CHAPTER 10

Wideband Channels

Up to this point, we have considered calculation path losses and/or received powers at a single frequency. Of course, any communication system uses a finite bandwidth about a center frequency. The previous calculations are reasonable only if the channel can be considered as independent of frequency over the communication bandwidth. Such a case would be called a "narrowband" communications channel, which is characterized by the propagation medium and its variation with frequency as compared to the bandwidth of the information transmitted. Same communication bandwidth may be treated as narrowband for some environments while it may become wideband for other environments.

Different fading at different frequencies result in distortion of the transmitted information, and can cause significant errors upon reception. It may be desirable to use a narrowband communication system to reduce such errors. However, there is an increasing demand for faster communication systems (primarily for high speed data transfer) which forces the systems to operate in wideband region. The characterization of such channels and methods to mitigate adverse affects of wideband channels are considered in this chapter.

10.1. Two Path Model

The simplest frequency fading channel consists of two rays: a direct ray from the transmitter to the receiver traveling a distance R, and a second ray scattered from an obstacle displaced laterally by H from the direct path, as shown in Fig. 10.1. For simplicity, assume that both rays have equal power.

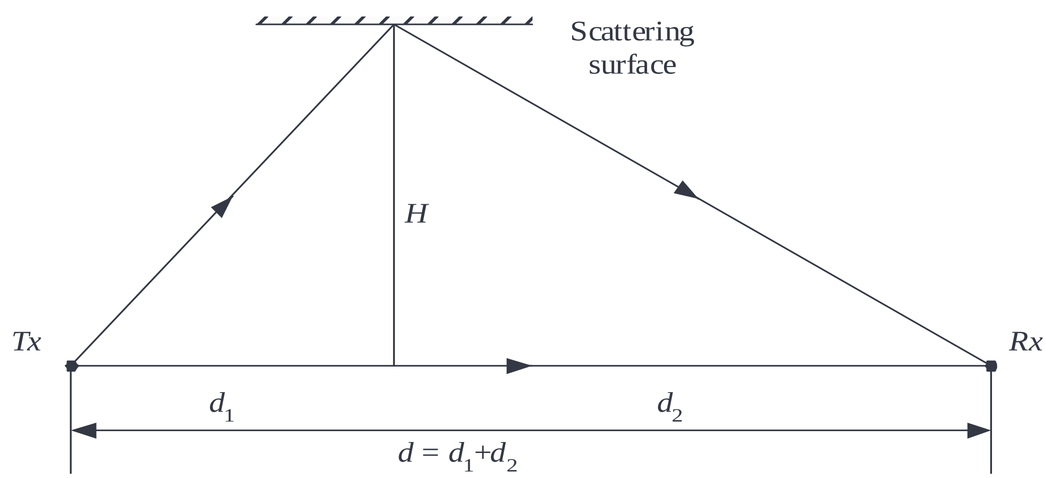

FIGURE 10.1. Two ray model.

If an impulse is transmitted by the transmitter, the receiver will observe two impulses separated in time by $\tau = \Delta R/c$ where ΔR is the path length difference and c is the speed of light. The impulse response of the channel is then

$$h(t) = \delta(t + t_0) + \delta(t + t_0 + \tau) \tag{10.1.1}$$

where $t_0 = d/c$. The transfer function of the channel is then

$$H(\omega) = e^{j\omega t_0}\left(1 + e^{j\omega\tau}\right) = 2\cos\frac{\omega\tau}{2}\exp\left(\frac{1}{2}j\omega(2t_0 + \tau)\right). \tag{10.1.2}$$

The magnitude of the transfer function is simply $|2\cos\omega t/2|$ which is shown in Fig. 10.2. Obviously, the transfer function is not constant and different frequency components will experience different amounts of attenuation, which will give rise to dispersion of the transmitted signals. The 3 dB bandwidth about a peak is π/τ. We may assume that the dispersion of a signal having a bandwidth smaller than the 3 dB bandwidth is negligible. That is, if the signal bandwidth is smaller than $1/\tau$, then the channel may be assumed to be dispersionless. Therefore, we define

$$f_{coh} = \frac{1}{\tau} \tag{10.1.3}$$

as the *coherence bandwidth* of the channel. For example, if the path length difference is 3 m, than $\tau = 10$ ns and $f_{coh} = 100$ MHz. If the signal bandwidth is comparable to 100 MHz the response will clearly be different for different frequencies and the channel is said to exhibit frequency selective fading. On the other hand, if the signal bandwidth is 10 MHz, the channel may be assumed to have a constant response.

10.2. Channel Characterization

10.2.1. Time Varying Channel Characterization. In real life, there are many scatterers and the scattered fields will not be of equal magnitude. The channel transfer function will not have nulls as in the two-ray model. The channel is time-varying due to the motion of transmitter, receiver or the scatterers. In such cases there will be a Doppler shift in the frequency of the received signal and the channel transfer function will be time-varying. It is convenient to use the complex envelope representation to represent real bandpass signals. We use the relation

$$s(t) = \text{Re}\left\{x(t)\, e^{j\omega_c t}\right\} \tag{10.2.1}$$

to define the complex envelope representation $x(t)$ of the real signal $s(t)$. For a linear time-varying channel, the complex envelope of the output is given by

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau;t)\, d\tau = \int_{-\infty}^{\infty} x(t)\, h(t-\tau;t)\, d\tau \tag{10.2.2}$$

where $h(\tau;t)$ is the *time-varying impulse response* of the channel at time t for an impulse applied at time τ. That is, if the input is $x(t) = \delta(t - t_0)$, the output will be $y(t) = h(t - t_0;t)$. Since a physical channel must be causal, $h(t - t_0;t)$ must be zero for $t < t_0$, or equivalently $h(\tau;t)$ must be zero for $\tau < 0$.

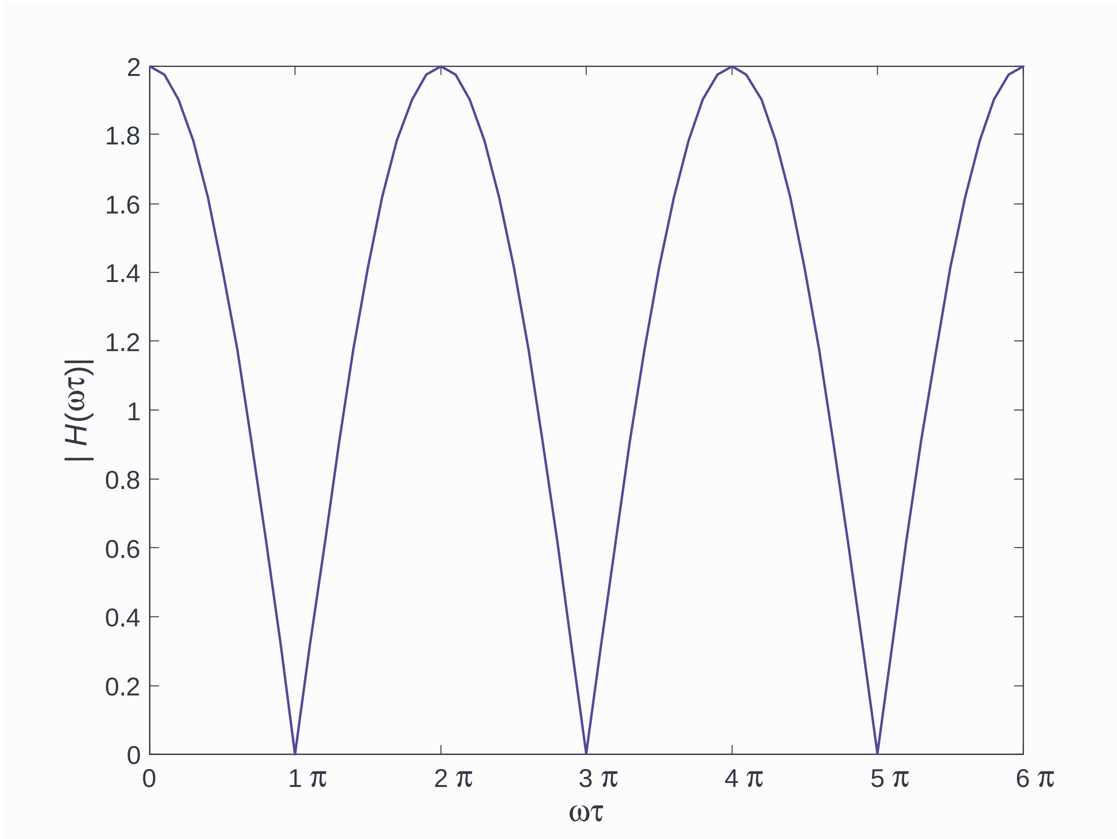

FIGURE 10.2. Magnitude of channel transfer function.

The Fourier transform of the transfer function with respect to the delay variable τ is defined as the *time-varying transfer function*, i.e.,

$$T(\omega;t) = \int_{-\infty}^{\infty} h(\tau;t)\, e^{-j\omega\tau} d\tau. \tag{10.2.3}$$

The complex envelope of the output can be written as

$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, T(\omega;t)\, e^{j\omega t} d\omega \tag{10.2.4}$$

where $X(\omega)$ is the Fourier transform of the input signal $x(t)$.

If we take the Fourier transform of $T(\omega;t)$ with respect to the t variable also, we get what is called as the *Doppler-spread function*:

$$H(\omega,\varpi) = \int_{-\infty}^{\infty} T(\omega;t)\, e^{-j\varpi t} dt. \tag{10.2.5}$$

With this definition, the complex envelope of the output can be written as

$$y(t) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(\omega)\, H(\omega,\varpi)\, e^{j(\omega+\varpi)t} d\omega d\varpi. \tag{10.2.6}$$

Using $\mu = \omega + \varpi$, we can write

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\mu-\varpi)\,H(\mu-\varpi,\varpi)\,d\varpi\right)e^{j\mu t}d\mu \tag{10.2.7a}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\mu)\,e^{j\mu t}d\mu \tag{10.2.7b}$$

where

$$Y(\mu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\mu-\varpi)\,H(\mu-\varpi,\varpi)\,d\varpi \tag{10.2.8}$$

is the Fourier transform of the output.

If the Fourier transform of $h(\tau;t)$ with respect to t variable is carried out first, we can express $y(t)$ as

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t-\tau)\,S(\tau;\varpi)\,e^{j\varpi t}d\varpi d\tau \tag{10.2.9}$$

where

$$h(\tau;t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\tau;\varpi)\,e^{j\varpi t}d\varpi. \tag{10.2.10}$$

The function $S(\tau;\varpi)$ is called the *delay-Doppler spread* function. The relation between these functions is shown schematically in Fig. 10.3.

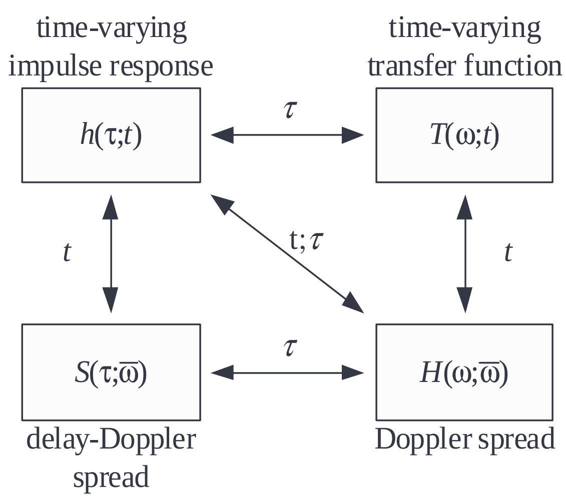

FIGURE 10.3. Relation between different definitiions.

10.2.2. Random Time Varying Channel Characterization. The definitions in the previous section apply to a deterministic channel. In real life, the channel is random. This means that, the impulse response is a random process. Typically, only a second order characterization of the channel is used. This means specification of the channel autocorrelation function. Such a characterization will be complete only for random processes of that have distributions with at most two parameters, including a Gaussian process. Corresponding to each function defined above, we define an autocorrelation function:

$$R_h(\tau_1,\tau_2;t_1,t_2) = \langle h(\tau_1;t_1)\, h^*(\tau_2;t_2)\rangle\,, \tag{10.2.11a}$$

$$R_T(\omega_1,\omega_2;t_1,t_2) = \langle T(\omega_1;t_1)\, T^*(\omega_2;t_2)\rangle\,, \tag{10.2.11b}$$

$$R_H(\omega_1,\omega_2;\varpi_1,\varpi_2) = \langle H(\omega_1;\varpi_1)\, H^*(\omega_2;\varpi_2)\rangle\,, \tag{10.2.11c}$$

$$R_S(\tau_1,\tau_2;\varpi_1,\varpi_2) = \langle S(\tau_1;\varpi_1)\, S^*(\tau_2;\varpi_2)\rangle\,. \tag{10.2.11d}$$

It must be noted that (τ_1, τ_2) are time delay variables with (ω_1, ω_2) being the corresponding frequency variables, while (t_1, t_2) are time variables with (ϖ_1, ϖ_2) being the corresponding frequency variables. Any of these autocorrelation functions may be used to obtain the autocorrelation of the output. For example, if $x(t)$ is a deterministic signal, we have

$$R_y(t_1,t_2) = \langle y(t_1)\, y^*(t_2)\rangle \tag{10.2.12a}$$

$$= \left\langle \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega_1)\, T(\omega_1;t_1)\, e^{j\omega_1 t_1} d\omega_1\right)\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega_2)\, T(\omega_2;t_2)\, e^{j\omega_2 t_2} d\omega_2\right)^*\right\rangle \tag{10.2.12b}$$

$$= \left\langle \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(\omega_1)\, X^*(\omega_2)\, T(\omega_1;t_1)\, T^*(\omega_2;t_2)\, e^{j(\omega_1 t_1 - t_2\omega_2)} d\omega_1 d\omega_2\right\rangle \tag{10.2.12c}$$

$$= \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(\omega_1)\, X^*(\omega_2)\, \langle T(\omega_1;t_1)\, T^*(\omega_2;t_2)\rangle\, e^{j(\omega_1 t_1 - t_2\omega_2)} d\omega_1 d\omega_2 \tag{10.2.12d}$$

$$= \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(\omega_1)\, X^*(\omega_2)\, R_T(\omega_1,\omega_2;t_1,t_2)\, e^{j(\omega_1 t_1 - t_2\omega_2)} d\omega_1 d\omega_2 \tag{10.2.12e}$$

A channel is said to be wide-sense stationary (WSS) when the correlation function does not depend on the absolute values of t_1 and t_2 but only on the time difference $\Delta t = t_2 - t_1$. In such a case, the autocorrelation functions become a function of the variable Δt, i.e.,

$$R_h(\tau_1,\tau_2;\Delta t) = \langle h(\tau_1;t_1)\, h^*(\tau_2;t_2)\rangle\,, \tag{10.2.13a}$$

$$R_T(\omega_1,\omega_2;\Delta t) = \langle T(\omega_1;t_1)\, T^*(\omega_2;t_2)\rangle\,. \tag{10.2.13b}$$

This gives rise to

$$R_H(\omega_1,\omega_2;\varpi_1,\varpi_2) = \delta(\varpi_1 - \varpi_2)\, P_H(\omega_1,\omega_2;\varpi_1)\,, \tag{10.2.14a}$$

$$R_S(\tau_1,\tau_2;\varpi_1,\varpi_2) = \delta(\varpi_1 - \varpi_2)\, P_S(\tau_1,\tau_2;\varpi_1) \tag{10.2.14b}$$

where

$$P_H(\omega_1,\omega_2;\varpi_1) = \int_{-\infty}^{\infty} R_T(\omega_1,\omega_2;\Delta t)\, e^{-j\varpi_1\Delta t} d(\Delta t)\,, \tag{10.2.15a}$$

$$P_S(\tau_1,\tau_2;\varpi_1) = \int_{-\infty}^{\infty} R_h(\tau_1,\tau_2;\Delta t)\, e^{-j\varpi_1\Delta t} d(\Delta t)\,. \tag{10.2.15b}$$

Equations (10.2.14) show that the Doppler shifts corresponding to different times are uncorrelated.

If the channel is such that contributions of scattering from elements having different time delays (for single scattering, different time delays can be thought of as arriving from different ellipsoids about the transmitter and receiver) is uncorrelated, then it is classified as uncorrelated scattering (US). This assumption is also equivalent to assuming that the correlation functions are wide-sense stationary with respect to the frequency variables ω_1 and ω_2 i.e., the correlation functions in ω_1 and ω_2 are dependent only on their difference $\Delta\omega = \omega_2 - \omega_1$:

$$R_T(\omega_1, \omega_2; t_1, t_2) = R_T(\Delta\omega; t_1, t_2), \tag{10.2.16a}$$
$$R_H(\omega_1, \omega_2; \varpi_1, \varpi_2) = R_H(\Delta\omega; \varpi_1, \varpi_2). \tag{10.2.16b}$$

In this case the correlation functions in the time-delay variable can be written as

$$R_h(\tau_1, \tau_2; t_1, t_2) = \delta(\tau_1 - \tau_2) P_h(\tau_1; t_1, t_2), \tag{10.2.17a}$$
$$R_S(\tau_1, \tau_2; \varpi_1, \varpi_2) = \delta(\tau_1 - \tau_2) P_S(\tau_1; \varpi_1, \varpi_2) \tag{10.2.17b}$$

where

$$\begin{aligned} P_h(\tau_1; t_1, t_2) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} R_T(\Delta\omega; t_1, t_2) e^{j\tau_1\Delta\omega} d(\Delta\omega), \\ P_S(\tau_1; \varpi_1, \varpi_2) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} R_H(\Delta\omega; \varpi_1, \varpi_2) e^{j\tau_1\Delta\omega} d(\Delta\omega). \end{aligned}$$

If the channel is WSS in both time and frequency variables then it is classified as WSSUS channel. This leads to

$$R_h(\tau_1, \tau_2; t_1, t_2) = \delta(\tau_1 - \tau_2) P_h(\tau_1; \Delta t), \tag{10.2.18a}$$
$$R_T(\omega_1, \omega_2; t_1, t_2) = R_T(\Delta\omega; \Delta t), \tag{10.2.18b}$$
$$R_H(\omega_1, \omega_2; \varpi_1, \varpi_2) = \delta(\varpi_1 - \varpi_2) P_H(\Delta\omega; \varpi_1), \tag{10.2.18c}$$
$$R_S(\tau_1, \tau_2; \varpi_1, \varpi_2) = \delta(\tau_1 - \tau_2) \delta(\varpi_1 - \varpi_2) P_S(\tau_1, \varpi_1). \tag{10.2.18d}$$

For a WSSUS channel we have

$$R_y(t, t+\Delta t) = \langle y(t)\, y^*(t+\Delta t)\rangle \tag{10.2.19a}$$

$$= \left\langle \left(\int_{-\infty}^{\infty} x(t-\tau)\, h(\tau;t)\, d\tau\right) \left(\int_{-\infty}^{\infty} x(t+\Delta t-\tau')\, h(\tau';t+\Delta t)\, d\tau'\right)^* \right\rangle \tag{10.2.19b}$$

$$= \left\langle \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t-\tau)\, x^*(t+\Delta t-\tau')\, h(\tau;t)\, h^*(\tau';t+\Delta t)\, d\tau' d\tau \right\rangle \tag{10.2.19c}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t-\tau)\, x^*(t+\Delta t-\tau')\, \langle h(\tau;t)\, h^*(\tau';t+\Delta t)\rangle\, d\tau' d\tau \tag{10.2.19d}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t-\tau)\, x^*(t+\Delta t-\tau')\, R_h(\tau,\tau';t,t+\Delta t)\, d\tau' d\tau \tag{10.2.19e}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t-\tau)\, x^*(t+\Delta t-\tau')\, \delta(\tau-\tau')\, P_h(\tau;\Delta t)\, d\tau' d\tau \tag{10.2.19f}$$

$$= \int_{-\infty}^{\infty} x(t-\tau)\, x^*(t+\Delta t-\tau)\, P_h(\tau;\Delta t)\, d\tau \tag{10.2.19g}$$

When the time separation of the observation, Δt, is zero, we write $P_h(\tau,0) = P_h(\tau)$ and the above equation simplifies to

$$R_y(t,t) = \int_{-\infty}^{\infty} x(t-\tau)\, x^*(t-\tau)\, P_h(\tau)\, d\tau \tag{10.2.20a}$$

$$= \int_{-\infty}^{\infty} |x(t-\tau)|^2\, P_h(\tau)\, d\tau \tag{10.2.20b}$$

$$\approx P_h(\tau). \tag{10.2.20c}$$

The above approximation is valid if the time duration of ^{2} is much smaller than the spread of $P_h(\tau)$ or equivalently, if the spectrum of is $|x(t)|^2$ is constant over the frequency interval where the spectrum of $P_h(\tau)$ is non-zero. The function $P_h(\tau)$ is known as the *power delay profile* of the channel. $P_h(\tau)$ can be considered as the scattering function $P_S(\tau,\varpi)$ averaged over all Doppler frequencies. If there are discrete scatterers, the power delay profile will consist of several impulses as shown in Fig. 10.4. Typically, the power delay profile will be a continuous curve with several impulsive peaks.

Two parameters are of practical interest and they are the mean delay, $\bar{\tau}$, and the r.m.s. delay spread, τ_{rms} defined as

$$\bar{\tau} = \frac{\int \tau P_h(\tau)\, d\tau}{\int P_h(\tau)\, d\tau}$$

$$\tau_{\text{rms}} = \sqrt{\frac{\int (\tau-\bar{\tau})^2 P_h(\tau)\, d\tau}{\int P_h(\tau)\, d\tau}}.$$

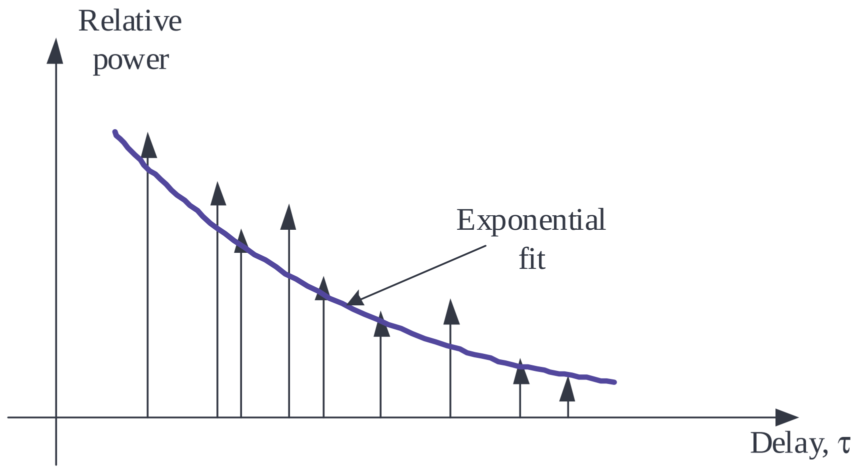

FIGURE 10.4. A discrete power delay profile.

Values of delay spread tend to depend on the environment. For instance, in macrocells that have cell radii ranging from 1 to 20 km, τ_{rms} could range from 0.1 μs 10 μs. In the case of a microcell which has cell radius ranging from 100 m to 1 km, τ_{rms} can range from 10 ns to 100 ns. Several models of delay spread are available to the user to carry out an initial system design. Table 10.1 shows the various exponential models used in the study of GSM systems while Table 10.2 gives typical values of τ_{rms} in various environments.

TABLE 10.1. Standard power delay profiles.

Environment	**Typical delay spread τ_{rms}**
Rural	$P_h(\tau) = \begin{cases} e^{-9.2\tau} & 0\,\mu\mathrm{s} < \tau < 0.7\,\mu\mathrm{s} \\ 0 & \text{otherwise} \end{cases}$
Hilly terrain	$P_h(\tau) = \begin{cases} e^{-3.5\tau} & 0\,\mu\mathrm{s} < \tau < 2\,\mu\mathrm{s} \\ 0.1e^{15-\tau} & 15\,\mu\mathrm{s} < \tau < 20\,\mu\mathrm{s} \\ 0 & \text{otherwise} \end{cases}$
Urban	$P_h(\tau) = \begin{cases} e^{-\tau} & 0\,\mu\mathrm{s} < \tau < 7\,\mu\mathrm{s} \\ 0 & \text{otherwise} \end{cases}$
Hilly urban	$P_h(\tau) = \begin{cases} e^{-\tau} & 0\,\mu\mathrm{s} < \tau < 5\,\mu\mathrm{s} \\ 0.5e^{5-\tau} & 5\,\mu\mathrm{s} < \tau < 10\,\mu\mathrm{s} \\ 0 & \text{otherwise} \end{cases}$

TABLE 10.2. Typical values of τ_{rms} in various environments

Environment	**Typical delay spread** τ_{rms}
Rural	200 ns
Hilly terrain	$5-10\,\mu$s
Urban	$1\,\mu$s
Hilly urban	$1-3\,\mu$s
Indoor	$10-50$ ns