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CHAPTER 2

Line-of-Sight Propagation

2.1. Fresnel Zone

The term line of sight (LOS) propagation is applied to the cases in which the power radiated by a transmitting antenna arrives at the receiver antenna without any obstruction. This condition implies that the Friis' transmission formula can be used.

If we draw a straight line from the transmitting antenna to the receiver antenna, this line should not intersect any obstacle. In fact, since the power is propagated as a wave, this condition is not enough. The neighborhood of the line between the antennas should also be clear of any obstacles.

One way to visualize the wave propagation is explained by Huygens. As the wave arrives at a point, it creates secondary sources, each of which behave like an isotropic radiator as shown in Fig. 2.1. Therefore, the field at a further away point will be the sum of waves radiated from these secondary sources. However, the path lengths will be different. Now consider the locus of points for which $R_2 - R_1 = \lambda_0/2$. These points define an ellipse in the plane (or an ellipsoid in 3-D space) and for any point inside this ellipse, the path length difference will be less than $\lambda_0/2$, implying that the phase difference will be less than 180⁰. This ellipse is the boundary of the first Fresnel zone. The ellipses defined by the loci of points for which $R_2 - R_1 = n\lambda_0/2$ form the boundaries of the Fresnel zones.

FIGURE 2.1. Visualization of the Huygens' principle.

Figure 2.2 shows the first few Fresnel zones for a communication system. The LOS condition means that at least a few of the Fresnel zones are clear of any obstacles. Since the phase of the wave propagating through adjacent Fresnel zones differ by 180^0 , the sum of these waves tend to cancel. Actually, in optical theory it is shown that adjacent higher order zones cancel each other, cancellation being more complete for the higher order zones. The aggregate effect of all higher order zones after pairwise cancellation is only about half the contribution from the first zone. Therefore, the first Fresnel zone bounds the volume contributing significantly to wave propagation and if the first Fresnel zone is clear of any obstacles, LOS propagation can be assumed.

FIGURE 2.2. Fresnel zones.

The intersection of the Fresnel ellipsoid with a plane perpendicular to the propagation path is a circle whose radius depends on the distance of the plane to the two antennas. In communication systems, we need to determine the radius of the Fresnel zones. With reference to Fig. 2.3, we have

$$
AX_n + X_nB = R_1 + R_2 + n\frac{\lambda_0}{2}.
$$
\n(2.1.1)

From the triangles AX_nX_0 and BX_nX_0 we have

$$
AX_n = \sqrt{R_1^2 + h_n^2} \approx R_1 + \frac{h_n^2}{2R_1}
$$
\n(2.1.2a)

$$
BX_n = \sqrt{R_2^2 + h_n^2} \approx R_2 + \frac{h_n^2}{2R_2}
$$
\n(2.1.2b)

since $h_n \ll R_1, R_2$ due to the fact that $R_1 \gg \lambda_0$ and $R_2 \gg \lambda_0$. Using (2.1.2) in (2.1.1) gives

$$
\frac{h_n^2}{2} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = n \frac{\lambda_0}{2}
$$
\n(2.1.3)

which can be solved for h_n to give

$$
h_n = \sqrt{\frac{R_1 R_2 n \lambda_0}{R_1 + R_2}}.\tag{2.1.4}
$$

The maximum is achieved when $R_1 = R_2 = R/2$ in which case the Fresnel radius is

$$
h_{n,\max} = \frac{1}{2}\sqrt{Rn\lambda_0}.\tag{2.1.5}
$$

For the first Fresnel radius, we must take $n = 1$ giving

$$
h_1 = \sqrt{\frac{R_1 R_2 \lambda_0}{R_1 + R_2}},\tag{2.1.6a}
$$

$$
h_{1,\max} = \frac{1}{2}\sqrt{R\lambda_0}.\tag{2.1.6b}
$$

FIGURE 2.3. Calculation of the radii of the Fresnel zones.

2.2. Obstructed LOS

When an obstacle is present in the first few Fresnel zones, the Friis' transmission formula will no more be valid. In general, it is quite difficult to calculate the effect of the obstacle on propagation. However, if the obstacle is totally opaque to the radio waves and has a sharply defined edge, the so called "knife edge" diffraction formulas can be used. Since electrical properties of the edge is not defined, the methods of physical optics are used.

Two possible cases for radio propagation over a knife edge are considered which are shown in Fig. 2.4. In case (a), the object does not cut the direct path but protrudes into first Fresnel zones. In case (b), the direct ray from the transmitter to the receiver is obstructed. The quantity h defined in the figure is assumed to be negative for case (a) and positive for case (b).

Using the theory of optical diffraction, $\mathbf{1}$, it can be shown that for free space the attenuation function is given by

$$
F_d(v) = \frac{1}{\sqrt{2}} \int_v^{\infty} \left(e^{j\frac{1}{2}\pi x^2} \right) dx = \frac{1}{\sqrt{2}} \left[C(v) + jS(v) \right]
$$
 (2.2.7)

where $C(v)$ and $S(v)$ are Fresnel integrals defined as

$$
C(v) = \frac{1}{2} - \int_0^v \cos\left(\frac{\pi x^2}{2}\right) dx,\tag{2.2.8a}
$$

$$
S(v) = \frac{1}{2} - \int_0^v \sin\left(\frac{\pi x^2}{2}\right) dx.
$$
\n
$$
(2.2.8b)
$$

The parameter v is given by

$$
v = \frac{\sqrt{2}h}{h_1} \tag{2.2.9}
$$

FIGURE 2.4. Radio propagation across a knife edge.

where h_1 is the radius of the first Fresnel zone at the location of the obstacle. The magnitude of $F_d(v)$ is plotted in Fig. 2.5. Although this formula can be worked out in a computer, we have a simple approximation given as

$$
20\log_{10}|F_d(v)| = \begin{cases}\n-6.9 - 20\log_{10}\left(\sqrt{1 + (v - 0.1)^2} + v - 0.1\right) & \text{for } v > -0.7 \\
0 & \text{for } v \le -0.7\n\end{cases}
$$
\n(2.2.10)

which is also shown in Fig. 2.5 .

When the obstacle does not protrude into the first Fresnel zone, we have $h \leq -h_1$, or equivalently, $v \le -\sqrt{2}$ and $|F_d(v)| \le 1$ dB in this range. Thus we can assume that the transmission is LOS as mentioned previously. If the line of sight is obstructed, the attenuation F_d due to diffraction should be added to the free space path loss.

2.3. Atmospheric Attenuation

In free space, the radio waves spread as spherical waves and the power density decreases by an inverse square law as given by the Friis' transmission formula. As already mentioned, this attenuation is given by the free space path loss. Although the earth's atmosphere is quite thin and has electrical properties close to that of free space, the attenuation caused by the gases and particles in the atmosphere is not negligible, especially over long distances.

Maxwell's equations show that the radio waves decay exponentially in a medium with finite conductivity. Thus, when radio waves propagate in lossy media, the attenuation can be described either by the factor $e^{-\alpha l}$ or by the factor $10^{-\delta l/20}$ where α and δ is the loss per unit length of the propagation path, and l is the total distance covered. In the first case α is in nepers, (Np) per unit distance (typically km), while δ is in dB per unit distance in the second case.

FIGURE 2.5. Attenuation function versus v .

The field at a distance R will then be given by

$$
E = \frac{\sqrt{30P_t G_t (\theta, \phi)}}{R} e^{-\alpha R} = \frac{\sqrt{30P_t G_t (\theta, \phi)}}{R} 10^{-\delta R/20}.
$$
 (2.3.1)

There are several mechanisms that contribute to attenuation of radio waves as they propagate in the atmosphere. These are considered in the following sections.

2.3.1. Molecular Absorption. The gases present in the atmosphere, mainly oxygen and uncondensed water vapor absorb radio frequencies at certain frequencies. As a result, there are various absorption lines in the centimeter and millimeter wavelength region separated by frequency bands of relatively much lower attenuation. Figure 2.6 shows the atmospheric attenuation in the frequency range between 10 and 300 GHz, [2]. Although the amount of water vapor is not specified, the behavior is typical. There are strong water vapor absorption lines at $22 \text{ GHz } (\lambda_0 = 1.35 \text{ cm})$ and $183 \text{ GHz } (\lambda_0 = 1.65 \text{ mm})$. At 60 GHz $(\lambda_0 = 0.5 \text{ cm})$ and 118 GHz $(\lambda_0 = 0.25 \text{ cm})$ we observe strong absorption due to oxygen. If the atmospheric attenuation is large, e.g., above 10 dB, the range of communication is severely restricted. It can be seen from Fig. 2.6 that there are "windows" that are relatively transparent to radio waves. One such window is the Ka band between 22 GHz and 60 GHz. On the other hand, the high attenuation frequencies may be used when the transmitted signal is not desired to reach far away receivers such as in short range secure communication systems. For example 60 GHz frequency is proposed for collision avoidance radars.

It must be noted that the atmospheric attenuation is dependent on the (uncondensed) water vapor content and therefore on temperature and height above sea level.

FIGURE 2.6. Atmospheric attenuation versus millimeter wavelengths. The IEEE radar bands are also shown, [2].

There appears a general increase in the attenuation with frequency in Fig. 2.6. However, this result is only true for this frequency range. At much higher frequencies, especially in the optical region, the attenuation decreases. The atmospheric opacity as a function of wavelength is shown in Fig. 2.7. It can be seen that atmosphere is fairly transparent to radio waves of wavelength 1 cm to 10 m and also in the visible region and parts of the infrared spectrum. In the infrared spectrum there are two main windows, namely 3-5 μ m and 8-12 μ m bands.

FIGURE 2.7. Atmospheric opacity as a function of wavelength.

2.3.2. Molecular Scattering. At very short wavelengths, especially in the optical range which includes the visible spectrum, the molecules of the atmospheric gases, namely $O₂$ and $N₂$, scatter the waves in different directions, thus reducing the energy propagated along a direction. Since the dimensions of these molecules (typically $3 \times 10^{-4} \mu$ m) are much smaller than the wavelength, classical Rayleigh scattering theory can be used to evaluate scattering from such particles. In the notation adopted, the attenuation due to Rayleigh scattering is

$$
\delta = \frac{4.34 \times 32\pi^3 \times 10^3}{3N\lambda_0^4} (n_0 - 1)^2 \text{ dB/km}
$$
\n(2.3.2)

where $N = 2.69 \times 10^{25}$ per m³ is the Avogadro's number under normal atmospheric conditions, and n_0 is the refractive index at the earth's surface, typically equal to 1.000325. From (2.3.2) it follows that the attenuation is proportional to the fourth power of frequency, that is the attenuation due to molecular scattering increases rapidly with increasing frequency.

The water molecules (H_2O) are about the same size of the O_2 and N_2 molecules, but the presence of water markedly affects atmospheric attenuation. Even if the water vapor content is far from saturation level, these molecules tend to condensate into tiny droplets many times larger in size than the molecules. These droplets are formed around the strongly hygroscopic molecules of salts of chlorine, magnesium, sodium, and sulphur which are always present in the atmosphere. At low air humidity, these droplets are crystals of size of about $1 \times 10^{-3} \mu m$ which are an order of magnitude larger than the sizes of atmospheric molecules. As the humidity increases, these crystals turn into droplets of solution, with radii 10^{-3} to $10^{-1} \mu$ m. At humidity levels close to saturation, the droplets get larger and may have a radius up to 1μ m. This may cause a haze to appear in the atmosphere and the attenuation of visible light increases drastically, reducing visibility down to a few kilometers. Experiments have shown that in the case of scattering by small droplets in a moist air (but in the absence of a haze) the attenuation is proportional to 1.75th power of frequency, i.e., not as rapidly as predicted by Rayleigh scattering.

Specific attenuation due to atmospheric gases is studied in detail in ITU-R Recommendation P676—8, and procedures to calculate gaseous attenuation at frequencies up to 1000 GHz are given in [3].

2.3.3. Attenuation by Solid Particles. Sometimes, very small (not over $1 \mu m$) dry particles such as dust, smoke, ash, and like are suspended in the atmosphere. If humidity is very low, they remain as dry particles, and act as scatterers producing dry haze. If the humidity is high, they may cause water vapor to condense around them and form haze. The atmospheric attenuation caused by such particles is mostly in the optical range and it is negligible for radio wave frequencies.

The three mechanisms that contribute to attenuation of radio waves discussed so far do not cause much attenuation in the radio wave frequency range. In the radio wave frequency range, the attenuation caused by precipitation particles is much more important.

2.3.4. Attenuation by Precipitation Particles. The precipitation particles that are considered in this section are rain, snow, hail and fog. Although fog is not precipitating, we will consider it in this class since its properties are quite similar to rain. There are basically two mechanisms that cause attenuation by such particles: absorption and scattering.

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2.3.4.1. *Attenuation by Rain.* Each water droplet can be considered as an imperfect conductor in which the radio waves induce displacement currents. Since the conductivity is finite, this causes ohmic losses in the particle, which means some of the energy of the wave is being absorbed, thus attenuating the wave. Besides, the induced currents behave like secondary sources, radiating (or scattering) the energy in different directions. This scattered energy results in the attenuation of the radio waves in the direction of propagation — instead of propagating in the right direction, the radio waves are partially scattered in all directions.

Assume that a drop of rain can be considered as a spherical dielectric of radius $a \ll \lambda_0$ with complex dielectric constant $\kappa = \kappa' - j\kappa''$. The incident electric field is assumed to propagate in the x direction as

$$
\mathbf{E}_i = E_0 \mathbf{a}_z e^{-jk_0 x}.\tag{2.3.3}
$$

Since $a \ll \lambda_0$ the incident field is essentially uniform over the extent of the drop. The polarization produced in the drop can be determined by solving a boundary value problem and shows that the dipole polarization P in the drop is constant given by

$$
\mathbf{P} = 3\frac{\kappa - 1}{\kappa + 2}\epsilon_0 E_0 \mathbf{a}_z.
$$
 (2.3.4)

The drop can be considered as a single dipole of dipole moment

$$
\mathbf{P}_0 = 4\pi a^3 \frac{\kappa - 1}{\kappa + 2} \epsilon_0 E_0 \mathbf{a}_z.
$$
 (2.3.5)

Since the incident field is time varying, the time derivative of the dipole moment gives the dipole moment of an equivalent Hertzian dipole, i.e., we can view the drop as a Hertzian dipole of moment $Idl = j\omega P_0$. The radiation from the drop can then be written as

$$
\mathbf{E}_s = -\omega Z_0 k_0 P_0 \sin \theta \frac{e^{-jkr}}{4\pi r} \mathbf{a}_\theta.
$$
 (2.3.6)

The total scattered power is given by the integral of the Poynting vector over a sphere enclosing the drop as

$$
P_s = \frac{1}{2} Y_0 \int_0^{2\pi} \int_0^{\pi} |\mathbf{E}_s|^2 r^2 \sin \theta d\theta d\phi = \frac{\omega^2 k_0^2 Z_0}{12\pi} |\mathbf{P}_0|^2
$$
 (2.3.7a)

$$
=\frac{\omega^2 k_0^2 Z_0}{12\pi} \left(4\pi a^3 \frac{\kappa-1}{\kappa+2} \epsilon_0 E_0\right)^2 \tag{2.3.7b}
$$

$$
= \frac{4}{3}\pi a^2 (k_0 a)^4 Y_0 \left| \frac{\kappa - 1}{\kappa + 2} \right|^2 E_0^2.
$$
 (2.3.7c)

This is the Rayleigh formula for the scattered power from which (2.3.2) can be derived. Notice that the total scattered power is proportional to the 4th power of frequency. In radar applications, the scattering cross section σ_s is defined as the ratio of the total scattered power to the incident power density. Hence

$$
\sigma_s = \frac{P_s}{\frac{1}{2}Y_0 E_0^2} = \frac{8}{3}\pi a^2 (k_0 a)^4 \left| \frac{\kappa - 1}{\kappa + 2} \right|^2.
$$
\n(2.3.8)

The power density in the backscattering direction (opposite to the propagation) is given by

$$
P_{\rm BS} = \frac{1}{2} Y_0 |E_s|^2 \bigg|_{\theta = \pi/2} = \frac{\omega^2 k_0^2 Z_0 P_0^2}{32\pi^2 r^2}.
$$
\n(2.3.9)

For a monostatic radar, this power would return back to the radar to be detected and is called the rain clutter. The radar backscatter cross section σ_{BS} is defined so that if scattering occurred isotropically, the same backscattered power would result. That is

$$
P_{\rm BS} = P_{inc} \frac{\sigma_{\rm BS}}{4\pi r^2}.
$$
\n(2.3.10)

Combining $(2.3.9)$ and $(2.3.10)$ yields

$$
\sigma_{\rm BS} = 4\pi a^2 \left(k_0 a \right)^4 \left| \frac{\kappa - 1}{\kappa + 2} \right|^2 = \frac{2}{3} \sigma_s. \tag{2.3.11}
$$

The power absorbed by the drop can be found by considering the ohmic losses. The polarization current inside the sphere is $J_p = j\omega P$ and is uniform. The total electric field inside the sphere is given by $\mathbf{P} = (\kappa - 1) \epsilon_0 \mathbf{E}$. The time average absorbed power is then

$$
P_a = \frac{1}{2} \operatorname{Re} \int_0^a \int_0^{2\pi} \int_0^{\pi} \mathbf{E} \cdot \mathbf{J}_p^* r^2 \sin \theta d\theta d\phi dr
$$
 (2.3.12a)

$$
=\frac{2}{3}\pi a^3 \operatorname{Re} \mathbf{E} \cdot \mathbf{J}_p^*
$$
 (2.3.12b)

$$
=6\pi a^3 k_0 Y_0 \frac{\kappa'' |E_0|^2}{|\kappa+2|^2}
$$
\n(2.3.12c)

The ratio of the absorbed power to the incident power density is defined as the absorption cross section, σ_a . Thus,

$$
\sigma_a = \frac{P_a}{\frac{1}{2}Y_0 |E_0|^2} = 12\pi a^2 (k_0 a) \frac{\kappa''}{|\kappa + 2|^2}.
$$
\n(2.3.13)

The extinction cross section σ_e is the sum of the absorption cross section and the scattering cross section

$$
\sigma_e = \sigma_s + \sigma_a \tag{2.3.14}
$$

and is a measure of the power removed from the propagating radio wave. The total power removed from the incident wave as scattered or absorbed power is given by the product of the incident power density with the extinction cross section σ_e .

In the case of precipitation, there are many particles. If we assume that there are N drops per unit volume, the power removed from an incident wave of power density P as the wave propagates in the x direction is

$$
\frac{dP}{dx} = -P\sigma_e N = -\frac{1}{2}Y_0 |E_0|^2 \sigma_e N.
$$
\n(2.3.15)

The solution of this equation is

$$
P(x) = P(0) e^{-\sigma_e N x}.
$$
\n(2.3.16)

Thus the attenuation will be $\sigma_e N$ Np per unit length. This shows that as the number of rain drops per unit volume increases (heavy rain) the attenuation will also increase. In fact,

| Name | R. |
|----------------------|---------------------------|
| Drizzle | $0.25 \,\mathrm{mm/hour}$ |
| Light rain | $1 \,\mathrm{mm/hour}$ |
| Moderate rain | $4 \,\mathrm{mm/hour}$ |
| Heavy rain | $16 \,\mathrm{mm/hour}$ |
| Thunderstorm | $35 \,\mathrm{mm/hour}$ |
| Intense thunderstorm | $100 \,\mathrm{mm/hour}$ |

TABLE 2.1. Rain rates.

the sizes of the drops in an actual rain will not all be the same. Since σ_e depends strongly on the drop size, this should also be taken into account. If $N(a) da$ is the number of drops per unit volume with radii between a and $a + da$, we have

$$
\frac{dP}{dx} = -P \int_0^\infty \sigma_e(a) N(a) da \qquad (2.3.17)
$$

and the decay rate will be

$$
A\left(x\right) = \int_0^\infty \sigma_e\left(a\right) N\left(a\right) da. \tag{2.3.18}
$$

The decay rate in $(2.3.18)$ is shown to be a function of x since the drop size distribution N (a) may change at different locations of rain.

The theory outlined above gives a general idea about the attenuation of radio waves by rain. However, we still need to know the extinction cross section of drops and the drop size distribution function. Such quantities are very difficult to measure and may depend on many different conditions. Usually, only the rain rate R in millimeters of water per hour can be measured, and in propagation calculations we need to relate decay rate to rain rate. There is no generally accepted definition scale for rain rates but Table (2.1) gives a general idea.

A detailed theoretical and experimental study given in [4] relates the specific attenuation to the decay rate in the form

$$
A = aR^b \text{ dB/km.}
$$
\n
$$
(2.3.19)
$$

The constants a and b are in turn given by

$$
a = G_a f^{E_a}; \quad b = G_b f^{E_b} \tag{2.3.20}
$$

and Table (2.2) gives the values of the relevant constants at temperature of 0° C. The rain rate in $(2.3.19)$ is in mm/h, while f in $(2.3.20)$ is in GHz.

| Frequency range | σ_a | $E_a\$ | Frequency range | | E_h |
|---|-----------------------|--------|--|-------|-----------|
| $< 2.9 \mathrm{GHz}$ | 6.39×10^{-5} | 2.03 | $< 8.5\,\mathrm{GHz}$ | 0.851 | 0.158 |
| $< 54\,\mathrm{GHz}$ $2.9\,\mathrm{GHz}$ | 4.21×10^{-5} | 2.42 | 8.5 GHz $< 25 \, \text{GHz}$ | 1.41 | -0.0779 |
| 180 GHz $54\,\mathrm{GHz}$ | 4.09×10^{-2} | 0.699 | $25\,\mathrm{GHz}$ $< 164 \mathrm{GHz}$ | 2.63 | -0.272 |
| $180\,\mathrm{GHz} <$ | 3.38 | | $164\,\mathrm{GHz} < 1$ | | 0.0126 |

TABLE 2.2. Table of G_a , E_a , G_b , and E_b values.

The rain attenuation calculated using $(2.3.19)$ and $(2.3.20)$ is shown for different frequencies in Fig. 2.8 as a function of rain rate, whereas Fig. 2.9 gives the variation of rain attenuation as a function of frequency for $R = 5 \,\text{mm/h}$. It can be seen that for moderate rain the attenuation is relatively small at X band (10 GHz), only 0.0738 dB/ km, but it rapidly increases with frequency reaching 0.85 dB/ km at 30 GHz, and 2.34 dB/ km at 50 GHz.

FIGURE 2.8. Attenuation by rain as a function of rain rate as calculated by aR^b formula.

The ITU-R Recommendation P.838–3, [5], gives the specific rain attenuation γ_R (dB/km) as

$$
\gamma_R = kR^\alpha \tag{2.3.21}
$$

where the frequency and polarization dependent coefficients k and α are also given up to $f = 1000$ GHz.

2.3.4.2. *Attenuation by Fog and Cloud.* The attenuation of radio waves by fog is essentially same as that due to rain. The fog differs from rain in that fog is a suspension of very small water droplets with radii in the range 0.01 to 0.05 mm. The attenuation (in dB/km) by fog is almost linearly proportional to the total water content per unit volume. Thus, defining fog attenuation coefficient, K_l , as the attenuation per unit density of liquid water per unit distance gives fog attenuation as $A = WK_l$ where W is the liquid water density. A concentration of $\tilde{W} = 0.032 \text{ g/m}^3$ corresponds to a light fog that reduces optical visibility down to about 600 m. A concentration of $W = 0.32 \text{ g/m}^3$ corresponds to medium fog with

FIGURE 2.9. Rain attenuation calculated by aR^b formula as a function of frequency for $R = 5$ mm/h.

optical visibility of around 120 m, while a concentration of $W = 2.35 \text{ g/m}^3$ corresponds to dense fog with optical visibility of around 30 m. The relation between visibility and water density is

$$
W = 0.0156V^{-1.43} \text{ g/m}^3 \tag{2.3.22}
$$

where V is the visibility in km .

The fog attenuation depends also on the temperature. A formula for fog attenuation coefficient, which is claimed to be valid in the temperature range from $-8\degree\text{C}$ to 20 $\degree\text{C}$, is given in [6] as

$$
K_{l} = \begin{cases} 6.0826 \times 10^{-4} f^{1.8963} \theta^{(7.8087 - 0.01565f - 3.073 \times 10^{-4} f^{2})}, & 5 \text{ GHz} \le f \le 150 \text{ GHz} \\ 0.07536 f^{0.9350} \theta^{(-0.7281 - 0.0018f - 1.542 \times 10^{-6} f^{2})}, & 150 \text{ GHz} < f \le 1000 \text{ GHz} \end{cases}
$$
(2.3.23)

where f is in GHz and $\theta = 300/T$ with T being the temperature in K. The fog attenuation coefficient in $(2.3.23)$ is in $\left(\frac{dB}{km}\right) / \left(\frac{g}{m^3}\right)$.

Since fog is a cloud formed near earth's surface, the attenuation in clouds are similar to the attenuation in fog. The ITU-R Recommendation ITU-R P.840—4, [7], provides methods to predict the attenuation due to clouds and fog on Earth-space paths.

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2.3.4.3. *Attenuation by Snow and Hail.* The complex dielectric constant $\kappa = \kappa' - j\kappa''$ of ice is significantly different from that of water. For ice, κ' is almost constant and is equal to 3.17 for temperatures between −30 °C and 0 °C throughout the frequency range of 1 GHz to 300 GHz. The imaginary part is very small (less than 4×10^{-3}). The small value of κ'' implies very small attenuation. However, snow and hail commonly consist of a mixture of ice crystals and liquid water.

The attenuation due to dry snow and hail are at least an order of magnitude smaller than that in rain for the same precipitation rate. However, attenuation by wet snow is comparable to that of rain. Thus, the attenuation depends highly on meteorological conditions which are hard to measure or estimate. Therefore, there is no simple expression that relates attenuation to precipitation rate and the subject will not be discussed any further here.

2.3.5. Effect of polarization. Most of the particles in the atmosphere either do not have any orientation or are randomly distributed and their conglomerate do not have directional properties. Therefore, the atmospheric attenuation is essentially independent of the wave polarization. The rain particles, on the other hand, are somewhat elongated in the horizontal direction due to the drag of the atmosphere. Still the attenuation is not much different for horizontally or vertically polarized waves and the formulas given in the foregoing sections are assumed to be correct. For different polarizations, the formulas in [5] can be used.

More importantly, due to the directional properties of the rain particles, a portion of the incident energy is transmitted in the cross polarized wave. This causes a loss in the copolar energy and performance degradation in dual polarization systems which are systems that efficiently utilize frequency resources by transmitting different information on two orthogonally polarized waves. When circular polarization is used, the rain can switch the right hand polarization to left hand, and vice versa.

2.4. Atmospheric Attenuation for Radar

The basic mechanisms of atmospheric attenuation in radar systems are the same. The main difference is that the radar signals undergo two way propagation, i.e., from radar to target and back, which doubles the attenuation. Therefore, the signal strength calculations can be done using the formulas given in previous sections.

In Section 2.3.4.1, it is mentioned that the atmospheric particles radiate some of the energy back to the radar as given by (2.3.9). This energy is also received by the radar and interferes with the target signal. Such undesired signals are called clutter. Although the particles are very small, they occupy a large volume in space and the clutter power may be very large obscuring the target. This issue is beyond the scope of this course and will not be further investigated.

2.5. Examples

EXAMPLE 6. Suppose that the transmitter and receiver antennas in a communication *link are located at* $h_t = 100 \text{ m}$ *and* $h_r = 100 \text{ m}$ *above earth. The link operates at* $f = 1 \text{ GHz}$ *. The distance between the antennas is* 20 km *and there is a hill of height* 50 m *with a sharply pointed peak, at a distance of* 5 km *from the transmitter. Determine the path loss between the transmitter and the receiver. Repeat the same problem if the hill height is* 120 m.

SOLUTION 6. *Using (2.1.4) we can find the radius of the first Fresnel zones as*

$$
h_1 = \sqrt{\frac{R_1 R_2 \lambda_0}{R_1 + R_2}} = 33.5 \,\mathrm{m}.
$$

Since the hill does not protrude into the first Fresnel zone, we may assume LOS propagation and obtain

$$
L = \left(\frac{4\pi Rf}{c}\right)^2 = 118 \, dB.
$$

For hill height of 120 m*, the line of sight path is obstructed. The parameter* v *can be calculated as*

$$
v = \frac{\sqrt{220}}{33.5} = 0.844.
$$

Using (2.2.7) or from Fig. 2.5 we find

$$
F_d = -12.8 \ dB
$$

and the total loss becomes $118 + 12.8 = 130.8$ *dB.*

EXAMPLE 7. *Repeat the previous example for* $h_t = 200$ m *and hill height of* 120 m.

SOLUTION 7. In this case the range is slanted and we must take this into account. The *geometry is shown in Fig. 2.10. The distance of hill top from the line of sight is*

$$
h = h_x \cos \theta = \frac{h_x}{\sqrt{1 + \left(\frac{h_t - h_r}{R}\right)}} = 46.9 \,\mathrm{m}.
$$

It must be noted that in most practical situations θ *will be very small and we may assume* h_x = h as in this example. We have $v = \sqrt{246.9/33.5} = -1.98$ and F_d is negligible.

FIGURE 2.10. Geometry for example (7).

EXAMPLE 8. Determine the field due to a wave freely propagated in a fog with a visibility *of about* 120 m *at a distance of* 10 km *from the transmitter if the temperature is* 5 ◦C*. The transmitter power is* $P_t = 40$ *dBm, the transmitter antenna gain is* $G_t = 500$ *, and the frequency is* $f = 10$ GHz.

SOLUTION 8. *Using (2.3.23) with* $\theta = 300/278$ *, we find the fog attenuation coefficient is*

$$
K_l = 8.56 \times 10^{-2} \, (dB / \, \text{km}) / (g / \, \text{m}^3)
$$

The liquid water density is obtained from visibility by using (2.3.22) as

 $W = 0.32 \text{ g/m}^3$.

The attenuation is then $A = 8.56 \times 10^{-2} \times 0.32 = 0.027$ (*dB*/km) *and the total loss due to fog is* 0.27 *dB. The electric field is then given by (see eq. (1.3.4))*

$$
E_{rms} = \frac{\sqrt{30P_tG_t}}{R} \times 10^{-\frac{0.27}{20}} = 37.5 \,\mathrm{mV/m}
$$

EXAMPLE 9. Suppose that a transmitter radiates radio waves at $f = 30$ GHz. The trans*mitter power is* $P_t = 100 \text{ W}$ *, antenna gain is* $G_t = 500$ *. The receiver is in the direction of the main beam and is* 50 km *away from the transmitter. There is rain between the transmitter and receivers that extends* 10 km *of rate* 10 mm/ h*. Determine the field strength at the location of the receiver if the wave is freely propagated in the rain.*

SOLUTION 9. The power density at a distance R from the transmitter is

$$
P = \frac{P_t G_t}{4\pi R^2}.
$$

We have

$$
10\log_{10}\frac{1}{4\pi R^2} = -105 \ dB
$$

Using $(2.3.19)$, $(2.3.20)$, and Table (2.2) we find the specific rain attenuation as $A = 1.74$ $dB/\,\mathrm{km}$ *. Since the rain extends by* 10 km*, the total attenuation due to rain is* $L_r = 17.4$ *dB. The specific attenuation due to atmospheric gases for the portion of path without rain is found to be about* 0.1 *dB*/ km *from Fig.* (5) *of ITU-R P.676-8*, [3], *for* $p = 1013$ *hPa*, $T = 15 \degree C$, and $W = 7.5 \degree g/m^3$. Thus, the power density at the location of receiver will be

 $P = 27 + 20 - 105 - 17.4 - 4 = -79.4$ $dBW/m²$

where 27 *dB is the antenna gain,* 20 *dBW is the transmitter power, and* 4 *dB is the attenuation due to the atmospheric gases. The field at the location of the receiver can be found as:*

$$
E_{rms} = \sqrt{Z_0 P} = 2.08 \,\mathrm{mV/m}.
$$