths by introducing the height-gain functions as

$$FL_t = \frac{F_{tr} + F_{rt}}{2} - 20\log_{10}\left(|U(y_t)|\,|U(y_r)|\right) \tag{5.5.4}$$

where y_t and y_r are the normalized heights of the transmitter and the receiver, respectively. Note that the heigt-gain functions depend on the ground parameters, and the parameters of the ground over which the antenna resides should be used. The height-gain functions can be approximated as

$$U(y) = 1 + jkh\Delta \tag{5.5.5}$$

where h is the height of the antenna Δ is the normalized surface impedance.

Figure 5.12 shows the attenuation factor as a function of distance for a three segment mixed path. The first and last sections have the parameters of sea ($\kappa' = 80$, $\sigma = 5\,\mathrm{S/m}$) while the middle section models wet ground ($\kappa' = 30$, $\sigma = 10^{-2}\,\mathrm{S/m}$). The middle region extends from 100 km to 300 km. The field decreases rapidly over the land section and increases beyond the second boundary. Millington coins the name land-to-sea recovery to this phenomenon, which has been experimentally verified, [**41**]. However, experiments generally do not show such a sharp increase as shown in Fig. 5.12 which is due to the fact that the separation between segments are not sharply defined in real paths.

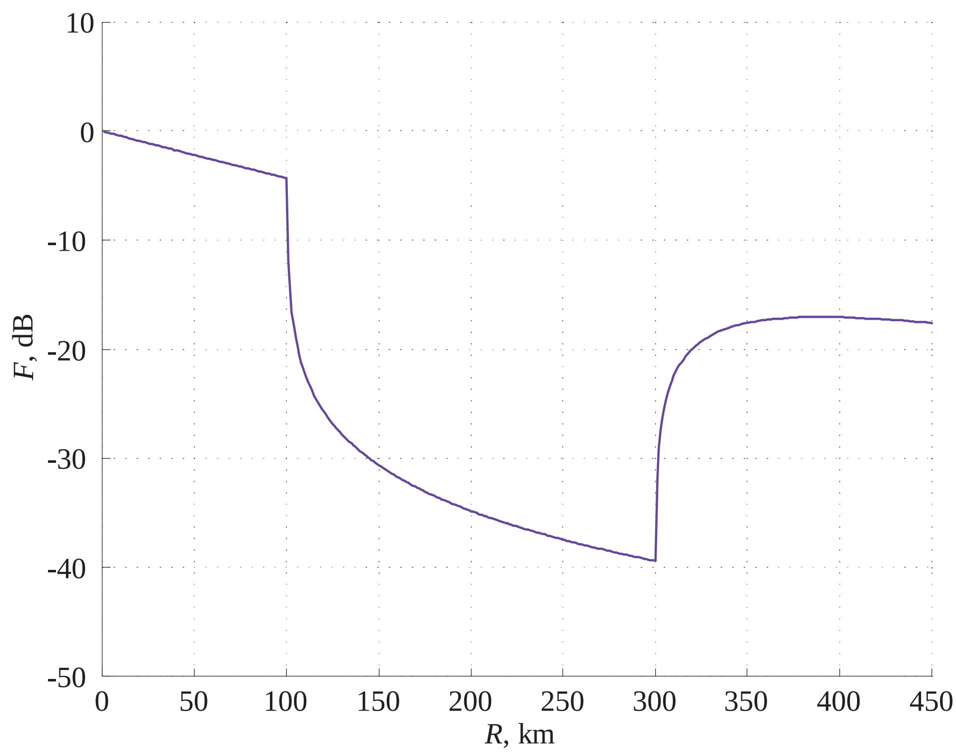

FIGURE 5.12. Surface wave attenuation factor for a three segment mixed path propagation medium.

5.6. Four Ray Model

In many cases, the line of sight between the transmitter and the receiver is obstructed. We have seen in section 2.2 that the signal may arrive at the receiver through diffraction. The presence of the earth casues ground reflected waves. The reflected waves also undergo diffraction and the four ray model is used in such cases.

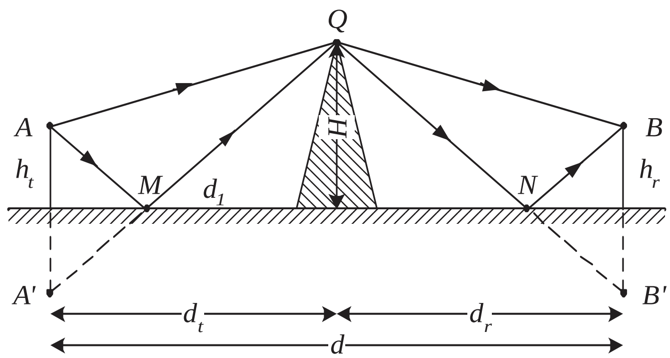

FIGURE 5.13. Four ray model geometry.

The geometry of the four ray model is shown in Fig. 5.13. The rays transmitted by the transmitter at point A arrive the receiver at point B through four paths:

(1) Directly from A to Q, then after diffraction directly to B, (path AQB),
(2) Directly from A to Q, then after diffraction, by reflection at point N to B, (path AQB'),
(3) From A to the reflection point M, then to Q, then after diffraction, directly to B, (path $A'QB$),
(4) From A to the reflection point M, then to Q, then after diffraction, by reflection at point N to B, (path $A'QB'$).

The field at the receiver is then the sum of the signals propagated through these four paths. It has been observed in certain cases that, after propagating through mountain ranges radio waves can exhibit a build up of the field, especially at microwave frequencies, [**42**]. This *obstacle gain* can be explained by the four ray model. It is important to understand what is meant by the gain mentioned here. The point at point B can be larger than it would be if the obstacle were not there, altough intuitively, one would expect the field to be smaller when there is an obstacle.

The field at point B can be calculated by using interference formulas. The attenuation factor at point B can be written as

$$F_T = \Big| F_{c,1} e^{-jk_0\left(R_{AQ}+R_{QB}\right)} + F_{c,2}\Gamma_M e^{-jk_0\left(R_{A'Q}+R_{QB}\right)} + F_{c,3}\Gamma_N e^{-jk_0\left(R_{AQ}+R_{QB'}\right)} + F_{c,4}\Gamma_M\Gamma_N e^{-jk_0\left(R_{A'Q}+R_{QB'}\right)} \Big| \quad (5.6.1)$$

where $F_{c,i}$, $i = 1, \ldots, 4$ is the complex attenuation factor of the four paths calculated including the attenuation due to obstruction, and Γ_M and Γ_N are the reflection coefficients at points M and N, respectively. The attenuation factor $F_{c,i}$ is given in (2.2.7) as

$$F_{c,i} = \frac{1}{\sqrt{2}}\left[C\left(v_i\right) + jS\left(v_i\right)\right] \quad (5.6.2)$$

TABLE 5.1. Quantities required for evaluation of $F_{c,i}$.

Ray	**Att. factor**	**Tx location**	**Rx location**	**Relative path length**	**Clearence height**	**Reflection coefficient**
1	$F_{c,1}$	A	B	0	$h_1 = H - \frac{h_t d_2 + h_r d_1}{d}$	1
2	$F_{c,2}$	A	B'	$\frac{2h_t H}{d_1}$	$h_2 = H - \frac{h_t d_2 - h_r d_1}{d}$	Γ_N
3	$F_{c,3}$	A'	B	$\frac{2h_r H}{d_2}$	$h_3 = H + \frac{h_t d_2 - h_r d_1}{d}$	Γ_M
4	$F_{c,4}$	A'	B'	$\frac{2h_t H}{d_1} + \frac{2h_r H}{d_2}$	$h_4 = H + \frac{h_t d_2 + h_r d_1}{d}$	$\Gamma_N \Gamma_M$

Defining h_i as the clearence height and h_1 as the radius of the first Fresnel zone at the location of the obstacle given by

$$h_F = \sqrt{\frac{d_1 d_2 \lambda_0}{d}},$$

we have $v_i = \sqrt{2} h_i / h_F$.

If the obstacle height is much greater than the antenna heights, we may assume that $F_{c,i}$ are all same, and simplify (5.6.1) as

$$F_T = F \left|1 + \Gamma_M e^{-jk_0 \Delta R_1}\right| \left|1 + \Gamma_N e^{-jk_0 \Delta R_2}\right| \tag{5.6.3}$$

where $F = |F_{c,i}|$, $\Delta R_1 = R_{A'Q} - R_{AQ}$, $\Delta R_2 = R_{QB'} - R_{QB}$. Furthermore, if $\Gamma_M = \Gamma_N = -1$, we have

$$F_T = F \left|1 - e^{-jk_0 \Delta R_1}\right| \left|1 - e^{-jk_0 \Delta R_2}\right|. \tag{5.6.4}$$

It can be easily seen that if $\Delta R_1 = (2n+1)\,\lambda_0/2$, and $\Delta R_2 = (2m+1)\,\lambda_0/2$, the attenuation factor becomes $F_T = 4F$.

To calculate the complex attenuation factors, we must include the diffraction loss due to the obstruction by using (2.2.7). With reference to Fig. 5.13, the quantities required for the calculation of propagation factors $Fc_{,i}$ are listed in Table 5.1.

As an example, consider a geometry for which $h_t = h_r = 30\,\text{m}$. Figure 5.14 shows the attenuation factor plotted against the obstacle height H for a propagation path of $d = 80\,\text{km}$. The maximum obstacle gain of 15 dB is obtained for an obstacle height of 2000 m. Note that as the obstacle height goes zero the four ray theory predicts the no obstacle attenuation factor as expected.

5.7. Examples

EXAMPLE 16. *Using the spherical earth model, calculate the attenuation factor F for a radar operating at* 3 GHz *sited at a height of $h_r = 30\,\text{m}$ above sea and observing a target at a distance of* 125 km *and a height of $h_t = 3000\,\text{m}$ above sea. The radar antenna has a beam width of* 3° *and is tilted above the horizon by* 0.5°*. The radar uses vertical polarization.*

SOLUTION 16. *Taking $a_e = 8500\,\text{km}$, we can determine the horizon distance R_h as*

$$R_h = \sqrt{2a_e h_r} + \sqrt{2a_e h_t} = 248\,\text{km}. \tag{5.7.1}$$

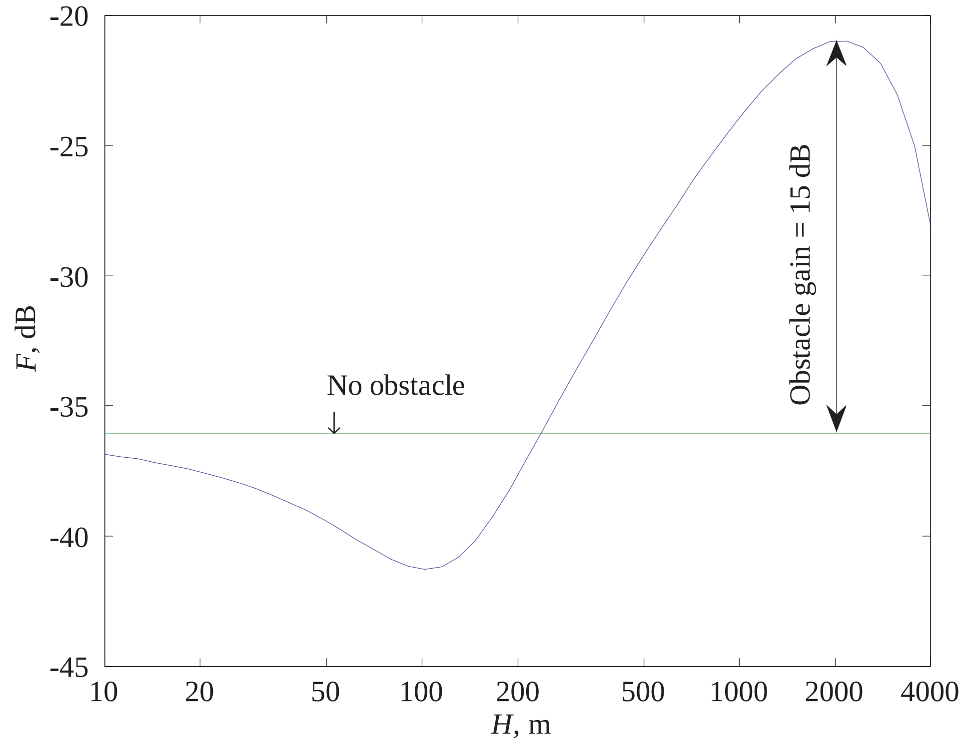

FIGURE 5.14. Attenuation factor versus obstacle height, H.

Since $R < 0.8R_h$ *we can use the interference region formulas and calculate*

$$p = 1.989 \times 10^5\,; \quad \Phi = 2.502\,\mathrm{rad}\,; \quad d_1 = 1745\,\mathrm{m}\,; \tag{5.7.2}$$

$$d_2 = 123\,255\,\mathrm{m}\,; \quad S_1 = 7.727 \times 10^{-2}; \quad S_2 = 0.546\,; \tag{5.7.3}$$

$$T = 0.1\,; \quad S = 0.503\,; \quad J = 0.698\,; \tag{5.7.4}$$

$$K = 0.705\,; \quad D = 0.988\,; \quad \Delta R = 1.005\ \mathrm{m}\,; \tag{5.7.5}$$

$$\tan\psi = 1.709\,; \quad \psi = 1.709 \times 10^{-2}\,\mathrm{rad} = 0.979\,^\circ. \tag{5.7.6}$$

The value of ΔR *being greater than* $\lambda_0/4$ *is another indicator for the validity of interference region formulas. For sea water we can take* $\kappa' = 80$ *and* $\sigma = 5\,\mathrm{S/\,m}$*, and the reflection coefficient for vertical polarization becomes*

$$\Gamma_v = -0.729 - 4.182 \times 10^{-2} j. \tag{5.7.7}$$

We must also take the antenna pattern effects into account. The direct wave angle ξ_d *can be found as*

$$\xi_d = \cos^{-1}\left(\frac{(a_e + h_t)^2 - (a_e + h_r)^2 - R_d^2}{2\,(a_e + h_r)\,R_d}\right) - \frac{\pi}{2} = 0.94\,^\circ \tag{5.7.8}$$

while the reflected wave angle is $-\psi$*. Since the radar beam is tilted above the horizon, the direct and reflected waves arrives at* $0.44\,^\circ$ *and* $-1.479\,^\circ$ *from bore sight of the antenna. Thus* *if $f(\theta)$ denotes the radar antenna field pattern, we must determine $f(0.44)$ and $f(-1.479)$. The patterns of antennas can be well approximated by a Gaussian function in their main beam, i.e., the antenna pattern can be written as*

$$f(\theta) = \exp\left(-2\ln 2\left(\frac{\theta}{\theta_{BW}}\right)^2\right). \tag{5.7.9}$$

where θ_{BW} is the 3dB beam width of the antenna. For our case

$$f(0.44) = 0.971; \quad f(-1.479) = 0.714. \tag{5.7.10}$$

Finally we find

$$F = \left|f(\xi_d) + f(\xi_r)\, D\Gamma_v e^{-jk_0\Delta R}\right| = 0.491 = -6.178\, dB. \tag{5.7.11}$$

This indicates that the field strength incident on the target is $6.178\, dB$ smaller than it would be under free space conditions with the antenna pointing directly at the target. If we ignore the antenna pattern, we would have

$$F = \left|1 + D\Gamma_v e^{-jk_0\Delta R}\right| = 0.354 \tag{5.7.12}$$

which is about 3dB different.

EXAMPLE 17. *A horizontally polarized radar operating at* $10\,\mathrm{GHz}$ *is sited at an elevation of* $20\,\mathrm{m}$ *and tracking an airplane flying over sea at a height of* $500\,\mathrm{m}$. *At which range, R_0, the plane will be in the maximum of the lowest lobe?*

SOLUTION 17. *The total horizon range is $R_h = \sqrt{2a_e h_r} + \sqrt{2a_e h_t} = 110.6\,\mathrm{km}$. The phase difference between the direct and reflected waves is*

$$\alpha = k_0\Delta R + \phi \tag{5.7.13}$$

where ϕ is the phase angle of the reflection coefficient. If ϕ changes as a function of range, this problem becomes very difficult to solve. However, since the propagation is over sea and horizontal polarization is used, we may assume that $\phi = \pi$. Counting from the lowest lobe, the maxima will occur when $\alpha = 2n\pi$ or $\Delta R = (2n-1)\lambda_0/2$. Thus to be at the lowest maximum we must have

$$\Delta R = \frac{\lambda_0}{2}. \tag{5.7.14}$$

Now we can calculate the value of the Q function as

$$Q = \frac{R_h\Delta R}{2h_r h_t} = 0.082\,95. \tag{5.7.15}$$

We also have $T = 0.2$. Since J, S, and T are implicitly related, we must work the equations in (3.2.12) backwards. However, this requires the solution of a highly nonlinear system. The solution can be done by using the graphs prepared for this purpose. We can read the value of S from Fig. 5.19 as $S = 0.89$ which gives

$$R_0 = SR_h = 98.4\,\mathrm{km}. \tag{5.7.16}$$

Another approach would be to determine ΔR for several values of R_0 and make a table as

$d\,(\mathrm{km})$	70	80	90	100
$\Delta R\,(\mathrm{cm})$	13.3	8.0	3.9	1.1

By linear interpolation we can find $R_{0,1} = 98.6\,\mathrm{km}$*. A second iteration gives* $R_{0,2} = 98.2\,\mathrm{km}$ *which is the correct value to one decimal place.*

EXAMPLE 18. *Assume that a communication link operates at* $f = 3\,\mathrm{GHz}$ *with vertical polarization above medium dry ground. The distance between the terminals is* $100\,\mathrm{km}$ *and the terminals are at* $50\,\mathrm{m}$ *and* $100\,\mathrm{m}$ *above earth. Determine how far below the free-space level the signal will be.*

SOLUTION 18. *The total horizon range is* $\sqrt{2a_eh_1} + \sqrt{2a_eh_2} = 70.4\,\mathrm{km}$*, which means that the terminals are well beyond the horizon and the single term expression can be used. The electrical parameters for medium dry ground are* $\kappa' = 15$*,* $\sigma = 10^{-3}\,\mathrm{S/\,m}$ *from Table (3.1). The normalized surface impedance is then*

$$\Delta = \frac{\sqrt{\kappa - 1}}{\kappa} = 0.249\ + 4.63 \times 10^{-5} j \tag{5.7.17}$$

We have

$$x = \left(\frac{k_0 a_e}{2}\right)^{1/3} \left(\frac{R}{a_e}\right) = 7.58 \tag{5.7.18}$$

which is greater than $10/3$ *and justifies the use of the single term expression. Next we determine*

$$q = -j \left(\frac{k_0 a_e}{2}\right)^{1/3} \Delta = 2.98 \times 10^{-2} - 161j. \tag{5.7.19}$$

Since $|q| \gg 1$ *we may assume that* z_n *are the zeros of* $u(z)$*. We have*

$$V(x) = 2.2 \times 10^{-6} = -113.1\ dB, \tag{5.7.20}$$

$$U(y_1) = 14.63 = 23.3\ dB, \tag{5.7.21}$$

$$U(y_2) = 87.47 = 39.0\ dB. \tag{5.7.22}$$

and

$$F = -50.8\ dB. \tag{5.7.23}$$

Hence the field strength will be 50.8 *dB below the free space level.*

EXAMPLE 19. *What should be the power of a transmitter operating at* $f = 1\,\mathrm{MHz}$ *with vertical polarization over wet ground to produce a field of* $E_{rms} = 40\,\mu\mathrm{V/\,m}$ *at a distance of* $500\,\mathrm{km}$*. The transmitter antenna is a vertical monopole with a gain of* 1.5*. Assume that the propagation is over medium ground.*

SOLUTION 19. *Using single term expression we find*

$$V(x) = 2.85 \times 10^{-2}.$$

Antenna height is not given in the problem but at $\lambda_0 = 300\,\mathrm{m}$*, it is reasonable to assume that* $U_1(y_t) = 1$*. Thus the attenuation factor is* $F = 2.85 \times 10^{-2}$*. We can write*

$$\frac{\sqrt{30 P_t G_t}}{R} F = 40 \times 10^{-6}.$$

Solving, we find $P_t = 10.95\,\mathrm{kW}$*.*

SOLUTION 20. $R = 500\,000$

EXAMPLE 20. *A microwave communication link is operating at $f = 3\,\mathrm{GHz}$ with horizontal polarization over "wet ground." The link antennas are mounted on towers at a height of $h = 50\,\mathrm{m}$. Assuming a smooth earth, determine the maximum distance d so that the signal level is not reduced below its free space value.*

SOLUTION 21. *An attenuation factor of $F = 1$ is required. The total horizon distance for this case is $R_h = 58.3\,\mathrm{km}$. Assuming $\Gamma_h = -1$, and $D = 1$ the attenuation factor is given by*

$$F = 2\left|\sin\frac{k_0 \Delta R}{2}\right|. \tag{5.7.24}$$

Setting $F = 1$ we find that the path length difference should be $\Delta R = \lambda_0/6$. If we use flat earth model, we have

$$\frac{2h^2}{d} = \frac{0.1}{6} \Longrightarrow d = 300\,\mathrm{km} \tag{5.7.25}$$

which is greater than the total horizon distance. Therefore, we must use spherical earth model. Since $\Delta R < \lambda_0/4$ we will take $D = 1$. The antennas being at equal height implies $d_1 = d_2 = d/2$ and $S_1 = S_2 = d/(2\sqrt{2a_e h})$ in the spherical earth formulas. Thus,

$$J(S,T) = \left(1 - S_1^2\right)^2 = \left(1 - \left(\frac{d/2}{\sqrt{2a_e h}}\right)^2\right)^2 \tag{5.7.26}$$

and we get

$$\frac{2h^2}{d}\left(1 - \frac{d^2}{8a_e h}\right)^2 = \frac{1}{60}. \tag{5.7.27}$$

The only unknown in this equations is d and can be solved to give $d = 45.5\,\mathrm{km}$ (the second real solution gives $d = 71\,\mathrm{km}$ which is larger than the total horizon range). At this distance the grazing angle is $\psi = 8.59 \times 10^{-4}\,\mathrm{rad}$ and $\Gamma_h = 0.999\,5\angle 180^\circ$ justifying the assumption made.

To solve (5.7.27) rewrite it as

$$d_{\mathrm{km}} = g\left(d_{\mathrm{km}}\right) = 10\sqrt{34}\sqrt{\left(1 - \sqrt{\frac{d_{\mathrm{km}}}{300}}\right)}. \tag{5.7.28}$$

Starting with an arbitrary value of d_0, use $d_{k+1} = g\left(d_k\right)$.

EXAMPLE 21. *For the microwave communication link of example 20, determine the attenuation factor at $d = 60\,\mathrm{km}$.*

SOLUTION 22. *The distance $d = 60\,\mathrm{km}$ is in the intermediate region. We will use interpolation method for this example. We choose two points in the interference region. The first point is where the first maximum occurs where $\Delta R = \lambda_0/2$ and $F = 1 + D\left|\Gamma\right|$. The second point is the first point of quadrature, $\Delta R = \lambda_0/4$ and $F = \sqrt{1 + D^2\left|\Gamma\right|^2}$. These points are* *found as*

$$\frac{2h^2}{d}\left(1-\frac{d^2}{8a_eh}\right)^2=\frac{1}{20}\Longrightarrow d_m=36.6\,\mathrm{km}\,;\qquad D_m=0.659\,;\qquad F_m=4.4\,dB, \tag{5.7.29}$$

$$\frac{2h^2}{d}\left(1-\frac{d^2}{8a_eh}\right)^2=\frac{1}{40}\Longrightarrow d_q=42.7\,\mathrm{km}\,;\qquad D_q=0.549\,;\qquad F_q=1.2\,dB. \tag{5.7.30}$$

Next we choose three points in the diffraction zone, at $d = 1.3R_h$, $1.5R_h$, *and* $2R_h$, *and calculate the attenuation factors by the single term expression. The normalized height of the antennas are* $y = 4.879$ *giving* $U_1(y_1) = U_2(y_2) = 23.3\,dB$. *The normalized distances and corresponding values of* $V(x)$ *are*

$$x_2 = 5.74\,;\qquad V(5.74) = -82.2\,dB, \tag{5.7.31}$$

$$x_{2.5} = 6.63;\qquad V(11.04) = -97.1\,dB, \tag{5.7.32}$$

$$x_3 = 8.83\,;\qquad V(8.83) = -134.6\,dB. \tag{5.7.33}$$

Using these results we can form the plot shown in Fig. 5.15. Note that $F = 1$ *is obtained at* $d = 44.3$ *which is fairly accurate. The difference from example 20 is due to the divergence factor being taken as* 1 *in that example.*

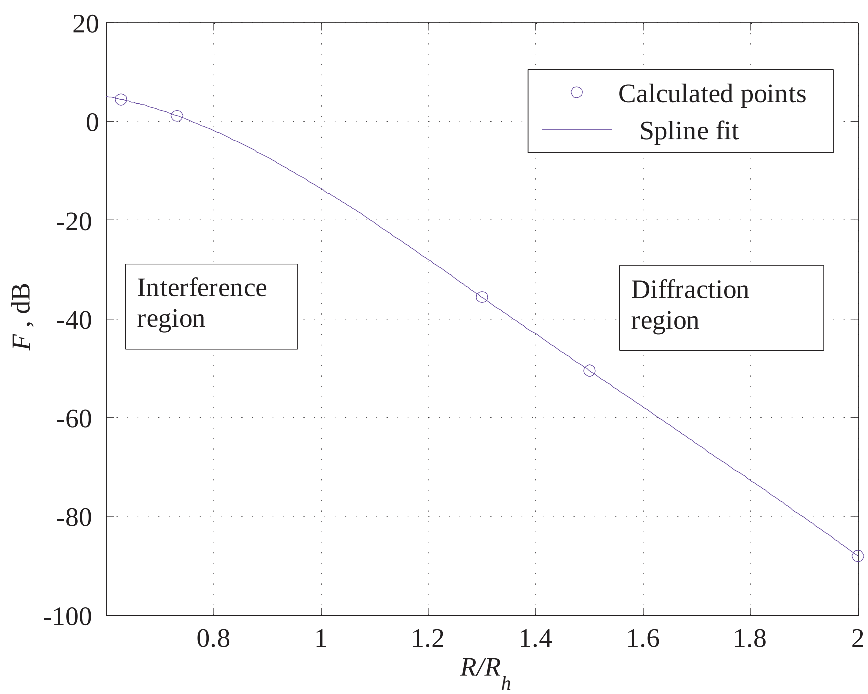

FIGURE 5.15. Attenuation factor in dB for $h_t = h_r = 50\,\mathrm{m}$, $f = 3\,\mathrm{GHz}$.

EXAMPLE 22. *Consider a citizen's band system operating at* 27 MHz *in rural environment (smooth earth assumption is valid). Both the transmitter and receiver antennas are very close* *to the ground and have unity gains. The transmitter power is* $5\,\mathrm{W}$*, and the receiver noise figure is* 10*, and the receiver bandwidth is* $5\,\mathrm{kHz}$*. Propagation is over medium ground. If a* 20 *dB SNR is required at the receiver, what would be the maximum range for both vertical and horizontal polarizations?*

SOLUTION 23. *The noise power at the receiver is*

$$P_n = kTBF = 1.38 \times 10^{-23} \times 300 \times 5000 \times 10 = 2.07 \times 10^{-16}\,\mathrm{W}. \tag{5.7.34}$$

The required signal power for 20 *dB SNR is then*

$$P_s = 2.07 \times 10^{-14}\,\mathrm{W} = -137\ dBm. \tag{5.7.35}$$

The received signal power for free space communication is

$$P_{r,fs} = 37 - 20\log_{10}\left(\frac{4\pi}{\lambda_0}\right) + 20\log_{10}(R) = 35.9 - 20\log_{10}(R)\ dBm. \tag{5.7.36}$$

Since both antennas are very close to the ground, the propagation is by means of the surface wave. The ground parameters are $\kappa' = 15$ *and* $\sigma = 10^{-3}\,\mathrm{S/m}$ *from Table (3.1). The numerical distance for vertical polarization is given by*

$$p = \left| -jk_0R\frac{(\kappa-1)}{2\kappa^2} \right| = 0.01758\,R. \tag{5.7.37}$$

At very large distances the surface wave attenuation factor (in dB) changes linearly with $\log_{10} p$ *as can be seen from Fig. 4.2, and we can write*

$$20\log_{10} F = -6 - 20\log_{10}(p) = -6 - 20\log_{10}(0.01758\,R) \tag{5.7.38}$$

$$= 29.1 - 20\log_{10}(R) \tag{5.7.39}$$

Now we can write the received signal as

$$P_r(dBm) = P_{r,fs}(dBm) + F(dB) = 65 - 40\log_{10}(R). \tag{5.7.40}$$

Equating this to -137 *dBm we find*

$$R = 112.2\,\mathrm{km}. \tag{5.7.41}$$

On the other hand, the flat earth formula hold up to $80/(27)^{1/3} = 26.7\,\mathrm{km}$ *range. Beyond this range the effect of spherical earth will cause the received signal to attenuate very rapidly. So it would be logical to assume that the maximum range will be slightly above* $26.7\,\mathrm{km}$*.*

With horizontal polarization the numerical distance is

$$p = \left| -jk_0R\frac{(\kappa-1)}{2} \right| = 439.8R. \tag{5.7.42}$$

Repeating similar calculations we get

$$P_r(dBm) = -23 - 40\log_{10}(R) \tag{5.7.43}$$

from which the maximum distance is determined to be only $0.7\,\mathrm{km}$*. This is due to the large surface impedance of the ground for horizontal polarization.*

EXAMPLE 23. *Assume that an antenna is* $30\,\mathrm{km}$ *from the shore and radiating a power of* $1\,\mathrm{kW}$ *at* $f = 500\,\mathrm{kHz}$ *using a vertical monopole antenna of gain* 1.5*. Determine the field at a distance of* $100\,\mathrm{km}$ *from the shore. Take the land parameters as medium ground.*

Solution 24. *With the definitions in section 5.5 we have*

$$\begin{aligned} F_1(d_1) &= -15.5 \ dB, \\ F_2(d_1) &\approx F_2(d_1+d_2) \approx F_2(d_2) \approx 0 \ dB, \\ F_1(d_2) &= -27.5 \ dB, \\ F_1(d_2+d_1) &= -30 \ dB. \end{aligned}$$

Then we can calculate

$$\begin{aligned} F_{tr} &= -15.5 \ dB, \quad F_{rt} = -2.5 \ dB, \\ F &= -9.0 \ dB. \end{aligned}$$

We can take the antenna height-gain as 1, *since the wavelength is large. Then,*

$$E_{rms} = \frac{\sqrt{30 P_t G_t}}{R} F = 0.58 \,\mathrm{mV/m}.$$

5.8. Appendix A: Calculation of the Roots of $u'(z) - qu(z) = 0$

The computation of the roots z_n appearing in the residue series expression is rather complicated. Fortunately, they are worked in the literature and series expansions for the roots are available, [**39**]. There are two series that can be used for the computations. The first one is in powers of q which is useful when q is small, and the second one is in inverse powers of q which is useful when q is large.

The first series which converges well for small values of q (typically for $|q| \ll \left|\sqrt[4]{\zeta_n \xi_n}\right|$) is

$$z_n = \sum_{m=0}^{\infty} a_m q^m \tag{5.8.1}$$

where $q = -j\left(\frac{k_0 a}{2}\right)^{1/3} \Delta$ as defined in (4.2.4), and $a_0(n) = \xi_n$ is the nth zero of $u'(z)$. The first 8 values of ξ_n are tabulated in Table 5.2. The values of $a_m(n)$ are

$$a_0(n) = \xi_n, \tag{5.8.2a}$$

$$a_1(n) = \frac{1}{a_0(n)}, \tag{5.8.2b}$$

$$a_2(n) = -\frac{a_1^2(n)}{2a_0(n)} = -\frac{1}{2a_0^3(n)}, \tag{5.8.2c}$$

$$a_3(n) = -\frac{3a_1(n)a_2(n) - 2a_2(n) - a_1(n)}{3a_0(n)} = \frac{3 - 2a_0(n) + 2a_0^3(n)}{6a_0^5(n)} \tag{5.8.2d}$$

$$a_{m+1}(n) = -\frac{1}{(m+1)a_0(n)} \sum_{k=1}^{m} (m-k+1) a_k(n) a_{m-k+1}(n), \quad m \geq 3. \tag{5.8.2e}$$

The value of ξ_n for $n > 8$ can be approximated by

$$\xi_n \approx \left[\frac{3\pi}{2}\left(n - \frac{3}{4}\right)\right]^{2/3} e^{-j\pi/3}. \tag{5.8.3}$$

Although this approximation is accurate only to 4% for $n > 8$, in most cases the residue series will be truncated before such terms are required.

TABLE 5.2. First 5 roots of $u(z)$ and $u'(z)$.

n	$\boldsymbol{\xi}_n$	$\boldsymbol{\zeta}_n$
1	$1.018793e^{-j\pi/3}$	$2.338\,107\,e^{-j\pi/3}$
2	$3.248198e^{-j\pi/3}$	$4.087949e^{-j\pi/3}$
3	$4.820099e^{-j\pi/3}$	$5.520560e^{-j\pi/3}$
4	$6.163307e^{-j\pi/3}$	$6.786708e^{-j\pi/3}$
5	$7.372177e^{-j\pi/3}$	$7.944134e^{-j\pi/3}$
6	$8.488487e^{-j\pi/3}$	$9.022651e^{-j\pi/3}$
7	$9.535449e^{-j\pi/3}$	$10.040174e^{-j\pi/3}$
8	$10.527660e^{-j\pi/3}$	$11.008524e^{-j\pi/3}$

The second series which converges well for large values of q (typically for $|q| \gg \left|\sqrt[4]{\zeta_n \xi_n}\right|$) is

$$z_n = \sum_{m=0}^{\infty} b_m q^{-m} \tag{5.8.4}$$

where $b_0(n) = \zeta_n$ is the nth zero of $u(z)$. The first 8 values of ζ_n are also tabulated in Table 5.2. The values of $b_m(n)$ are

$$\begin{array}{ccc} b_1(n) = 1 & b_2(n) = 0 & b_3(n) = \frac{b_0(n)}{3} \\ b_4(n) = \frac{1}{4} & b_5(n) = \frac{b_0^2(n)}{5} & b_6(n) = \frac{7b_0(n)}{18} \\ b_7(n) = \frac{b_0^3(n)+1.25}{7} & b_8(n) = \frac{29b_0^2(n)}{60} & b_9(n) = \frac{b_6(n)+b_0^4(n)+b_1^2(n)+2b_0(n)b_4(n)+2b_3(n)b_1(n)}{9} \end{array}. \tag{5.8.5}$$

The value of ζ_n for $n > 8$ are given approximately by

$$\zeta_n \approx \left[\frac{3\pi}{2}\left(n - \frac{1}{4}\right)\right]^{2/3} e^{-j\pi/3}. \tag{5.8.6}$$

When $q \approx \left|\sqrt[4]{\zeta_n \xi_n}\right|$, neither of the series converge. Therefore, convergence of the series must be checked.

5.9. Appendix B: Graphs for Spherical Earth

The graphs for spherical earth calculations are given.

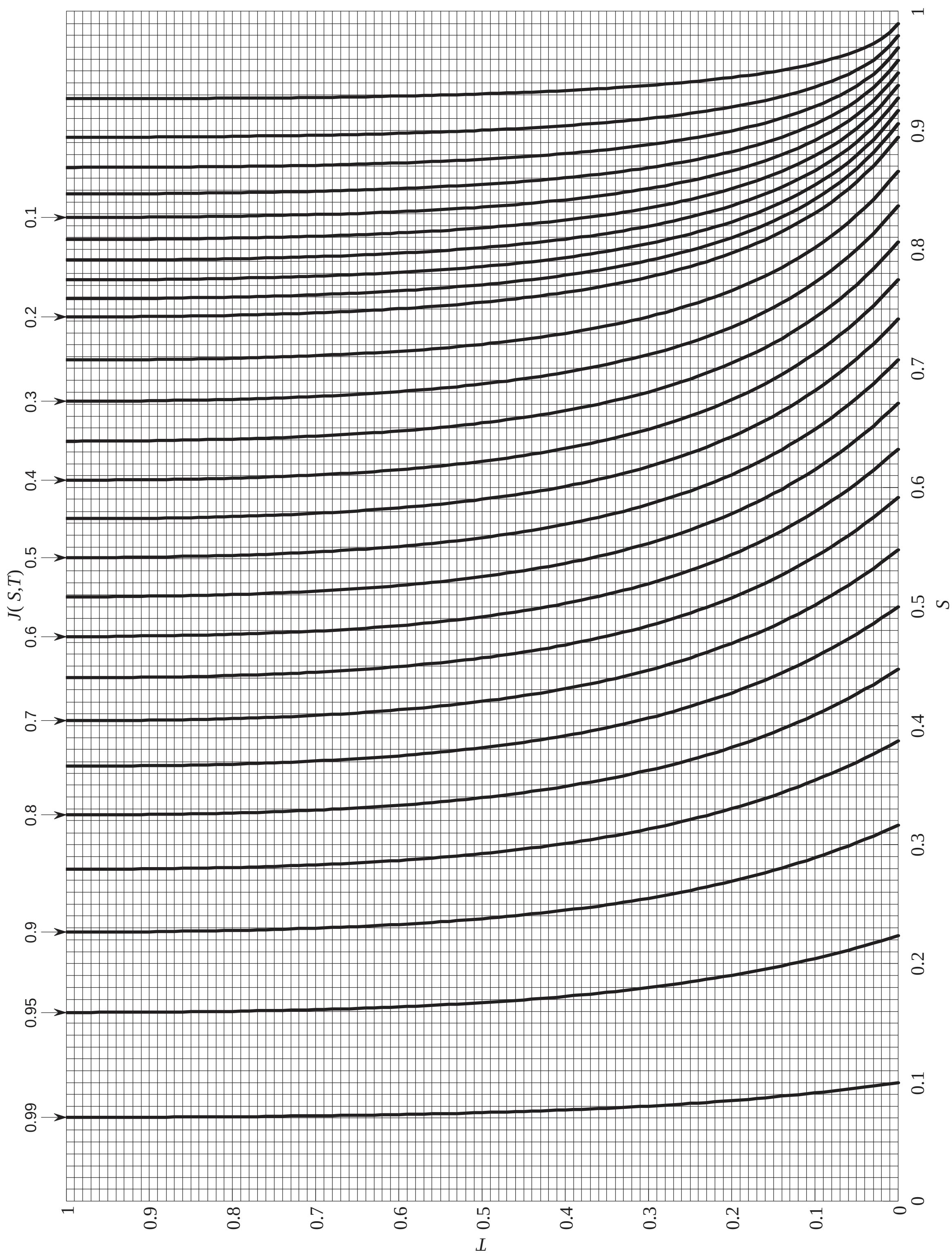

FIGURE 5.16. Correction factor J used in curved-earth pattern-propagation factor calculation, plotted as a function of parameters S and T for standard atmosphere.

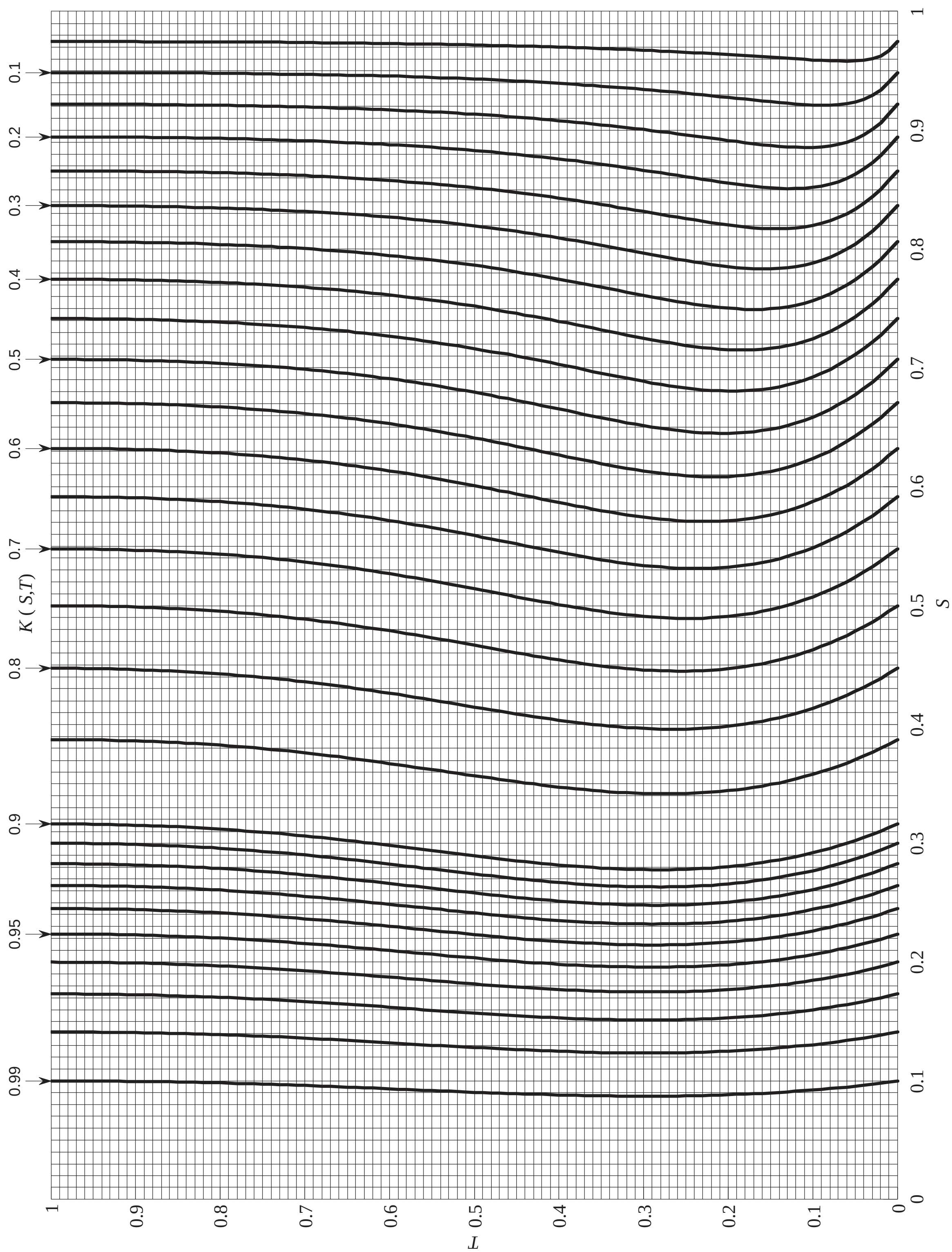

FIGURE 5.17. Correction factor K used in curved-earth pattern-propagation factor calculation, plotted as a function of parameters S and T for standard atmosphere.

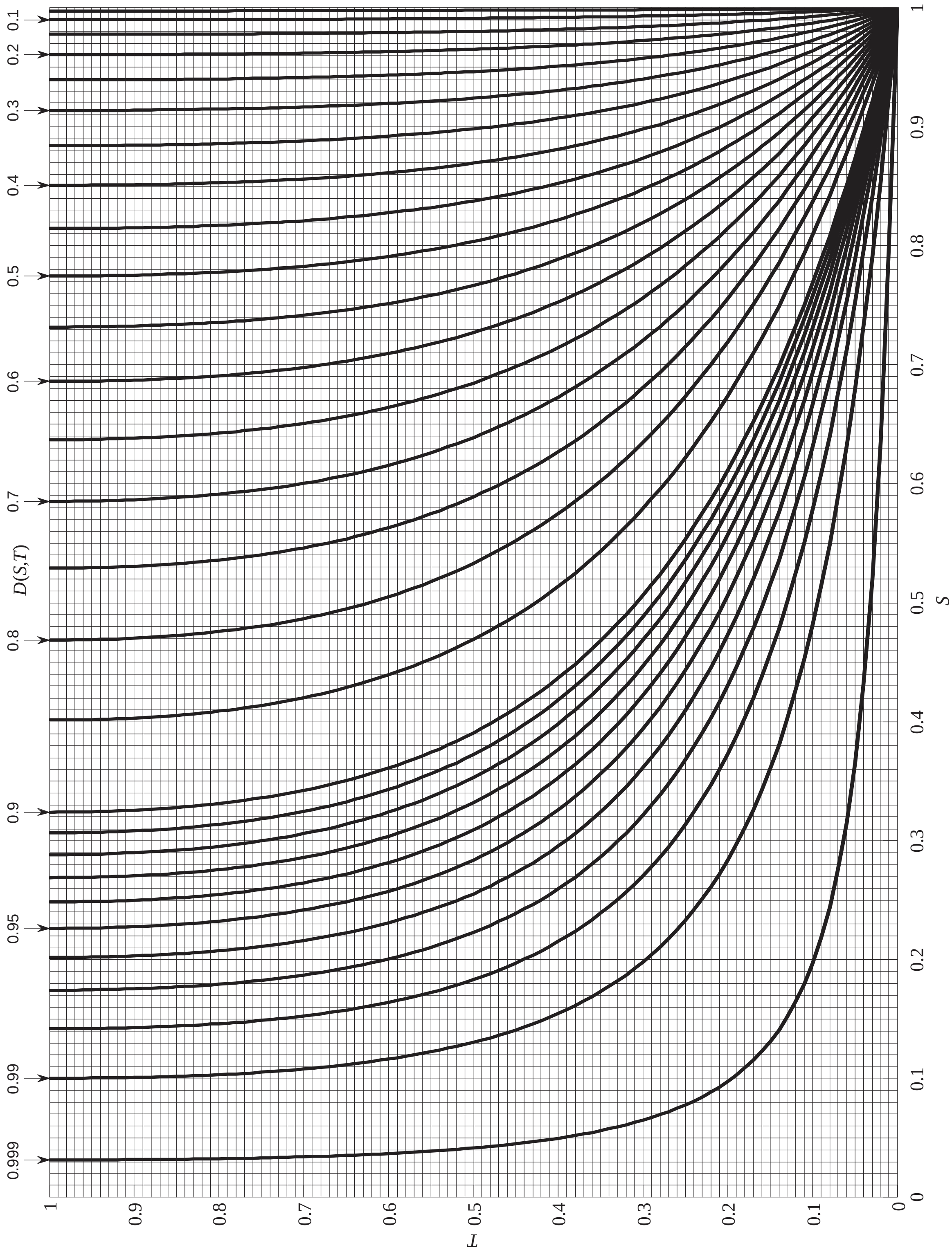

FIGURE 5.18. Divergence factor D used in curved-earth pattern-propagation factor calculation, plotted as a function of parameters S and T for standard atmosphere.

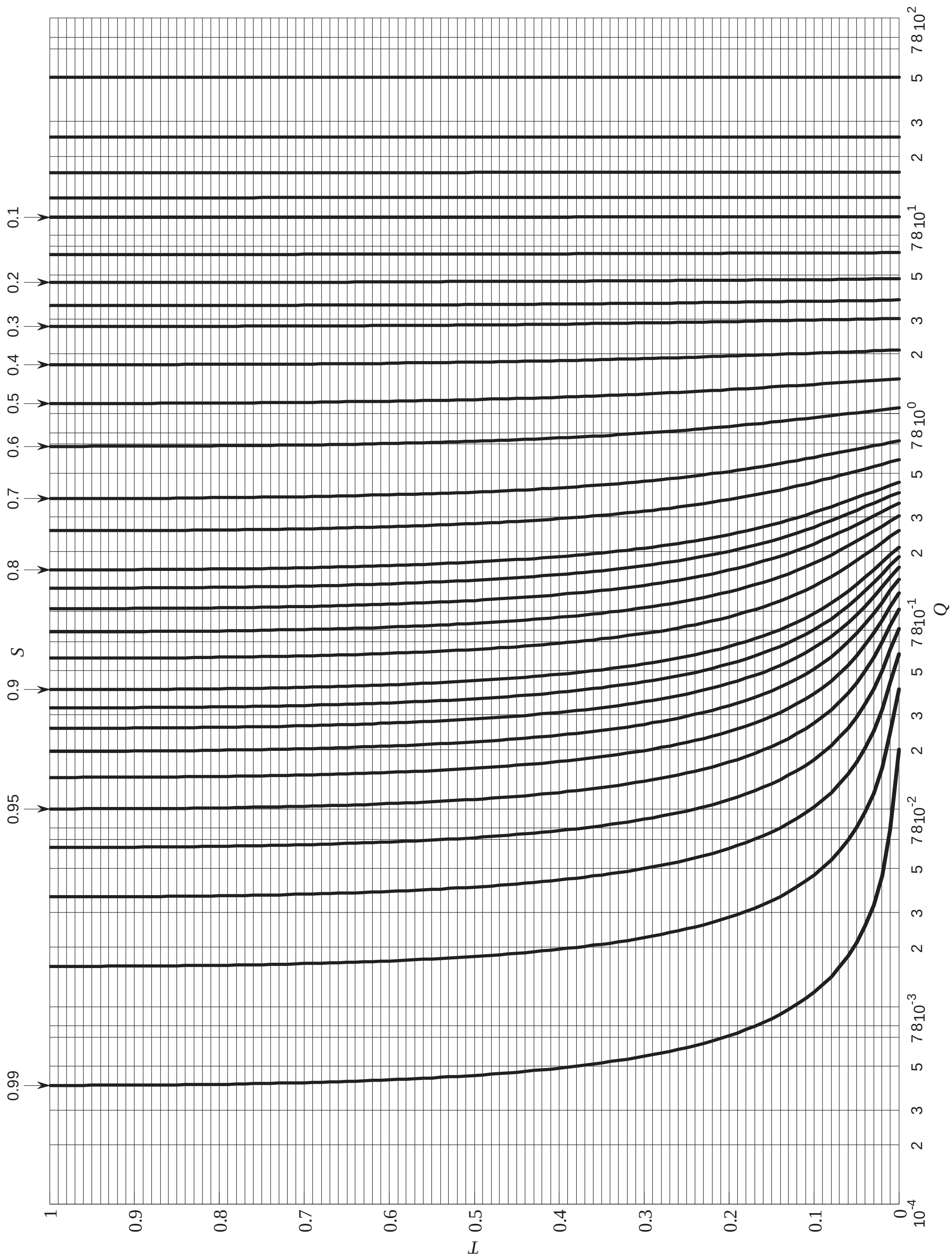

FIGURE 5.19. The function Q plotted as a function of parameters S and T for standard atmosphere.