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CHAPTER 3

Propagation over Earth (Interference Region)

When the antennas of a communication link are placed over the earth, the propagation is strongly influenced by the presence of the earth. There will be a wave reflected from the surface of the earth and the signal arriving at the receiver will be the sum of the direct and ground reflected signals. In this chapter we will consider the interference phenomena associated with antennas placed over the earth.

3.1. Propagation over a Flat Earth

The basic features of the propagation in the presence of earth can be understood by considering antennas located above a flat earth. Figure 3.1 shows the basic geometry for propagation over flat earth. The transmitting antenna is at a height h_t and the receiving antenna is at a height h_r with a distance R between them. A direct ray and a reflected wave arrives at the receiver which combine to form the total signal. The electric field strength at the receiving antenna location due to the direct ray will be proportional to

$$E_d \propto f_t\left(\theta_d\right)\frac{e^{-jk_0R_d}}{R_d} \tag{3.1.1}$$

while that due to indirect (reflected) ray will be proportional to

$$E_r \propto f_t\left(\theta_r\right)\Gamma\frac{e^{-jk_0R_r}}{R_r}. \tag{3.1.2}$$

f_t is the *field strength pattern* of the transmitting antenna, and Γ is the complex reflection coefficient at the ground. It must be noted that the voltage induced at the receiving antenna terminals will be proportional to

$$V_d \propto f_t\left(\theta_d\right)f_r\left(\theta'_d\right)\frac{e^{-jk_0R_d}}{R_d} \tag{3.1.3}$$

while that due to indirect ray will be proportional to

$$V_r \propto f_t\left(\theta_r\right)f_r\left(\theta'_r\right)\Gamma\frac{e^{-jk_0R_r}}{R_r} \tag{3.1.4}$$

where f_r are is the *field strength pattern* of the receiving antenna. In practical cases, h_t and h_r are much smaller than the separation R, so the angles θ_d, θ'_d, θ_r, and θ'_r are very small, and the radiation patterns can be assumed to be constant. An exception is when one of the antennas is located aboard a high flying aircraft and the antennas are highly directive. The total voltage induced at the receiver antenna terminals is proportional to the superposition of the two signals and will be proportional to

$$V_t \propto f_t\left(\theta_d\right)f_r\left(\theta'_d\right)\frac{e^{-jk_0R_d}}{R_d} + f_t\left(\theta_r\right)f_r\left(\theta'_r\right)\Gamma\frac{e^{-jk_0R_r}}{R_r}. \tag{3.1.5}$$

Since the two antennas are assumed to be in the far field of each other we can take $R_d \approx R_r$ in the denominator. However, their difference will be in the same order of wavelength and this approximation is not valid in the phase terms. The total field at the location of the receiver can then be written as

$$E_t = f_t(\theta_d)\frac{e^{-jk_0R_d}}{R_d}\left(1+\Gamma e^{-jk_0(R_r-R_d)}\right) \tag{3.1.6a}$$

$$= f_t(\theta_d)\frac{e^{-jk_0R_d}}{R_d}F. \tag{3.1.6b}$$

The factor F is called the *attenuation factor* and shows how the the field at the receiving antenna differs from the value it would have under free space conditions. If the angles involved are small, the attenuation factor can be written as

$$F = 1+\Gamma e^{-jk_0\Delta R} \tag{3.1.7}$$

where $\Delta R = R_r - R_d$ is the path length difference.

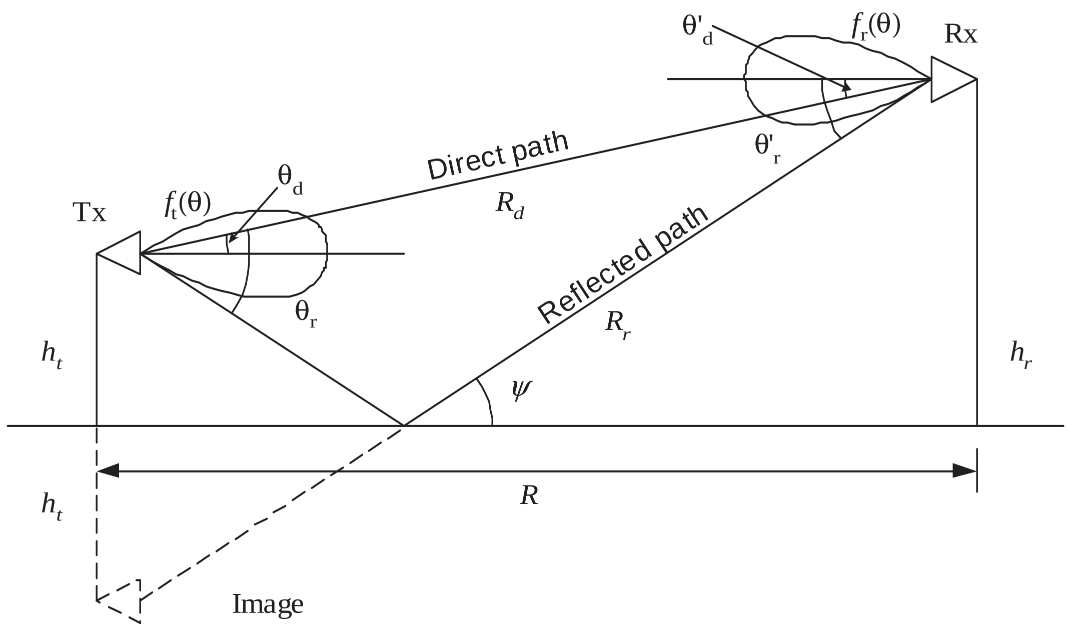

FIGURE 3.1. Geometry for propagation over flat earth.

With reference to Fig. 3.1 we can write

$$R_d = \sqrt{d^2+(h_r-h_t)^2}, \tag{3.1.8a}$$

$$R_r = \sqrt{d^2+(h_r+h_t)^2} \tag{3.1.8b}$$

since the reflected wave can be considered as a wave emerging from the image of the transmitting antenna. When $h_r, h_t \ll R$, we can write

$$R_d = R + \frac{(h_r - h_t)^2}{2R}, \tag{3.1.9a}$$

$$R_r = R + \frac{(h_r + h_t)^2}{2R} \tag{3.1.9b}$$

which gives

$$\Delta R = R_r - R_d = R + \frac{(h_r + h_t)^2}{2R} - R - \frac{(h_r - h_t)^2}{2R} \approx \frac{2h_r h_t}{R}. \tag{3.1.10}$$

Referring to Fig. 3.1, the *grazing angle*, ψ, can be determined as

$$\psi = \tan^{-1} \frac{h_t + h_r}{R_r} \approx \frac{h_t + h_r}{R}. \tag{3.1.11}$$

The approximation is quite accurate for most practical cases since both h_t and h_r are much smaller than R and the grazing angle is very small. The location of the reflection point can also be determined by simple geometry.

3.1.1. Earth as a Perfect Electric Conductor. If we suppose that the earth behaves like a perfect electrical conductor, (which is a good approximation for most cases as will be discussed later) the reflection coefficient will be -1 and we can write

$$|F| = \left|1 - e^{-jk_0 \Delta R}\right| = |1 - \cos k_0 \Delta R + j \sin k_0 \Delta R| = 2 \left|\sin \frac{k_0 h_r h_t}{R}\right|. \tag{3.1.12}$$

This result shows that the total field strength may increase up to twice the value of the field strength that would occur under free space conditions, and may also fall down to zero. Figure 3.2 shows the magnitude of the attenuation factor as a function of distance for $h_t = h_r = 100\,\mathrm{m}$ and $\Gamma = -1$ for $f = 1\,\mathrm{GHz}$. As the distance varies, the attenuation factor passes through a number of maxima when $\sin(k_0 h_r h_t / R) = 1$ and a number of minima when $\sin(k_0 h_r h_t / R) = 0$. The distances for which maxima and minima occur can be found as

$$R_{\max,n} = \frac{4h_r h_t}{(2n+1)\,\lambda_0}, \qquad n = 0, 1, 2, \ldots \tag{3.1.13a}$$

$$R_{\min,n} = \frac{2h_r h_t}{n\lambda_0}, \qquad n = 1, 2, \ldots \tag{3.1.13b}$$

The first maxima (as counted from the large distance end) occurs at $R_{\max} = 4h_r h_t / \lambda_0$, and beyond this distance $|F|$ decreases monotonically. When the distance is sufficiently large, we may approximate the sine function by its argument and write

$$|F| \approx \frac{4\pi h_r h_t}{\lambda_0 R}. \tag{3.1.14}$$

The error in this approximation is less than 1% for $R > 4.5 R_{\max}$.

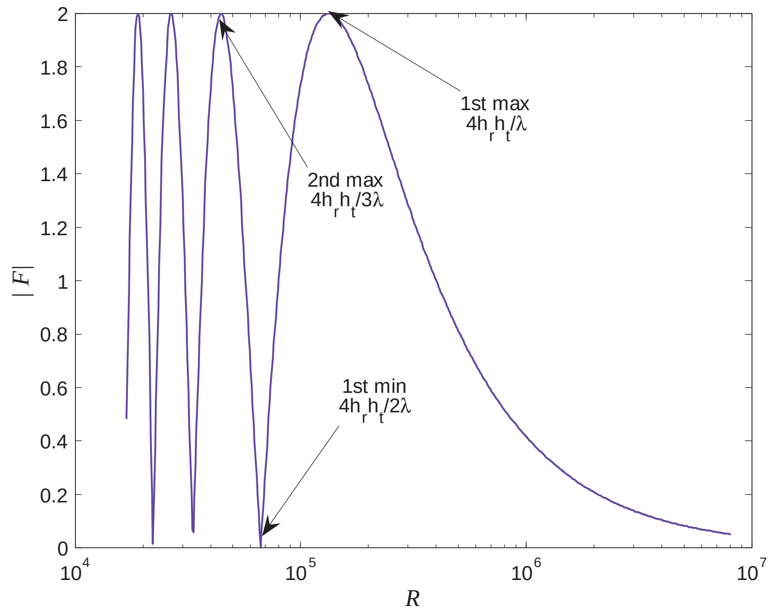

FIGURE 3.2. Attenuation factor versus distance for $\Gamma = -1$, $h_t = h_r = 100\,\text{m}$, and $f = 1\,\text{GHz}$.

Of particular significance is the nulls occurring in the total field as the distance between the two antennas is changed. Depending on the height of transmitting and receiving antennas there will be locations at which no signal will arrive to the receiver. In other words, a broadcasting antenna will not be able to cover all the locations inside a circle of maximum distance.

A coverage diagram is a plot of the field strength as a function of the location of the receiver. Generally, contours of constant field strength are plotted, although this is not necessarily the case. If we assume that the transmitter antenna is isotropic, the coverage diagram is obtained as the locus of points (R, h_r) for which the equality

$$|F| = 2\left|\sin\left(\frac{2\pi h_t}{\lambda_0}\tan\psi\right)\right| = c\frac{R}{R_f} \tag{3.1.15}$$

holds. In (3.1.15), R_f is a reference distance, c is a factor that determines the field strength relative to the field strength at the reference distance, and $\tan\psi = h_r/R$ is the elevation angle of the receiver. Several curves for different values of c may be plotted on the same graph. In that case $c = 1/\sqrt{2}, 1, \sqrt{2}, 2$ are typically chosen values which gives 3 dB difference between successive curves. Figure 3.3, the coverage diagram for a transmitter antenna located $50\lambda_0$ above earth is shown. The coverage diagram is actually an antenna field strength pattern drawn in different coordinates and including the effect of interference with ground reflected wave. The pattern of an antenna above earth can be considered as the pattern of the array formed by the antenna and its image.

Another way to show the coverage diagram is to use color for field strength as shown in Fig. 3.4.

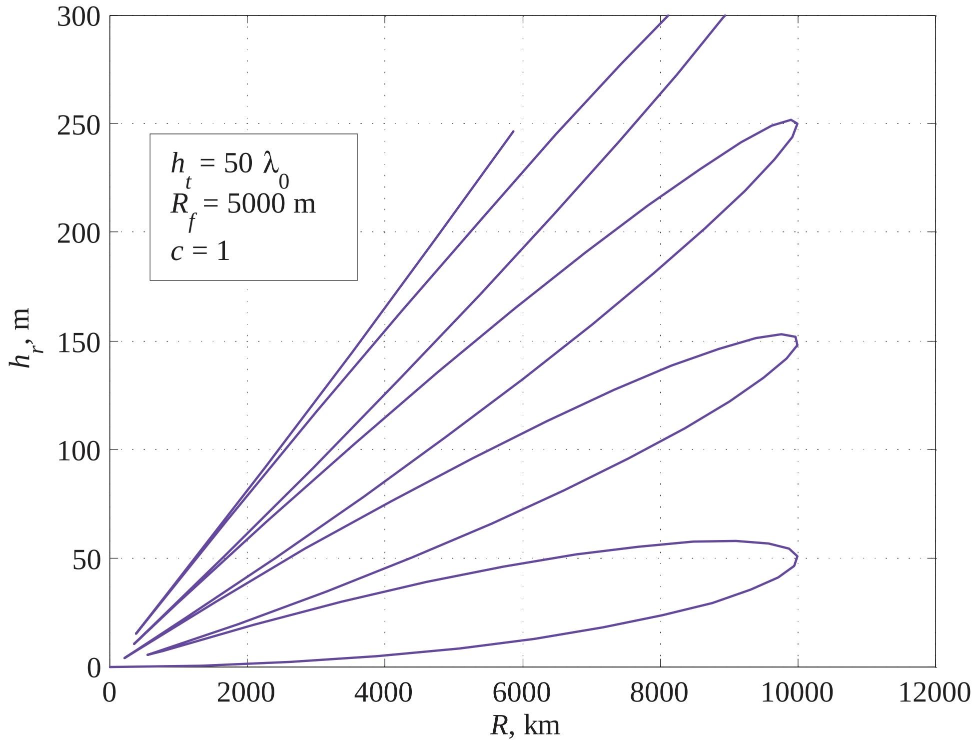

FIGURE 3.3. Coverage diagram for flat earth and $\Gamma = -1$.

3.1.2. Effect of Finite Conductivity. In the preceding section, we have considered the reflection coefficient to be equal to -1. Actually, the ground is not a perfect electric conductor and the reflection coefficient is given by Fresnel formulas. The two possible cases are reflection of a plane TEM wave polarized with the electric field in the plane of incidence (vertical polarization) and reflection of a plane TEM wave polarized with the electric field perpendicular to the plane of incidence (horizontal polarization). If the ground conductivity is σ, the permittivity is $\kappa'\epsilon_0$, and ψ is the grazing angle of incidence, then

$$\Gamma_v = \frac{\kappa \sin\psi - \sqrt{\kappa - \cos^2\psi}}{\kappa \sin\psi + \sqrt{\kappa - \cos^2\psi}}, \qquad \text{vertical polarization} \tag{3.1.16a}$$

$$\Gamma_h = \frac{\sin\psi - \sqrt{\kappa - \cos^2\psi}}{\sin\psi + \sqrt{\kappa - \cos^2\psi}}, \qquad \text{horizontal polarization} \tag{3.1.16b}$$

where $\kappa = \kappa' - j\kappa'' = \kappa' - j\sigma/\omega\epsilon_0$ is the complex dielectric constant. Thus, calculations of ground wave propagation requires knowledge of the electrical parameters of the earth. Typical values of electrical characteristics of the surface of the earth as given by ITU-R Recommendation P.527-3, [**8**], are given in Table (3.1). These parameters depend on frequency and detailed information can be found in [**8**]. Obviously, ground conductivity is different at different locations on earth and a map of the typical values is given in ITU-R Recommendation P.832-2, [**9**].

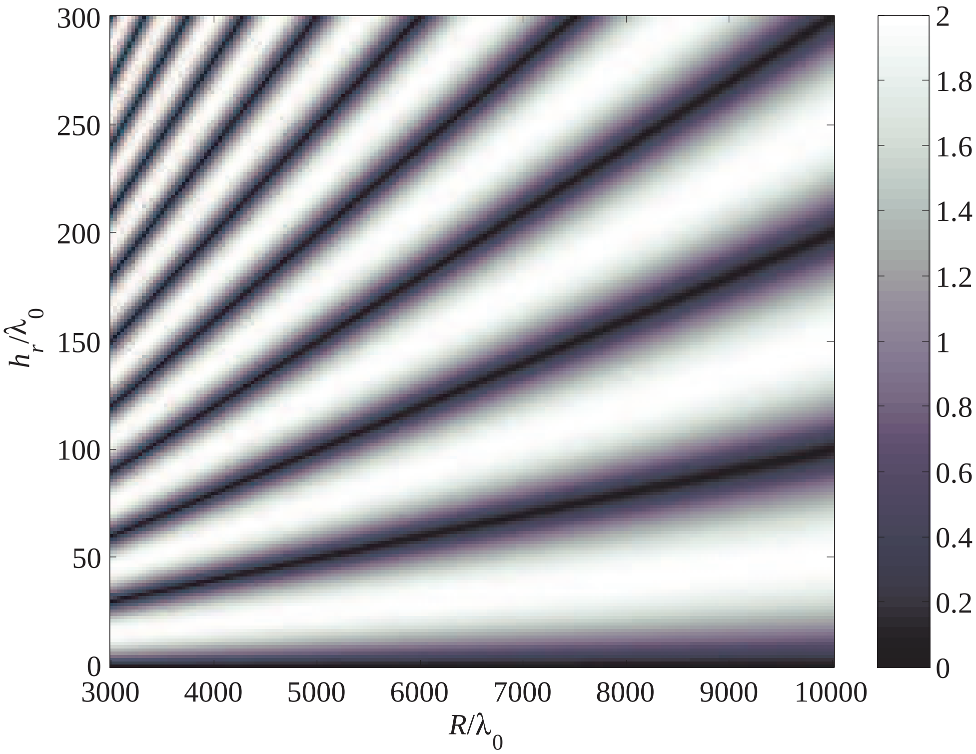

FIGURE 3.4. Coverage diagram for an isotropic antenna at a height $h_t = 50\lambda_0$ above flat earth and $\Gamma = -1$.

TABLE 3.1. Electrical characteristics of the surface of the earth.

	$100\,\mathrm{kHz} < f < 1\,\mathrm{GHz}$		$100\,\mathrm{GHz} < f < 1000\,\mathrm{GHz}$	
Surface	κ'	σ, S/m	κ'	σ, S/m
Sea water	80	5	7	70
Fresh water	80	3×10^{-2}	7	70
Ice	3.7	$10^{-5} - 2 \times 10^{-4}$	10	—
Wet ground	30	10^{-2}	4	18
Medium dry ground	15	10^{-3}	4	18
Very dry ground	3	10^{-4}	3	1

Figures 3.5 and 3.6 show the variation of the magnitude and phase of the reflection coefficient for vertical polarization as a function of the grazing angle, respectively, for $\kappa' = 15$ and $\sigma = 10^{-2}\,\mathrm{S/m}$. The Brewster angle effect for vertical polarization is obvious, which causes $|\Gamma_v|$ to have a minimum and the phase angle changes rapidly from $180\,^\circ$ to $360\,^\circ$. The variation of the magnitude and phase of the reflection coefficient for horizontal polarization are shown in Fig.s 3.7 and 3.8, respectively, for the same ground parameters. It is important to note that both Γ_v and Γ_h approaches -1 as the grazing angle ψ approaches zero.

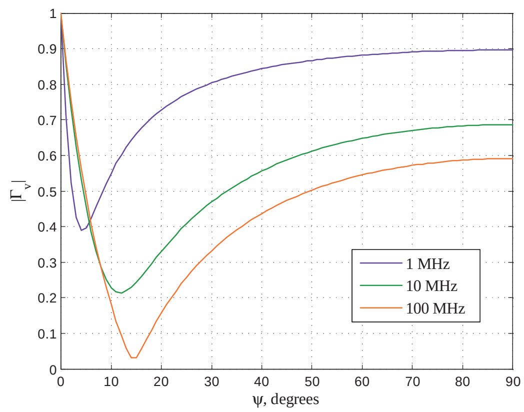

FIGURE 3.5. $|\Gamma_v|$ vs. grazing angle.

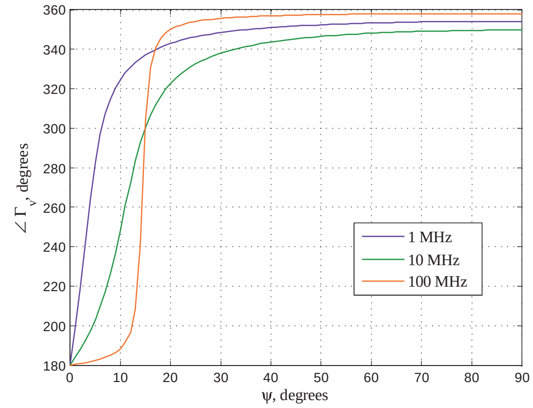

FIGURE 3.6. $\angle\Gamma_v$ vs. grazing angle

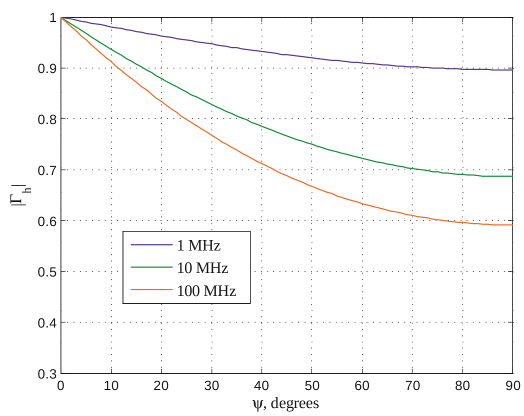

FIGURE 3.7. $|\Gamma_h|$ vs. grazing angle.

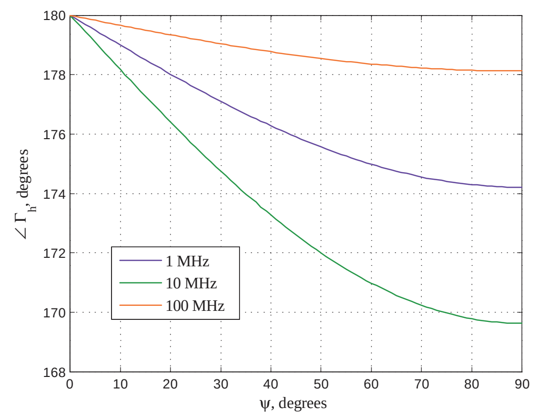

FIGURE 3.8. $\angle\Gamma_h$ vs. grazing angle

In most practical cases in propagation calculations, the grazing angle is very small and both Γ_v and Γ_h can be taken as -1. Still, one must be careful, especially for vertical polarization, due to the rapid change in phase angle. The best practice is to calculate these values from (3.1.16).

The total field at the location of the receiver is given by (3.1.7). The effect of Γ being different from -1 is that the total field will vary between $1 - |\Gamma|$ and $1 + |\Gamma|$ instead of 0 and 2. Thus, the maxima will be smaller while the nulls are filled, improving the coverage of the transmit antenna. Still, since Γ is very close to -1, the improvement in the coverage will only be marginal.

3.1.3. Reflection from a Rough Surface. The theoretical discussions of the preceding sections assume a smooth reflecting surface. In practice, this condition is rarely met. The effect of surface roughness on the reflection coefficient is more important than the electrical properties of the surface.

In the case of radio waves, the reflection occurs not from a single point but from a surface determined by the cross section of the Fresnel ellipsoid with the surface of the earth, as shown in Fig. 3.9. The size of the first Fresnel zone in a reflecting plane can be obtained by using this geometry. The ellipse shown in the figure represents the cross section of the first Fresnel ellipsoid with the plane of incidence, where the actual transmitter is replaced by its image. In practical cases we can take $|A'B| \approx R$ since the heights of the antennas are much smaller than their separation.

Using the similarity of the triangles $A'OP$ and BOQ, we can easily determine the distance $l_t = |PO|$ of the reflection point to the transmitter as

$$\frac{l_t}{h_t} = \frac{R - l_t}{h_r} \Rightarrow l_t = \frac{h_t}{h_r + h_t} R. \tag{3.1.17}$$

Similarly, the distance of the reflection point to the receiver is

$$l_r = |OQ| = \frac{h_r}{h_r + h_t} R. \tag{3.1.18}$$

To determine the distance MN, we need to find the intersection of the Fresnel ellipse with the horizontal line PQ. Using a Cartesian coordinate system in which the $x-$axis is directed along the major axis of the ellipse and choosing the coordinate center as the midpoint of the line segment $A'B$, we can write the equation of the ellipse as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{3.1.19}$$

where $a = R/2 + \lambda/4R/2$ is the major semi-axis of the ellipse. The minor semi-axis of the ellipse is determined by the maximum radius of the first Fresnel zone given by (2.1.6b), i.e., $b \approx \sqrt{R\lambda}/2$. The straight line MN is described by the equation

$$y = -\left(x - c\right) \tan \psi \tag{3.1.20}$$

where

$$c = |OO'| = \left(\frac{h_r}{h_r + h_t} R - \frac{R}{2} \right). \tag{3.1.21}$$

Solving (3.1.19) and (3.1.20) together, we find the abscissa of the points M and N. Since the angle ψ is very small, the difference between the two solutions can be taken to be equal to $|MN|$, which is the major axis of the reflection ellipse on the reflecting plane, and is given by

$$2ba \frac{\sqrt{b^2 + (a^2 - c^2) \tan^2 \psi}}{b^2 + a^2 \tan^2 \psi} = d \frac{\sqrt{(d\lambda + 4h_t h_r)\, d\lambda}}{d\lambda + (h_t + h_r)^2} \tag{3.1.22}$$

The minor semi-axis of the reflection ellipse is the Fresnel radius at the reflection point. The reflection essentially occurs from this reflection ellipse. If we restrict ourselves to the case where $h_r = h_t$, c becomes zero and the length of the major axis of the ellipse is

$$|MN| = \frac{2ab}{\sqrt{b^2 + a^2 \tan^2 \psi}}. \tag{3.1.23}$$

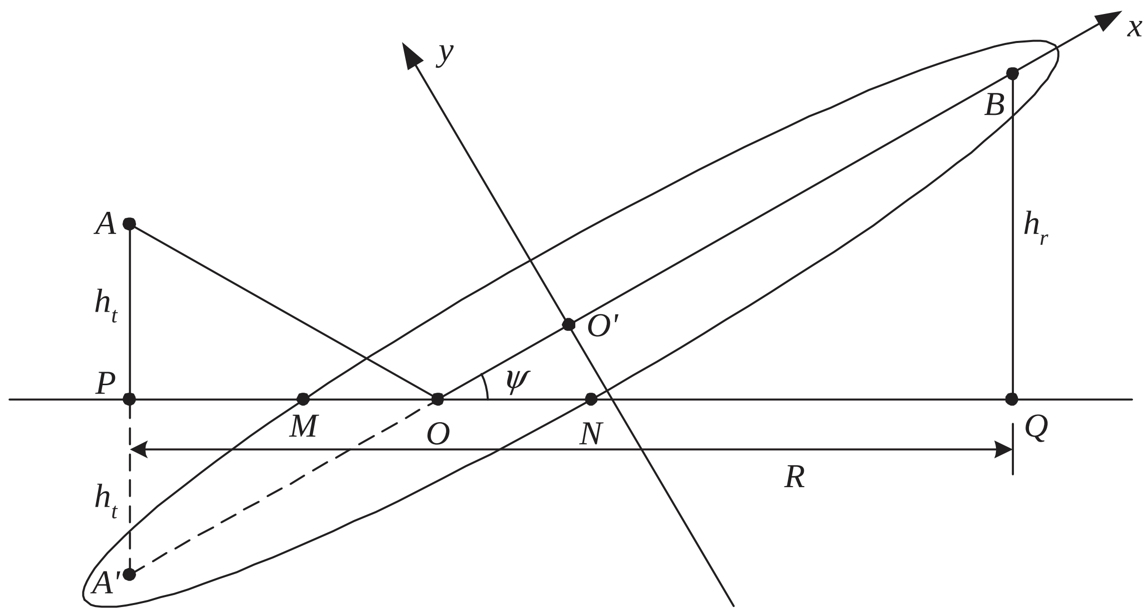

FIGURE 3.9. The size of the first Fresnel zone on the reflecting surface.

The area of the reflection ellipse is generally quite large. As an example, for $h_r = h_t = 50\,\text{m}$, $R = 50\,\text{km}$, and $f = 3\,\text{GHz}$, semi-major axis is about 14.4 km and the minor semi-axis is about 35 m.

The reflection coefficients calculated by (3.1.16) can be assumed to be valid only if the first Fresnel ellipse is sufficiently smooth. A measure of *surface roughness* may be obtained by considering the effective wavelength of the incident wave in the direction perpendicular to the surface. The effective wavelength λ_v in this direction is

$$\lambda_v = \frac{\lambda_0}{\sin\psi}. \tag{3.1.24}$$

If the average height of the surface irregularities is in this order, the surface can no longer be assumed as a smooth surface. Therefore it is reasonable to define a *surface roughness parameter* as $(\sigma_h \sin\psi)/\lambda_0$ where σ_h is the rms value of the surface height, (see Fig. 3.10). An expression for the magnitude of the reflection coefficient Γ_r of a rough, perfectly conducting surface was given by Ament, [**10**], as

$$\Gamma_r = \Gamma \exp\left(-2k_0^2\sigma_h^2 \sin^2\psi\right). \tag{3.1.25}$$

where Γ is the complex reflection coefficient of a smooth surface. However, measurements over sea has shown that this formula underestimates the experimental data when the surface roughness parameter exceeds 0.1. A more accurate formula given in [**11**] is

$$\Gamma_r = \Gamma \exp\left(-2k_0^2\sigma_h^2 \sin^2\psi\right) I_0\left(2k_0^2\sigma_h^2 \sin^2\psi\right) \tag{3.1.26}$$

where $I_0(x)$ is the modified Bessel function of order zero. Figure 3.11 shows the plots of the two formulas. The experimental data is within 10% of the modified formula, [**11**].

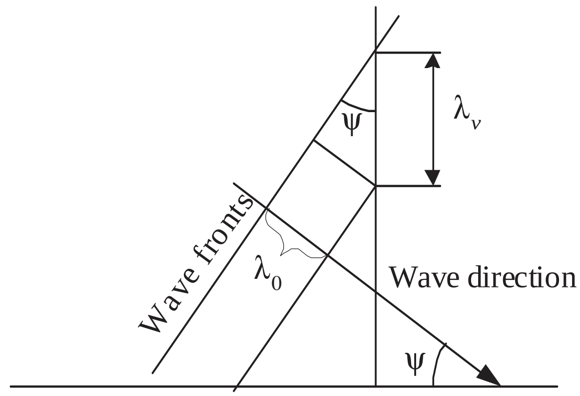

FIGURE 3.10. Description of vertical wavelength.

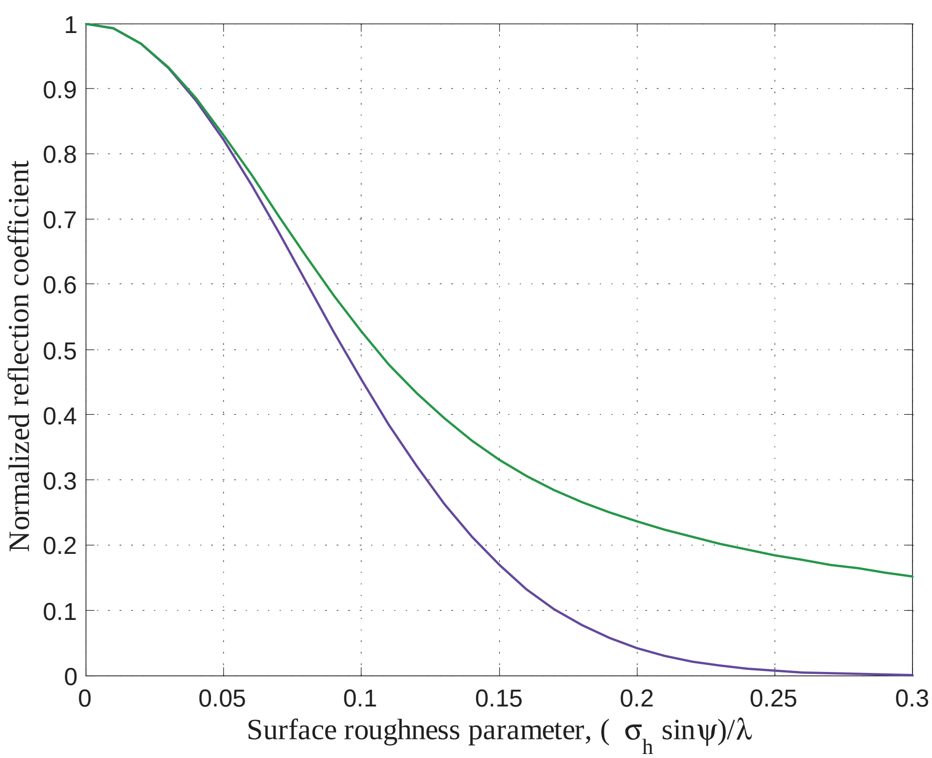

FIGURE 3.11. Normalized surface reflection coefficient as a function of the surface roughness parameter.

The foregoing discussions are concerned only with the *specular*, or *coherent*, component of the scattered signal. There is also a *diffuse*, or *incoherent*, component of reflection which has random amplitude and phase and occurs over a wider range of angles. For a more accurate calculation, diffuse scattering should also be taken into account.

TABLE 3.2. Beaufort wind scale.

Beaufort number	**Description**	**Wind speed**	**Wave height**
0	Calm	$< 1\,\mathrm{km/h}$	$0\,\mathrm{m}$
1	Light air	$1.1 - 5.5\,\mathrm{km/h}$	$0 - 0.2\,\mathrm{m}$
2	Light breeze	$5.6 - 11\,\mathrm{km/h}$	$0.2 - 0.5\,\mathrm{m}$
3	Gentle breeze	$12 - 19\,\mathrm{km/h}$	$0.5 - 1\,\mathrm{m}$
4	Moderate breeze	$20 - 28\,\mathrm{km/h}$	$1 - 2\,\mathrm{m}$
5	Fresh breeze	$29 - 38\,\mathrm{km/h}$	$2 - 3\,\mathrm{m}$
6	Strong breeze	$39 - 49\,\mathrm{km/h}$	$3 - 4\,\mathrm{m}$
7	Moderate gale	$50 - 61\,\mathrm{km/h}$	$4 - 5.5\,\mathrm{m}$
8	Gale	$62 - 74\,\mathrm{km/h}$	$5.5 - 7.5\,\mathrm{m}$
9	Strong gale	$75 - 88\,\mathrm{km/h}$	$7 - 10\,\mathrm{m}$
10	Storm	$89 - 102\,\mathrm{km/h}$	$9 - 12.5\,\mathrm{m}$
11	Violent storm	$103 - 117\,\mathrm{km/h}$	$11.5 - 16\,\mathrm{m}$
12	Hurricane	$\geq 118\,\mathrm{km/h}$	$\geq 14\,\mathrm{m}$

3.1.4. Reflection from Sea. The electrical parameters of sea depend on the salinity of the sea water, temperature, and also frequency. An empirical formula that gives the complex permittivity of the sea as a function of these parameters is given in [**12**]. Although these formulas fit the experimental data better, much simpler expressions given in [**8**] are commonly used.

The roughness parameter of the surface can be calculated using the Beaufort scale which is an empirical measure for describing wind speed based mainly on observed sea conditions, given in Table (3.2).

Diffuse scattering from sea surface is discussed in [**13**].

3.2. Propagation over a Spherical Earth

Flat earth assumption can only be used for relatively low distances. For distances approaching the maximum horizontal line of sight range (horizon distance) the spherical earth model must be used. The horizon distance for an elevated antenna can be found using the geometry of Fig. 3.12. If the earth radius is a_e the horizon distance of an antenna at a height h_t is

$$R_1 = \sqrt{(a_e + h_1)^2 - a_e^2} = \sqrt{2a_e h_1 + h_1^2} \approx \sqrt{2a_e h_1}. \tag{3.2.1}$$

The maximum line of sight distance is then given by

$$R_1 + R_2 = \sqrt{2a_e h_1} + \sqrt{2a_e h_2} = \sqrt{2a_e}\left(\sqrt{h_1} + \sqrt{h_2}\right). \tag{3.2.2}$$

If the separation between the transmitting and receiving antennas is comparable to the maximum line of sight distance, the spherical earth model must be used for calculating the reflection from earth's surface. The pertinent geometry is shown in Fig. 3.13.

There are several complications in the calculations arising from the spherical surface. The known parameters are the two antenna heights h_1 and h_2 and the total range d. Applying

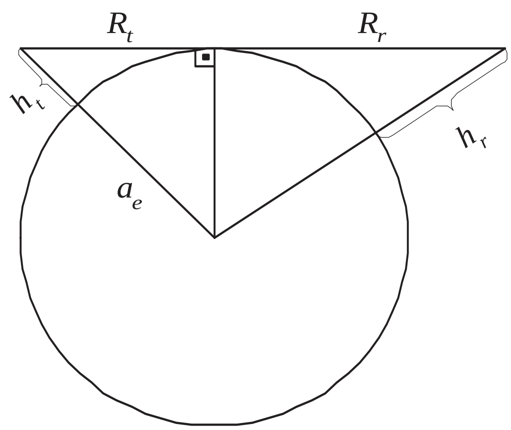

Figure 3.12. Determination of the maximum line of sight range.

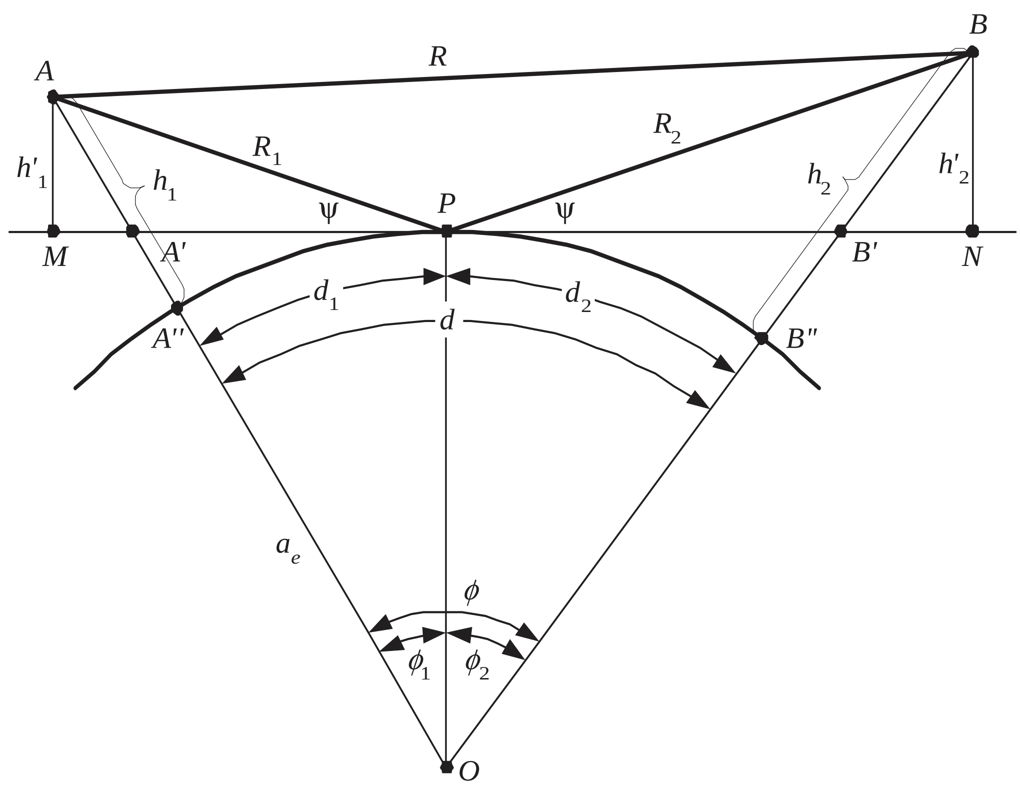

Figure 3.13. Spherical earth geometry.

the law of sines to the triangles AOP and BOP, we can write

$$\phi_1 + \psi = \cos^{-1}\left(\frac{a_e}{a_e + h_1}\cos\psi\right), \tag{3.2.3a}$$

$$\phi_2 + \psi = \cos^{-1}\left(\frac{a_e}{a_e + h_2}\cos\psi\right). \tag{3.2.3b}$$

Adding the two equations and using the fact that $\phi_1 + \phi_2 = \phi$ we get

$$\psi = \frac{1}{2}\left(\cos^{-1}\left(\frac{a_e}{a_e + h_1}\cos\psi\right) + \cos^{-1}\left(\frac{a_e}{a_e + h_2}\cos\psi\right) - \phi\right) \tag{3.2.4}$$

which is an equation in the unknown grazing angle. The two equations in (3.2.3) can also be put into the form of a quartic equation. In either case the solution requires the use of a computer. In fact, a simple and very good approximate solution is available which gives d_1 as, [**14**],

$$d_1 = \frac{d}{2} + p\cos\left(\frac{\Phi + \pi}{3}\right) \tag{3.2.5}$$

where

$$p = \frac{2}{\sqrt{3}}\left[a_e\left(h_1 + h_2\right) + \frac{d^2}{4}\right]^{1/2}, \tag{3.2.6a}$$

$$\Phi = \cos^{-1}\left(\frac{2a_e d\left(h_1 - h_2\right)}{p^3}\right) \qquad h_1 \leq h_2. \tag{3.2.6b}$$

Note that if $h_1 > h_2$ the same equations can be used to determine d_2.

Knowing the angle $\phi_1 = d_1/a_e$, the grazing angle ψ can be determined from (3.2.3a), and R_1 is found by using the law of sines as

$$R_1 = \frac{\sin\phi_1}{\cos\psi}\left(a_e + h_1\right). \tag{3.2.7}$$

A similar calculation will give R_2 as

$$R_2 = \frac{\sin\phi_2}{\cos\psi}\left(a_e + h_2\right) \tag{3.2.8}$$

and the path length difference between direct and reflected paths can be calculated as $\Delta R = R_1 + R_2 - R$. Actually, there is an easier approximate method to calculate the path length difference, [**15**]. If we consider the tangent MN to the earth at the reflection point, we may consider the reflection as occurring over flat earth with *effective antenna heights* h_1' and h_2' shown in Fig. 3.13. The angles $\widehat{MAA'}$ and $\widehat{NBB'}$ are very small, and we can take $h_1' \approx |AA'|$ and $h_2' \approx |BB'|$. Since d_1 is the horizon distance of an antenna of height $|A'A''|$ we have

$$|A'A''| = \frac{d_1^2}{2a_e} \tag{3.2.9}$$

and thus

$$h_1' = h_1 - \frac{d_1^2}{2a_e} \tag{3.2.10}$$

and similarly

$$h_2' = h_2 - \frac{d_2^2}{2a_e}. \tag{3.2.11}$$

As a result, we can use (3.1.10) to obtain the path length difference with the actual heights replaced by the effective heights.

A systematic (and more accurate) procedure for calculation of the path length difference is also available and is given in [**14**] as

$$S_1 = \frac{d_1}{\sqrt{2a_e h_1}}, \tag{3.2.12a}$$

$$S_2 = \frac{d_2}{\sqrt{2a_e h_2}}, \tag{3.2.12b}$$

$$T = \sqrt{h_1/h_2} < 1, \tag{3.2.12c}$$

$$S = \frac{S_1 T + S_2}{1+T} = \frac{d}{\sqrt{2a_e h_1} + \sqrt{2a_e h_2}}, \tag{3.2.12d}$$

$$J(S,T) = \left(1 - S_1^2\right)\left(1 - S_2^2\right), \tag{3.2.12e}$$

$$K(S,T) = \frac{\left(1 - S_2^2\right) + T^2\left(1 - S_1^2\right)}{1+T^2}, \tag{3.2.12f}$$

$$\Delta R = \frac{2h_1 h_2}{d} J(S,T), \tag{3.2.12g}$$

$$\tan\psi = \frac{h_1 + h_2}{d} K(S,T), \tag{3.2.12h}$$

$$D = \left[1 + \frac{4S_1 S_2^2 T}{S\left(1 - S_2^2\right)(1+T)}\right]^{-1/2}. \tag{3.2.12i}$$

In these equations, J and K are shown as functions of S and T only, which is not obvious. The geometric problem has three parameters, h_t, h_r, and d. It is not possible to find these parameters from a knowledge of S and T only. However, if the distance is scaled by α and heights by α^2, the values of S and T will not change, and for J and K to be same, S_1 and S_2 must not change. To see this, we must consider (3.2.5) and (3.2.6), which shows that the scaling will increase p and d_1 by a factor of α, while Φ will remain same. This means that S_1 and S_2 will not change with such a scaling. Therefore, if we know any two of the parameters S_1, S_2, S, T, J, and K, the others are implicitly defined.

For reflection from a spherical surface, the reflected rays have a greater divergence than the incident rays, as depicted in Fig. 3.14. This effect can be taken into account by multiplying the reflected field by a factor D called the *divergence factor* and its value can be calculated using (3.2.12i). With this correction (3.1.7) becomes

$$F = 1 + D\Gamma e^{-jk_0 \Delta R}. \tag{3.2.13}$$

However, this correction is insignificant and may be neglected in propagation calculations. The errors introduced by the inaccurate knowledge of surface characteristics and surface irregularities more than absorb the effect of the ray divergence.

The divergence factor is a quantity between zero and one and it approaches zero as the grazing angle ψ approaches zero. This in fact is not correct. The derivation of D is based completely on geometrical calculations (i.e., ray optics). As the grazing angle approaches zero, the diffracted fields become more important and an electromagnetic calculation is required, as will be discussed in the next chapter. A good practice is to use D from (3.2.12i) if $\Delta R \geq \lambda_0/4$ and take $D = 1$ if $\Delta R < \lambda_0/4$, [**16**]. As the grazing angle approaches zero ΔR becomes smaller and $D = 1$ assumption will give zero field, while (3.2.12i) gives $D = 0$ and

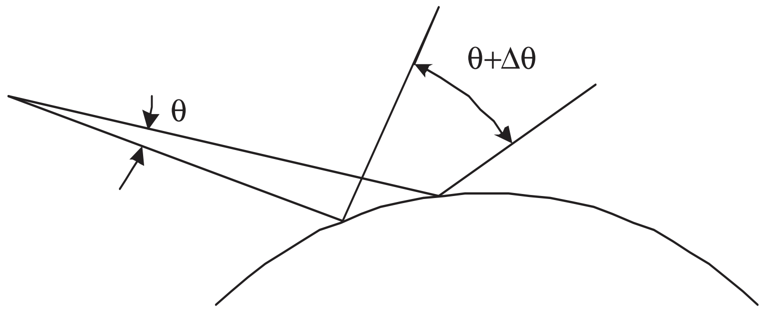

FIGURE 3.14. Divergence of ray upon reflection from a spherical surface.

the total field approaches the free space value. Although both of these results are incorrect, the latter is unacceptable, since $\Delta R = 0$ defines the shadow boundary at which the geometric optic field should go to zero.

3.3. Atmospheric Refraction

The refractive index of the atmosphere changes with height above the surface. At greater heights the atmosphere is less dense resulting in a smaller index of refractivity. This causes the rays to curve downward as they propagate in the atmosphere. This effect can be understood by considering the atmosphere as divided into layers, with constant values of refractive index over each layer as shown in Fig. 3.15. The Snell's law gives

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = \cdots = n_k \sin\theta_k = \cdots \tag{3.3.1}$$

Since the refractive index decreases, the angles θ_k must decrease resulting in a downward curvature.

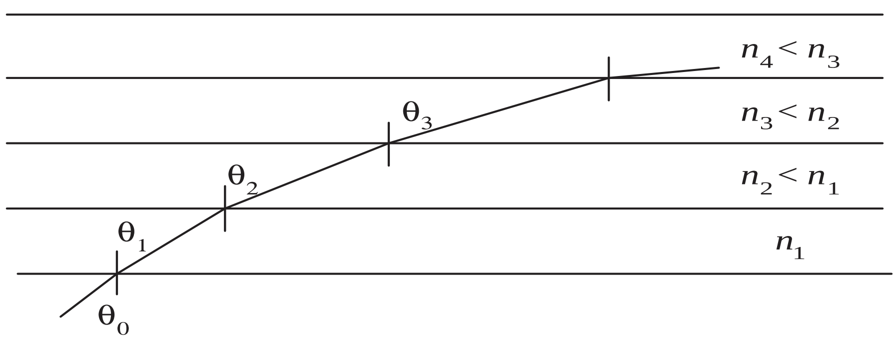

FIGURE 3.15. Curving of rays in a stratified medium.

It is fairly easy to calculate the curvature of the rays for a stratified atmosphere. Considering the geometry shown in Fig. 3.16, a ray incident on the lower surface at an angle θ is refracted along the distance dh and impinge on the upper surface at an angle $\theta + d\theta$. The angle subtended by the curve AB at the center of the curvature is also $d\theta$. The radius of curvature R is then given by

$$R = \frac{AB}{d\theta} \tag{3.3.2}$$

where AB is the arc length. From the triangle ABC' we get

$$AB = \frac{dh}{\cos\theta} \tag{3.3.3}$$

and

$$R = \frac{dh}{\cos(\theta)\, d\theta}. \tag{3.3.4}$$

The Snell's law gives

$$\begin{aligned} n \sin\theta &= (n + dn)\sin(\theta + d\theta) \\ &= n\sin\theta\cos d\theta + n\cos\theta\sin d\theta + dn\sin\theta\cos d\theta + dn\cos\theta\sin d\theta \\ &\approx n\sin\theta + n\cos(\theta)\, d\theta + \sin(\theta)\, dn. \end{aligned} \tag{3.3.5}$$

The approximation is obtained by expanding the right-hand side, assuming $\cos d\theta \approx 1$ and $\sin d\theta \approx d\theta$, and neglecting the second order terms. This equation yields

$$\cos(\theta)\, d\theta = -\sin\theta \frac{dn}{n}. \tag{3.3.6}$$

Substituting this result in (3.3.4) gives

$$R = \frac{n}{\sin\theta\left(-\frac{dn}{dh}\right)} \approx \frac{n}{-dn/dh} \tag{3.3.7}$$

where we have assumed that $\sin\theta \approx 1$, since for most of the propagation problems on earth (ground wave propagation), the angle θ is very close to $\pi/2$.

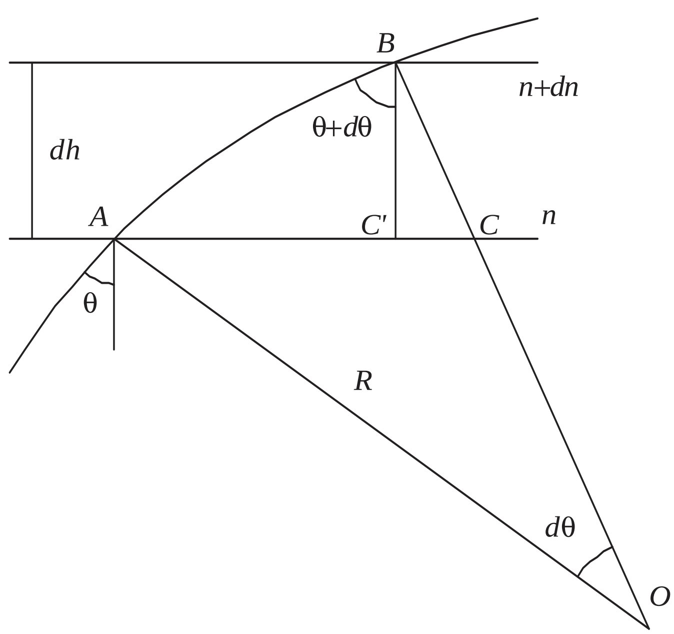

FIGURE 3.16. Radius of curvature of the rays.

The refractive index of the atmosphere is very close to unity and therefore a more practical definition is

$$N = (n-1) \times 10^6 \tag{3.3.8}$$

where N is called the *excess index of refraction* or the *refractivity*. The actual value of N depends on atmospheric pressure, water content and temperature. A formula for N is given in [**17**] as

$$N = \frac{77.6}{T}\left[p + 4810\frac{e}{T}\right] \tag{3.3.9}$$

where p is the barometric pressure in mbar, e is the partial pressure of water vapor in mbar, and T is the temperature in K. The variation of refractivity with height is generally modeled by an exponential function as, [**18**],

$$N = N_0 \exp\left(-h/h_0\right) \tag{3.3.10}$$

where N_0 is the refractivity at the sea level, h is the height above sea level and h_0 is a reference height. ITU-R recommends the values $N_0 = 315$, and $h_0 = 7.5\,\mathrm{km}$ to be used for reference calculations. This model is called the *reference atmosphere*.

In terms of refractivity, (3.3.7) can be written as

$$R \approx \frac{10^6}{-dN/dh} \tag{3.3.11}$$

where $n \approx 1$ is used. For the reference atmosphere we have

$$\frac{dN}{dh} = \frac{d\left(N_0 \exp\left(-h/h_0\right)\right)}{dh} = -\frac{N_0}{h_0}\exp\left(-h/h_0\right) \tag{3.3.12}$$

which, upon substitution in (3.3.11) yields

$$R \approx 10^6 \frac{h_0}{N_0}\exp\left(h/h_0\right). \tag{3.3.13}$$

The value of dN/dh varies between $-3.2 \times 10^{-2}\,\mathrm{m}^{-1}$ and $-4.2 \times 10^{-2}\,\mathrm{m}^{-1}$ for $0 \le h \le 2000\,\mathrm{m}$. A commonly used approach is to assume dN/dh to be a constant given by $-4 \times 10^{-2}\,\mathrm{m}^{-1}$. This gives the classical method of accounting for the atmospheric refraction by replacing the earth with a larger sphere and using straight lines for rays. In doing so, the relative curvature of the rays and the earth's surface are kept the same. In analytic geometry, the relative curvature is defined as the difference $1/a - 1/R$. In our case, a is the radius of earth and R is the ray curvature given by (3.3.11). Since we want to use straight lines (curvature ∞) for the rays, we must have

$$\frac{1}{a} - \frac{-dN/dh}{10^6} = \frac{1}{a_e} - \frac{1}{\infty} \tag{3.3.14}$$

which gives

$$a_e = \frac{a}{1 + a\frac{dN}{dh}10^{-6}}. \tag{3.3.15}$$

Assuming $dN/dh = -4 \times 10^{-2}\,\mathrm{m}^{-1}$, and using $a = 6378\,\mathrm{km}$ yields

$$a_e = \frac{a}{1 + a\frac{dN}{dh}10^{-6}} = 8562\ \mathrm{km} \tag{3.3.16}$$

or

$$a_e \approx \frac{4}{3}a. \tag{3.3.17}$$

This value of a_e is called the *effective earth radius*. In all the equations we have derived in Sec. 3.2, we have used a_e for the radius of earth. Under fairly general conditions the formulas will give quite accurate results if $a_e = 4a/3 = 8562$ km is used. An atmosphere that has a constant refractivity gradient equal to $-4 \times 10^{-2}\,\mathrm{m}^{-1}$ will be referred to as a *standart atmosphere* in this text, which should not be confused with the *reference atmosphere* defined by (3.3.10) with $N_0 = 315$, and $h_0 = 7.5\,\mathrm{km}$.

It must be kept in mind that for rays that go through very different heights, the effective earth radius will not be very accurate since the refractivity gradient cannot be assumed to be constant over such a large range of heights. Furthermore, if the atmospheric conditions change, the refractivity gradient may take a value other than $-4 \times 10^{-2}\,\mathrm{m}^{-1}$ which will give a different effective earth radius. Other meteorological phenomena may result in different types of propagation (ducting) as will be discussed later in Chapter (6).

3.4. Range-Angle-Height Charts

The curving of the rays in the atmosphere makes it rather difficult to generate coverage diagrams such as the one shown in Fig. 3.3. The range-angle-height charts are generated for this purpose. The basic idea in these charts is to use a transformed coordinate system in which the rays propagate along straight lines. An example of a range-angle-height chart is shown in Fig. 3.17. To generate a range-angle-height chart, it is necessary to solve for the paths of different rays, i.e., a *ray tracing algorithm* is required.

3.4.1. Ray Tracing for Spherically Layered Atmosphere. General formulas for ray tracing in inhomogeneous media are available, [**19, 20**], but in propagation calculations, ray tracing algorithms for spherically layered atmosphere are used since the refractivity model is a function of height only. In fact, this is not true but the variation of refractivity as a function of location is quite difficult to know or measure and we content ourselves with these models.

Suppose that there is a spherically layered atmosphere with a known refractivity profile and consider two adjacent infinitesimal layers as shown in Fig. 3.18. Using Snell's law we have

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \tag{3.4.1a}$$
$$n_2 \sin\theta_3 = n_3 \sin\theta_4. \tag{3.4.1b}$$

Multiplying (3.4.1a) by r_1 and (3.4.1b) by r_2, where r_1 and r_2 are the distances to earth's center at points P_1 and P_2, respectively, we get

$$n_1 r_1 \sin\theta_1 = n_2 r_1 \sin\theta_2, \tag{3.4.2}$$
$$n_2 r_2 \sin\theta_3 = n_3 r_2 \sin\theta_4. \tag{3.4.3}$$

Since P_1 and P_2 lie on the same line, we have

$$r_1 \sin\theta_2 = r_2 \sin\theta_3 = p \tag{3.4.4}$$

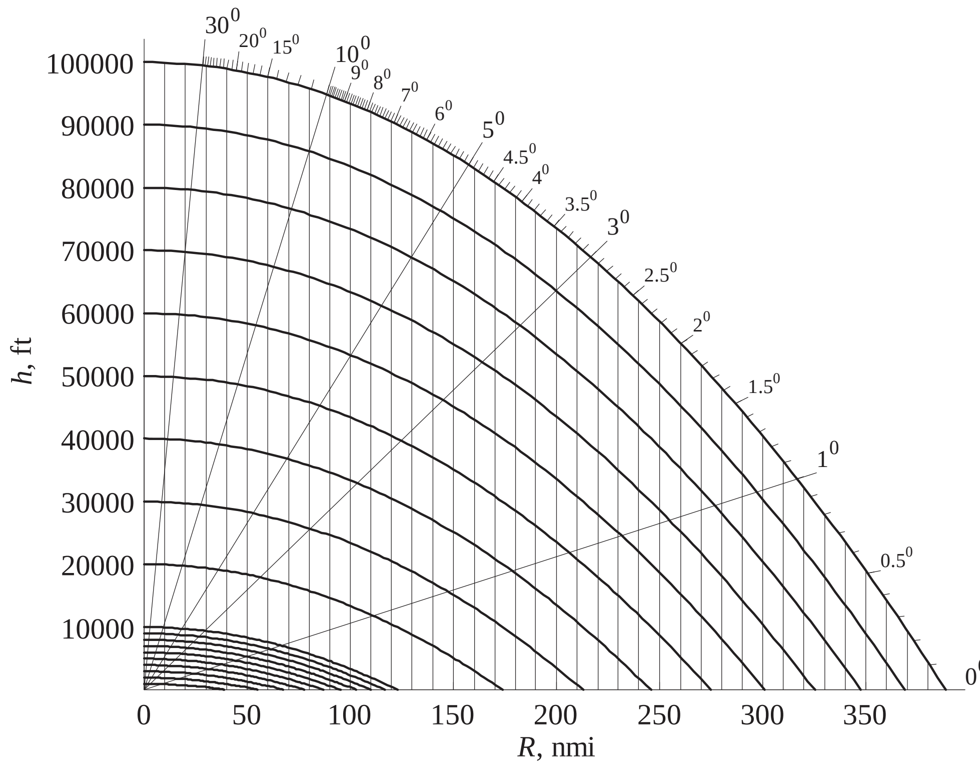

FIGURE 3.17. Range-angle-height diagram based on the standard atmosphere model

which shows that

$$n_1 r_1 \sin\theta_1 = n_2 r_1 \sin\theta_2 = n_2 r_2 \sin\theta_3 = n_3 r_2 \sin\theta_4. \tag{3.4.5}$$

We can consider more layers, but the result will be the same, i.e., the product $rn(r)\sin\theta$ is a constant at any point of the curve, called the *refractive invariant*, or equivalently

$$\frac{rn(r)}{a}\sin\theta = C \tag{3.4.6}$$

defines the ray path, where a is the radius of the earth.

In the following derivations we will use the geometry shown in Fig. 3.19. The constant C can be evaluated at any known point of the ray. In propagation problems, we know the ray direction at the transmitter location, thus C is known. It is customary to use the angle measured from the local horizon instead of the angle with the outward radial. Denoting the angle measured from the local horizon by α, and $q = rn(r)/a$ we can write

$$\left.\begin{array}{c} q\cos\alpha = C \\ q\sin\alpha = \pm\sqrt{q^2 - C^2} \end{array}\right\} \Rightarrow \tan\alpha = \frac{\pm\sqrt{q^2 - C^2}}{C} = \frac{dr}{rd\theta}. \tag{3.4.7}$$

If we change the variables r and θ to the more convenient variables h and s defined by

$$r = a + h; \quad \theta = \frac{x}{a} \tag{3.4.8}$$

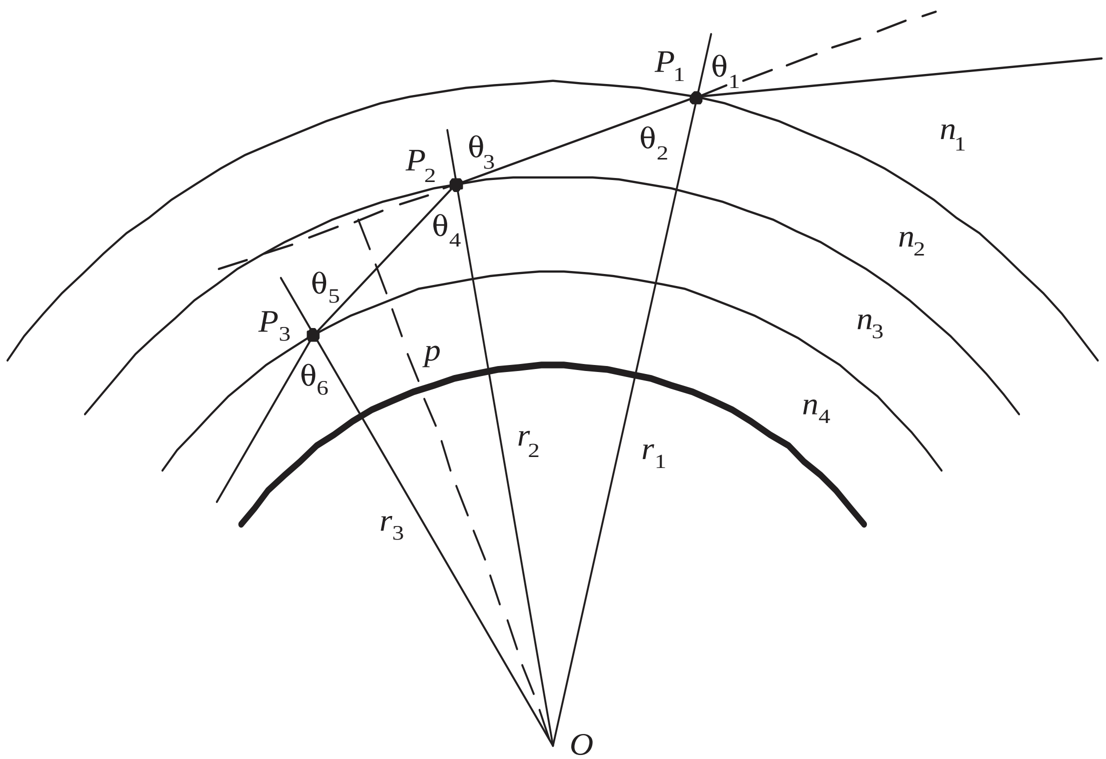

FIGURE 3.18. Refractive invariant.

where a is the radius of the earth, h defines height above earth's surface and x defines distance on the earth's surface, we get

$$\frac{\pm\sqrt{q^2 - C^2}}{C} = \frac{dh}{\left(1 + \frac{h}{a}\right) dx}. \tag{3.4.9}$$

In practical situations $h \ll a$ and we can approximate this equation as

$$dx = \frac{C dh}{\pm\sqrt{q^2(h) - C^2}}. \tag{3.4.10}$$

Notice that since dx is always positive (the ray is moving away from the transmitter) the sign of the radical must chosen to be the same as the sign of dh.

This differential equation can easily be integrated to give

$$x = C \int_{h_1}^{h} \left| \frac{dh'}{\sqrt{q^2(h') - C^2}} \right| \tag{3.4.11}$$

where we have used the variable h' as the dummy variable of integration. The absolute value is included to emphasize that dh' and the radical have the same signs. If the ray bends downward after making a peak at P_0, for points beyond P_0 the integral becomes

$$d = C \left(\int_{h_1}^{h_0} \frac{dh'}{\sqrt{q^2(h') - C^2}} + \int_{h}^{h_0} \frac{dh'}{\sqrt{q^2(h') - C^2}} \right). \tag{3.4.12}$$

where we have replaced the distance variable x by the more usual variable d. Equation (3.4.12) can be used to determine the height of the ray path at any given distance from the transmitter, thus providing the ray path.

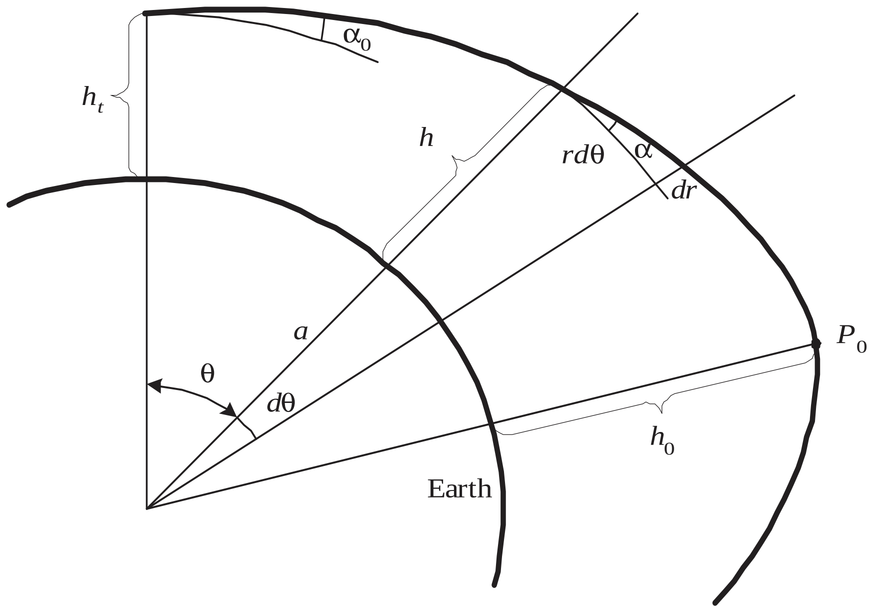

FIGURE 3.19. Geometry for ray tracing derivations.

For the standart atmosphere for which refractivity gradient is constant, a simpler method exists that makes use of the effective earth radius concept. Consider a curve of constant height h above the earth's surface. Since it defines a circle in the plane of the curve, with reference to Fig. 3.20, its equation can be written as

$$(y + a_e)^2 + x^2 = (a_e + h)^2 . \tag{3.4.13}$$

After expanding and using $y \ll a_e$ and $h \ll a_e$ we can simplify this equation to

$$y = h - \frac{x^2}{2a_e} \tag{3.4.14}$$

which defines a parabola. This derivation shows that the constant height contours above the earth's surface can be approximated as parabolic curves, which simplifies the generation of range-angle-height charts greatly.

There are many published papers on ray tracing methods including the computer generation of range-angle-height charts,a few of which can be found in the references, [**21, 22**].

3.5. Examples

EXAMPLE 10. *Determine the attenuation factor, free space path loss, field at the receiver, and the received power for* $P_t = 20\,\mathrm{W}$, $f = 450\,\mathrm{MHz}$, $G_1 = 100$, $G_2 = 100$, $h_t = 80\,\mathrm{m}$,

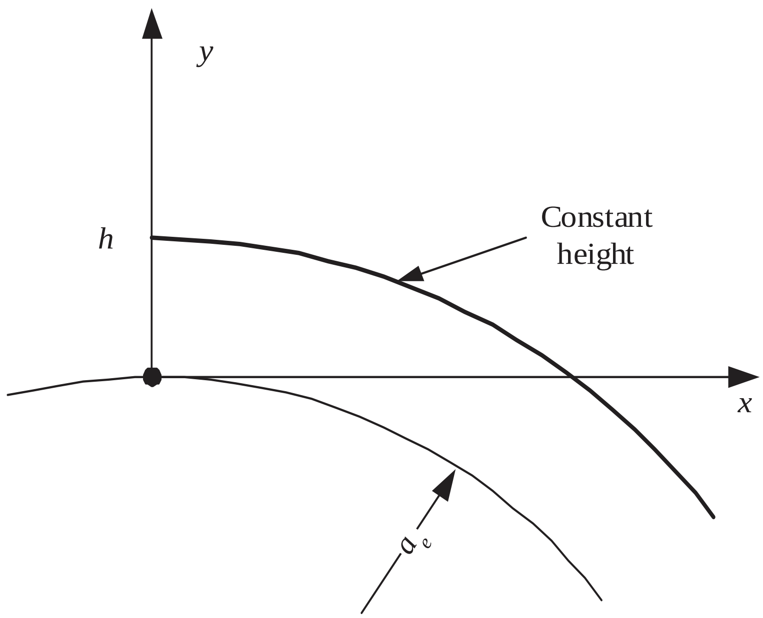

FIGURE 3.20. Geometry used for constant height curves.

$h_r = 20\,\mathrm{m}$, *and* $R = 1.25\,\mathrm{km}$. *The transmission is over medium dry ground. Consider vertical and horizontal polarization. Assume a flat earth.*

SOLUTION 10. *We can find* $R_r = \sqrt{R^2 + (h_t + h_r)^2} = 1254\,\mathrm{m}$ *and* $\psi = \tan^{-1}\frac{80+20}{1254} = 7.96 \times 10^{-2}\,\mathrm{rad}$. *If we use the approximation* $\psi = (h_t + h_r)/R$ *we get* $\psi = 0.08\,\mathrm{rad}$, *which is very close. By (3.1.10),* $\Delta R = 2.56\,\mathrm{m}$. *From Table (3.1) we have* $\kappa' = 15$ *and* $\sigma = 10^{-3}\,\mathrm{S/m}$ *for medium dry ground. Inserting these values in (3.1.16) we get*

$$\begin{aligned}\Gamma_v &= 0.515\angle 180^\circ \\ \Gamma_h &= 0.958\,\angle 180^\circ\end{aligned}$$

Since $R_{\max} = 4h_t h_r/\lambda_0 = 21.3\,\mathrm{km} > R$, *we must use (3.1.12). Thus*

$$\begin{aligned}|F_v| &= 0.85 \text{ or } |F_v| = 20\log_{10}(0.85) = -1.5 \ dB, \\ |F_h| &= 0.94 \text{ or } |F_h| = 20\log_{10}(0.94) = -0.5 \ dB.\end{aligned}$$

The free space path loss is $L = 20\log_{10}\left(\frac{4\pi R}{\lambda_0}\right) = 87.4 \ dB$. *The field at the receiver is*

$$\begin{aligned}E_{rms} &= \frac{\sqrt{30 \times 20 \times 100}}{1250} \times 0.85 = 166\,\mathrm{mV/m} \text{ for vertical polarization,} \\ E_{rms} &= \frac{\sqrt{30 \times 20 \times 100}}{1250} \times 0.94 = 184\,\mathrm{mV/m} \text{ for horizontal polarization.}\end{aligned}$$

The received power is

$$\begin{aligned}P_r &= 43 + 20 + 20 - 87.4 - 1.4 = -5.9 \ dBm, \text{ vertical} \\ P_r &= 43 + 20 + 20 - 87.4 - 0.5 = -4.9 \ dBm, \text{ horizontal}\end{aligned}$$

Example 11. *Assuming a flat earth, determine the attenuation factor and the electric field at the reception point for* $P_t = 1\,\mathrm{W}$, $f = 2\,\mathrm{GHz}$, $G_t = 20$, $h_t = 25\,\mathrm{m}$, $h_r = 10\,\mathrm{m}$, *and* $R = 10\,\mathrm{km}$. *The antennas are horizontally polarized and the propagation is over wet ground.*

Solution 11. *The grazing angle is* $\psi = (h_t + h_r)/R = 0.003\,5\,\mathrm{rad}$. *The reflection coefficient is* $\Gamma_v = -0.999$ *which can be taken as* -1. *Thus we can use (3.1.12) and* $F = 1.73 > 1$ *which shows that the signal strength is actually increased due to constructive interference. The field at the reception point is*

$$E_{rms} = \frac{\sqrt{30 \times 1 \times 20}}{10000} \times 1.73 = 4.\,2\,\mathrm{mV/\,m}.$$

Example 12. *Repeat example (11) for* $f = 10\,\mathrm{GHz}$, *and* $R = 16.5\,\mathrm{km}$. *Assume a flat earth.*

Solution 12. *The grazing angle is* $\psi = (25 + 10)/16500 = 2.\,1 \times 10^{-3}\,\mathrm{rad}$ *and therefore we may assume that the reflection coefficient is* -1 *(its actual value is* $\Gamma_h = -0.992$*). The path length difference is* $\Delta R = 2 \times 25 \times 10/16500 = 3\,\mathrm{cm}$. *This results in almost complete cancellation giving* $F = 0.06$ *or* -23 *dB. The field at the reception point is*

$$E_{rms} = \frac{\sqrt{30 \times 1 \times 20}}{16500} \times 0.06 = 94\,\mu\mathrm{V/\,m}.$$

Example 13. *Repeat example (12) using spherical earth model but use the actual earth radius* $a = 6371\,\mathrm{km}$.

Solution 13. *We apply the spherical earth formulas and get*

$$\begin{aligned} p &= \frac{2}{\sqrt{3}}\left[a\,(h_t + h_r) + \frac{d^2}{4}\right]^{1/2} = 19699.\,3, \\ \Phi &= \cos^{-1}\left(\frac{2ad\,(h_r - h_t)}{p^3}\right) = 1.\,996\,\mathrm{rad}, \\ d_r &= \frac{d}{2} + p\cos\left(\frac{\Phi + \pi}{3}\right) = 5467.\,2\,\mathrm{m}. \end{aligned}$$

Notice that since $h_r < h_t$ *we have used the formulas to calculate the distance of the reflection point to the receiver.*

$$\begin{aligned} S_1 &= \frac{d_r}{\sqrt{2ah_r}} = 0.484, \\ S_2 &= \frac{d_t}{\sqrt{2ah_t}} = 0.618, \\ T &= \sqrt{h_r/h_t} = 0.632, \\ S &= \frac{S_1T + S_2}{1 + T} = 0.566, \end{aligned}$$

$$\begin{aligned}
J(S,T) &= \left(1-0.484^2\right)\left(1-0.618^2\right)=0.473,\\
K(S,T) &= \frac{\left(1-S_2^2\right)+T^2\left(1-S_1^2\right)}{1+T^2}=0.660,\\
\Delta R &= \frac{2h_1h_2}{d}J(S,T)=1.43\,\mathrm{cm},\\
\psi &= \tan^{-1}\left(\frac{h_1+h_2}{d}K(S,T)\right)=1.4\times10^{-3}\,\mathrm{rad}.
\end{aligned}$$

In both flat earth and spherical models, the grazing angle is very small to justify the use of reflection coefficient as -1. *However, the path length differences are quite different in terms of wavelength (the difference in* ΔR *calculated by the two models is almost half wavelength). Since* $\Delta R \approx \lambda_0/4$ *we will take* $D=1$ *and obtain*

$$F = 1.995.$$

Notice the large difference in F *obtained by the two methods. The maximum line of sight distance for the two antennas is*

$$R_{\max} = \sqrt{2ah_t}+\sqrt{2ah_r}=29136\,\mathrm{km}.$$

Although the separation is only half this distance, we see that the affect of earth's curvature is not negligible. Finally the received field is

$$E_{rms} = \frac{\sqrt{30\times1\times20}}{16500}\times1.995=2.96\,\mathrm{mV/m}.$$

The path length difference can also be calculated using the idea of effective heights. The effective heights of the antennas are

$$\begin{aligned}
h_t' &= h_t-\frac{d_t^2}{2a}=15.4\,\mathrm{m},\\
h_r' &= h_r-\frac{d_r^2}{2a}=7.6\,\mathrm{m}
\end{aligned}$$

which gives $\Delta R = 2h_t'h_r'/R = 1.43\,\mathrm{cm}$.

Example 14. *Repeat example (13) using the effective earth radius* $a=8562\,\mathrm{km}$.

Solution 14. *The calculation steps is same except for the value of* a. *We find* $\Delta R = 1.8\,\mathrm{cm}$, $\psi = 1.6\times10^{-3}\,\mathrm{rad}$ *and* $F=1.901$. *The the field strength is calculated as* $E_{rms}=2.83\,\mathrm{mV/m}$. *Notice that there is* 0.4 *dB difference in the field calculations.*

Example 15. *Suppose a man of height* $1.80\,\mathrm{m}$ *is standing at the shore on a very calm day and observes that he can see the top of a ship whose height he knows to be* $10\,\mathrm{m}$. *How far is the ship to the shore?*

Solution 15. *The rays also bend at the optical frequencies but since index of refraction is a function of frequency, the effective earth radius will be different. At optical frequencies, the commonly accepted value of refractivity gradient is* $dN/dh=-2.2\times10^{-2}$ *which gives* $a_e\approx 7a/6$. *Using this value of effective earth's radius we get*

$$R=\sqrt{2h_ma_e}+\sqrt{2h_sa_e}=17.4\,\mathrm{km}.$$