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CHAPTER 1

Fundamental Concepts

1.1. Radio Waves

The term radio wave refer to the part of the electromagnetic spectrum corresponding to radio frequencies – that is, frequencies lower than around 300 GHz (or, equivalently, wavelengths longer than about 1 mm). Since the radio frequencies occupy a very large range in the spectrum, the spectrum is divided into smaller sections called bands. There are several designations for the radio frequency bands. The most commonly used band designation is the IEEE US definitions given in Table 1.

Table 1. IEEE US Radio spectrum designations

Band	**Frequency range**	**Origin of name**
HF band	3 to 30 MHz	**H**igh **F**requency
VHF band	30 to 300 MHz	**V**ery **H**igh **F**requency
UHF band	300 to 1000 MHz	**U**ltra **H**igh **F**requency
L band	1 to 2 GHz	**L**ong wave
S band	2 to 4 GHz	**S**hort wave
C band	4 to 8 GHz	**C**ompromise between S and X
X band	8 to 12 GHz	Used in WWII for fire control, unknown
Ku band	12 to 18 GHz	**K**urz **u**nder
K band	18 to 27 GHz	**K**urz (German)
Ka band	27 to 40 GHz	**K**urz **a**bove
V band	40 to 75 GHz	
W band	75 to 110 GHz	W follows V in the alphabet
mm band	110 to 300 GHz	

The ITU radio bands are designations defined in the ITU Radio Regulations and is given in Table 2

The usage of the radio spectrum is regulated by the government in most countries. The governments in turn obey the international regulations.

Radio waves are typically generated by moving charges in antennas. Antennas are devices that couple the electromagnetic energy generated by a system into the surrounding space. The electromagnetic energy than propagates to be received by another antenna at some other location.

1.2. Radiation from an Antenna

Consider a point source of energy as shown in Fig. 1. The total energy per second emanating from the source is the radiated power, measured in Watts, (W). We will denote

Table 2. ITU Radio spectrum designations

Band Number	Symbol	Frequency range	Wavelength range
1	ELF	3 to 30 Hz	10000 to 100000 km
2	SLF	30 to 300 Hz	1000 to 10000 km
3	ULF	300 Hz to 3 kHz	100 to 1000 km
4	VLF	3 to 30 kHz	10 to 100 km
5	LF	30 to 300 kHz	1 to 10 km
6	MF	300 kHz to 3 MHz	100 m to 1 km
7	HF	3 to 30 MHz	10 to 100 m
8	VHF	30 to 300 MHz	1 to 10 m
9	UHF	300 MHz to 3 GHz	10 cm to 1 m
10	SHF	3 to 30 GHz	1 to 10 cm
11	EHF	30 to 300 GHz	1 to 10 mm

this quantity by P_t which stands for transmitted power. A source that radiates energy uniformly in all directions is called an isotropic source. For such a source, energy propagates outward spherically. If we consider a spherical surface of radius R and center at the source, the energy crossing this surface at a unit time should be equal to the transmitted power, P_t, since the energy must be conserved. Since we are going to consider the energy received by some other antenna, we will be interested in the energy that propagates toward the receiving antenna, instead of the total energy. Therefore, it is convenient to define energy crossing a surface of unit area per unit time, which is called the *power density*, P. This quantity has the unit of $(\mathrm{W/m^2})$ and is called the energy density. For the isotropic source, the power density at a distance R from the source will then be

$$P = \frac{P_t}{4\pi R^2}. \tag{1.2.1}$$

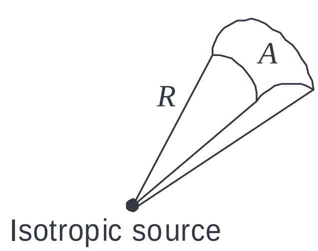

Figure 1. An isotropic source.

The above discussion implicitly assumes that the power (or energy) propagates along straight lines (which can be shown to be true by using Maxwell's equations). Therefore, it is sometimes more convenient to consider the flow of power through unit solid angle[1], which is called the *radiation intensity*, U. Unlike the power density, this quantity will remain constant

[1]Solid angle is defined similar to the planar angle; the solid angle subtended by an object is the area of its projection on the unit sphere.

as we move away from the radiator. Thus we have

$$U = \frac{P_t}{4\pi} = R^2 P. \tag{1.2.2}$$

It must be noted that the above discussions are only valid in the so called far field region of an antenna.

An isotropic radiator is a fictitious source. Real antennas cannot be isotropic. Actually, the antennas are desired to radiate more in preferred directions. The pattern of a typical antenna is shown in Fig. 2. The antenna radiates most of the power in a certain direction (generally the bore-sight direction). In practice, it is impossible to confine the radiated energy in a single direction and there will be some radiation in undesired directions. These radiations are called sidelobes of the antenna pattern. The directivity of an antenna is defined as

$$D = \frac{U_m}{U_0} = \frac{\text{maximum radiation intensity}}{\text{average radiation intensity}} \tag{1.2.3}$$

where D is the directivity. Multiplying numerator and denominator of (1.2.3) by 4π gives

$$D = \frac{4\pi U_m}{4\pi U_0} = \frac{4\pi U_m}{P_t} = \frac{4\pi\,(\text{maximum radiation intensity})}{\text{total power radiated}}. \tag{1.2.4}$$

The definition of directivity is based entirely on the shape of the radiated power pattern. The power input and antenna efficiency are not involved. The (power) gain of an antenna is defined as

$$G = \frac{4\pi\,(\text{maximum radiation intensity})}{\text{total power input to the antenna}}. \tag{1.2.5}$$

Notice that if the antenna is lossy, the total radiated power will differ from the total input power by a factor $\eta < 1$ called the efficiency of the antenna. Thus we have

$$G = \eta D. \tag{1.2.6}$$

The directivity and gain imply the maximum values for an antenna. These ideas can be generalized to any angle as

$$D(\theta,\phi) = \frac{4\pi U(\theta,\phi)}{P_t} = \frac{4\pi\,(\text{radiation intensity in the direction of }(\theta,\phi))}{\text{total power radiated}}, \tag{1.2.7a}$$

$$G(\theta,\phi) = \frac{4\pi\,(\text{radiation intensity in the direction of }(\theta,\phi))}{\text{total power input to the antenna}}. \tag{1.2.7b}$$

Again the gain and directivity patterns are related by the efficiency of the antenna as

$$G(\theta,\phi) = \eta D(\theta,\phi). \tag{1.2.8}$$

With these definitions, the power density at a distance R from the antenna in the direction (θ,ϕ) will be

$$P = \frac{P_t G(\theta,\phi)}{4\pi R^2}. \tag{1.2.9}$$

The quantity $P_t G(\theta,\phi)$ in (1.2.9) is called the *effective radiated power* and is abbreviated as ERP.

FIGURE 2. A typical antenna pattern.

1.3. Electromagnetic Wave Propagation Concepts

The above discussions ignore the vector nature of the electromagnetic waves. In reality, electromagnetic energy is carried by the electric and magnetic field components. In free space and at points sufficiently far away from the source, these fields are perpendicular to each other and to the direction of the propagation. Such a wave is referred to as a TEM (Transverse ElectroMagnetic) wave. In this course, we will only consider TEM waves. The electric and magnetic field vectors both oscillate at the frequency of the transmitted radiation. It is customary to drop the known sinusoidal time variation form the field expressions. Thus, the field vectors are in fact phasors. The field vectors may oscillate in a single direction, in which case the field is said to be linearly polarized. In certain cases, the tip of the electric (or magnetic) field vector will traverse an ellipse in the plane perpendicular to the propagation. Then the field is said to be elliptically polarized. As a special case of elliptic polarization, we may have circular polarization. For a linearly polarized field, if the direction of the electric field vector is parallel to the earth's surface the wave is said to be *horizontally polarized*, and if it is perpendicular, the wave is said to be *vertically polarized*. Although it is possible to have other directions, any field can be decomposed into a sum of a vertically polarized and a horizontally polarized waves. Therefore, we will confine ourselves to the study of these two waves. It must be emphasized that a circularly polarized wave can also be decomposed in a similar manner.

The cross product of the electric and magnetic field vectors is another vector called the Poynting Vector, and is denoted by $\mathbf{S}$.

$$\mathbf{S}\left(\mathrm{W/m^2}\right) = \mathbf{E}\left(\mathrm{V/m}\right) \times \mathbf{H}\left(\mathrm{A/m}\right). \tag{1.3.1}$$

For TEM waves this vector points in the direction of propagation. Its magnitude gives the power density. It must be noted that the field quantities in (1.3.1) are rms values so that $|\mathbf{S}|$ gives the average power. For TEM fields in free space, the magnitude of electric and magnetic field vectors are related by the equation

$$|\mathbf{E}| = Z_0 |\mathbf{H}| \tag{1.3.2}$$

where

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi = 377\Omega \tag{1.3.3}$$

is called the intrinsic impedance of the free space, $\mu_0 = 1.25664 \times 10^{-6}\,\mathrm{H\,m^{-1}}$ is the permeability and $\varepsilon_0 = 8.854187817 \times 10^{-12}\,\mathrm{F\,m^{-1}}$ is the permittivity of the free space. In some applications, the electric field is also required. Using (1.3.1) and (1.3.2) in (1.2.9) we can obtain

$$E_{rms} = \frac{\sqrt{30 P_t G_t(\theta, \phi)}}{R} \ \left(\mathrm{V/m}\right). \tag{1.3.4}$$

The peak field is then given by

$$E_m = \frac{\sqrt{60 P_t G_t(\theta, \phi)}}{R} \ \left(\mathrm{V/m}\right). \tag{1.3.5}$$

and the instantaneous field by

$$E(t) = \frac{\sqrt{60 P_t G_t(\theta, \phi)}}{R} \cos\left(\omega t - \varphi\right) \tag{1.3.6}$$

where φ is a constant phase. Generally, instead of trigonometric notation, (1.3.6) is written as

$$E(t) = \frac{\sqrt{60 P_t G_t(\theta, \phi)}}{R} e^{j(\omega t - \varphi)}$$

and the exponential is dropped in phasor notation. Unless otherwise stated, the field strength will be given in rms phasor notation without any subscript.

Another important quantity that can be derived from Maxwell's equations is the velocity of propagation of the radio waves which is given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.998 \times 10^8\,\mathrm{m/s}. \tag{1.3.7}$$

In a medium other than the free space, the intrinsic impedance and velocity of propagation can be found using the same formulas with permeability and permittivity replaced by that of the propagation medium. It is customary to define

$$\varepsilon = \varepsilon_r \varepsilon_0 \tag{1.3.8a}$$

$$\mu = \mu_r \mu_0 \tag{1.3.8b}$$

where ε_r and μ_r are called the *relative permittivity* and the *relative permeability*, respectively. In most media, $\mu_r = 1$ and $n = \sqrt{\varepsilon_r}$ is defined as the *refractive index* of the medium. Then we get

$$Z = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r \mu_0}{\varepsilon_r \varepsilon_0}} = \frac{1}{\sqrt{\varepsilon_r}} \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{Z_0}{n}, \tag{1.3.9a}$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\varepsilon_r \mu_0 \varepsilon_0}} = \frac{c}{n}. \tag{1.3.9b}$$

Another important quantity related to wave propagation is the *wavelength*. Wavelength is the space equivalent of the period of a periodic signal. Since the waves propagate as they move, the snapshot of a sinusoidal wave will have sinusoidal variation in space and the distance between the peaks (or nulls) of the wave is the wavelength, denoted by λ. Inverse of wavelength would be the space equivalent of frequency. In practice, we use the *wavenumber* defined as

$$k = \frac{2\pi}{\lambda}. \tag{1.3.10}$$

An important property of a wave is that the wavelength and frequency are related to the velocity by

$$v = \lambda f. \tag{1.3.11}$$

The constant phase term in (1.3.6) can now be expressed explicitly as follows. The phase reference is taken as the transmitter and since it takes R/c seconds for the wave to travel a distance of R, the field at a distance of R from the transmitter becomes

$$E(t) = \frac{\sqrt{60 P_t G_t(\theta, \phi)}}{R} \cos\omega\left(t - \frac{R}{c}\right) = \frac{\sqrt{60 P_t G_t(\theta, \phi)}}{R} \cos(\omega t - kR). \tag{1.3.12}$$

1.4. Receiving Antennas and Reciprocity

Antennas are also used to receive energy from a propagating radio wave. The antenna couples the surrounding environment to the receiving system. It is convenient to characterize a receiving antenna by an effective receiving area A_e so that the power received by the antenna is given by the incident power density multiplied by the effective area, i.e.,

$$P_r = A_e P \tag{1.4.1}$$

where P denotes the incident power density (W/m^2), A_e is the effective (equivalent) area (m^2), and P_r is the received power (W). Such an equation would be valid only if a plane wave is incident on the antenna. Otherwise the relation would be much more complicated. With this definition, it can be shown that under matched polarization and impedance conditions the effective area A_e is related to the gain G as

$$A_e = \frac{\lambda_0^2}{4\pi} G. \tag{1.4.2}$$

This relationship is true for any type of antennas, even if the antenna itself does not have a real aperture as in the case of dipole antennas. In the case of aperture antennas, the effective area is proportional to the physical area. In general, effective area will be smaller than the physical area and we have

$$A_e = \eta_a A_p \tag{1.4.3}$$

where A_p is the physical are and $\eta_a < 1$ is called the aperture efficiency.

The relation is a result of the reciprocity principle, and therefore can only be applied to reciprocal antennas. However, most practical antennas are reciprocal. Reciprocity implies that receiving and transmitting properties of the same antenna will be similar. Thus, an antenna responds differently to waves incident from different directions. As in the case of directivity or gain, we can define effective area as a function of direction by the relation

$$A_e(\theta, \phi) = \frac{\lambda_0^2}{4\pi} G(\theta, \phi). \tag{1.4.4}$$

The effective area by itself is used to imply the maximum value for an antenna.

In calculating the power received by an antenna, one must take polarization into account. If the polarization of the antenna and the incident wave are different, the antenna will not receive any signal. The electromagnetic waves impinging on an antenna, causes the charges on the antenna to oscillate back and forth and the receiving circuitry senses the resulting currents. Now, consider a dipole antenna. If the electric field of the radio wave is along the antenna, there is a long conductor along which the charges can move. However, if the electric field is perpendicular to the antenna, the charges do not have much space to move in that direction and the received power will be very small. Thus, a dipole antenna will receive the waves polarized in its direction while it will not respond to orthogonally polarized waves. Due to reciprocity, the dipole antenna will radiate electromagnetic waves polarized along its direction when used as a transmitting antenna. Equation (1.4.4) implies that polarization characteristics of an antenna is the same when receiving or transmitting. However, when using (1.4.1) the polarization properties of the antenna and incident wave must be same. If this is not the case, (1.4.1) should be modified as

$$P_r = A_e P \cos\alpha \tag{1.4.5}$$

where $\cos\alpha$ is the polarization mismatch factor.

1.5. Friis' Transmission Formula

The definitions in the previous sections make it relatively easy to derive an expression for the received signal in a line-of-sight communication link. With reference to Fig. 3, the power density radiated by the transmitter at the location of the receiving antenna will be

$$P = \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2}. \tag{1.5.1}$$

The power received by the receiving antenna will then be

$$P_r = A_{er}(\theta_r, \phi_r) P. \tag{1.5.2}$$

Using the reciprocity relation given in (1.4.4) along with these equations, we get

$$P_r = \frac{\lambda_0^2}{4\pi} G_r(\theta_r, \phi_r) \frac{P_t G_t(\theta_t, \phi_t)}{4\pi R^2}. \tag{1.5.3}$$

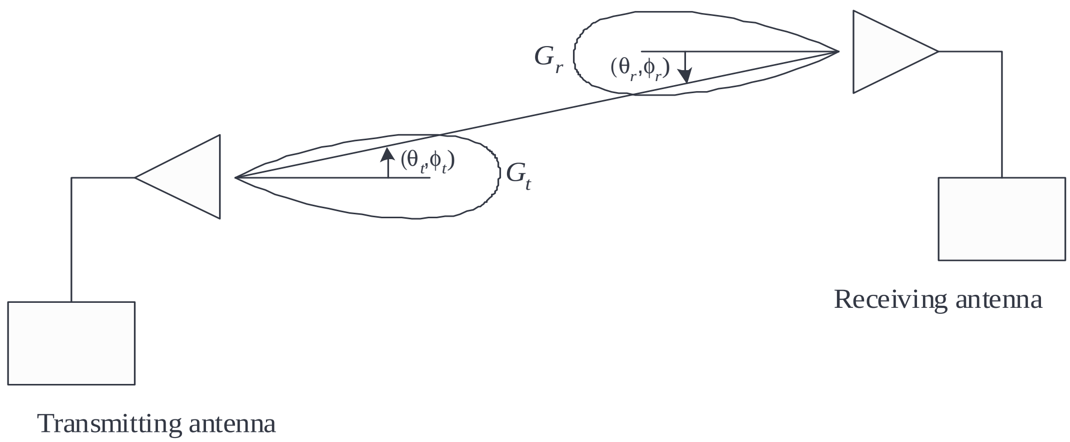

FIGURE 3. A transmitting and receiving communication system.

In practice, there may be impedance mismatch at the transmitter and receiver. If Γ_t is the reflection coefficient in the feed line for the transmitter, then only $\left(1-|\Gamma_t|^2\right)$ factor of the input power will be radiated. Similarly, if Γ_r is the input reflection coefficient for the receiver, only $\left(1-|\Gamma_r|^2\right)$ factor of the power will be transferred to the receiver. Furthermore, if there is a polarization mismatch, then the received signal should be multiplied by the mismatch factor. Thus (1.5.3) should be written as

$$P_r = \frac{P_t\lambda_0^2}{16\pi^2R^2}G_r\left(\theta_r,\phi_r\right)G_t\left(\theta_t,\phi_t\right)\left(1-|\Gamma_t|^2\right)\left(1-|\Gamma_r|^2\right)\cos\alpha. \tag{1.5.4}$$

Generally, the impedance and polarization mismatch factors are not considered as will be explained shortly. In communication systems, the antennas are aligned to look at each other and (1.5.3) is often written as

$$P_r = P_tG_tG_r\frac{\lambda_0^2}{16\pi^2R^2}. \tag{1.5.5}$$

It is customary to write (1.5.5) in logarithmic form as

$$10\log\left(P_r\right) = 10\log\left(P_t\right) + 10\log\left(G_t\right) + 10\log\left(G_r\right) - 20\log\frac{4\pi R}{\lambda_0}. \tag{1.5.6}$$

If P_r and P_t are given in watts, $10\log\left(P_r\right)$ and $10\log\left(P_t\right)$ are the powers in dBW, that is they are the powers expressed in decibels with respect to 1 watt. In communication systems, the decibel powers are generally expressed with respect to 1 mW and expressed as dBm. $10\log\left(G\right)$ is the antenna gain in decibels with respect to an isotropic antenna, since the gain of an isotropic antenna is 1. These quantities are expressed by dBi (decibels over isotropic). Therefore, (1.5.6) can be written as

$$P_r\left(\mathrm{dBm}\right) = P_t\left(\mathrm{dBm}\right) + G_t\left(\mathrm{dBi}\right) + G_r\left(\mathrm{dBi}\right) - 20\log_{10}\frac{4\pi R}{\lambda_0}. \tag{1.5.7}$$

The last term in (1.5.7) is called the *free space path loss* expressed in dB. It represents the loss of power due to spherical spreading. In words, this final equation says that

received power = transmitted power + antenna gains − losses.

Thus the impedance and polarization mismatch terms can be included in the losses along with the free space path loss. Actually any other losses in the system such as the atmospheric absorption loss, fading loss, etc., can also be included in this last term.

1.6. The Radar Equation

Radar detects objects by receiving the power reflected from them. If a target is at a distance R from the radar, the power density at the location of the target is

$$P_{\text{target}} = \frac{P_t G_t\left(\theta_t, \phi_t\right)}{4\pi R^2}. \tag{1.6.1}$$

where P_t is the transmitted power and $G_t\left(\theta_t, \phi_t\right)$ is the transmit antenna power pattern. The target intercepts a portion of the incident power and reradiates in various directions. The amount of the power intercepted by the target and reradiated back in the direction of the radar is measured by the radar cross section (back scattering cross section) of the target, denoted by σ and is defined by the relation

$$P_{\text{radar}} = \frac{P_t G_t\left(\theta_t, \phi_t\right)}{4\pi R^2}\frac{\sigma}{4\pi R^2}. \tag{1.6.2}$$

The radar cross section has the units of area and its a characteristic of the target only. It is a measure of the size of the target as seen by the radar. The power received by the radar is then

$$P_r = \frac{P_t G_t\left(\theta_t, \phi_t\right)}{4\pi R^2}\frac{\sigma}{4\pi R^2}A_r\left(\theta_t, \phi_t\right) = \frac{\lambda_0^2 P_t G_t^2 \sigma}{\left(4\pi\right)^3 R^4}. \tag{1.6.3}$$

The right most term is obtained by using (1.4.4). It is assumed that the radar uses the same antenna for both transmitting and receiving, and that the target is in the direction of antenna pattern maximum.

1.7. Signal to Noise Ratio

Signal reception is determined not only by the signal strength at the receiver but also by noise. In fact, the ratio of the signal power to noise power is defined as the signal to noise ratio (SNR) and is what determines the reception. In this course we will mainly deal with the signal strength. However, we will also consider the sources of noise.

The noise power present at the input of a receiver is the sum of internal noise and external noise. Internal noise is basically the thermal noise generated within the receiver and is covered in texts on radio receivers. We will be concerned with external noise. The main types of external noise are man-made interference, atmospheric noise, cosmic noise, and also thermal noise which is produced by the heated atmosphere and earth surface.

The thermal noise is generated by the random motion of conduction electrons in the ohmic parts of the receiver circuitry. This noise is directly proportional to the temperature of ohmic parts of the receiver and the receiver bandwidth. The available thermal noise power at the input of a receiver of noise band with B_n (Hz) at a temperature T (K) is given by

$$N_i = kTB_n \tag{1.7.1}$$

where $k = 1.380658 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ is the Boltzmann's constant. The noise power in practical receivers is often greater than this value due to reasons beyond the scope of this course. The noise figure F of a receiver is defined as

$$F = \frac{\text{noise power at the output of the practical receiver}}{\text{noise power at the output of an ideal receiver}} = \frac{N_o}{kTB_nA} \tag{1.7.2}$$

which may also be written as the ratio of the

$$F = \frac{N_o}{N_iA} = \frac{S_iN_o}{AS_iN_i} = \frac{S_i/N_i}{S_o/N_o}. \tag{1.7.3}$$

In communication systems and in radar, the minimum detectable signal, or *sensitivity* is the value of S_i that corresponds to the minimum ratio of output SNR necessary for detection and is denoted by $S_{\min}$. The SNR necessary for detection depends on how the receiver processes the signal.

Once the SNR for good reception and the noise power is known, one can easily find the required signal strength at the input of the receiver, thus all the pertinent characteristics of a radio link system can be determined. Radio wave propagation deals basically with the calculation of the signal strength.

1.8. Examples

EXAMPLE 1. *Find the free space path loss for two cases: (a)* $r = 10\,\mathrm{km}$, $f = 15\,\mathrm{kHz}$ *and (b)* $R = 10^8$, $f = 10\,\mathrm{GHz}$.

SOLUTION 1. *The free space path loss is given by*

$$L = \left(\frac{4\pi R}{\lambda_0}\right)^2 = \left(\frac{4\pi Rf}{c}\right)^2$$

where $c = 3 \times 10^8\,\mathrm{m/s}$ *is the velocity of light. Substituting the numerical values gives:*

(a) $L = 39.5$ *or* 15.9 *dB,*
(b) $L = 1.75 \times 10^{27}$ *or* 272.4 *dB.*

The first case corresponds to ULF communication. The second case corresponds to space communication. The distance of Venus from earth is about $10^8\,\mathrm{km}$.

EXAMPLE 2. *Suppose that the sensitivity (minimum received power for reliable communication) of a receiver is* $S_{\min} = -110$ *dBm. Assuming a line of sight communication link, find the required transmitter power for* $R = 400\,\mathrm{km}$ *and* $f = 1.5\,\mathrm{GHz}$, $G_t = G_r = 30$ *dB.*

SOLUTION 2. *The free space path loss is* $L = 148$ *dB. Using (1.5.7) we have*

$$-110 = P_t + 30 + 30 - 148$$

which gives $P_t = -22$ *dBm or* $P_t = 6.3\,\mu\mathrm{W}$.

Such a small value is basically due to high receiver sensitivity and free space communication.

EXAMPLE 3. *Suppose that a communication system uses* $15\,\mathrm{kHz}$ *bandwidth and requires* 10 *dB SNR for reliable detection. The receiver has a noise figure of* $F = 4$ *dB. Assuming a line of sight communication link, find the required transmitter power for* $R = 50\,\mathrm{km}$ *and* $f = 15\,\mathrm{MHz}$, $G_t = G_r = 6$ *dB.*

SOLUTION 3. *The noise power per* Hz *at room temperature* (20 °C) *is*

$$kT = 1.380658 \times 10^{-23}\,\mathrm{J\,K^{-1}} \times 293\,\mathrm{K} = 4.04 \times 10^{-21}.$$

This value is commonly taken as -203 *dBW or* -173 *dBm. The receiver sensitivity can be found as*

$$S_{\min} = kTBF\left(\frac{S_o}{N_o}\right)_{\min} = -173 \ dBm + 42 + 4 + 10 = -117 \ dBm$$

where $42 = 10\log_{10}(15000)$ *is the bandwidth in dB. Using same steps as in example (3) we get*

$$P_t = -117 - 6 - 6 + 90 = -39 \ dBm$$

or $P_t = 1.2 \times 10^{-4}\,\mathrm{mW}$, *where* 90 *dB is the free space path loss.*

EXAMPLE 4. *Voyager 1 entered deep space and is currently at a distance of about* 110 *AU (1 AU is the distance of earth to sun which is* 149, 597, 871 km*). It uses a 3.7 meter parabolic antenna at X band* (3.6 cm)*. Assuming a transmitter power of* 20 W *determine the power received by an earth receiver using a 34 meter parabolic antenna.*

SOLUTION 4. *The distance of Voyager 1 to earth is* $R = 110 \times 1.49 \times 10^{11} = 1.65 \times 10^{13}\,\mathrm{m}$. *The free space path loss is then* $L = 315$ *dB. The gain of a parabolic antenna is given by*

$$G = \varepsilon_{ap}\left(\frac{\pi d}{\lambda_0}\right)^2$$

where ε_{ap} *is the aperture efficiency which can be taken as* 0.75 *for most practical antennas. Thus*

$$G_t = 10\log_{10}\left(0.75\left(\frac{370\pi}{3.6}\right)^2\right) = 48.9 \ dB,$$

$$G_r = 10\log_{10}\left(0.75\left(\frac{3400\pi}{3.6}\right)^2\right) = 68.2 \ dB.$$

Using (1.5.7) gives

$$P_r = 43 + 48.9 + 68.2 - 315 = -154.9 \ dBm.$$

Comparing this value to the sensitivity of practical antennas, it is not surprising that communication with Voyager 1 is still possible.

EXAMPLE 5. *The distance from Earth to the edge of the observable universe is about 14 billion parsecs* (4.6×10^{10} *light years). Assuming that Voyager 1 reaches that distance, what would be the radius of the receiving antenna that would receive a power of* -173 *dBm?*

SOLUTION 5. *A light year is the distance light travels in a year. Thus*

$$1 \ light \ year = 365 \times 24 \times 60 \times 60 \times 3 \times 10^8\,\mathrm{m} = 9.46 \times 10^{15}\,\mathrm{m}.$$

Thus the distance from Earth to the edge of the observable universe is $R = 4.35 \times 10^{26}\,\mathrm{m}$. *The free space path loss is then* $L = 583$ *dB. Using (1.5.7) gives*

$$-173 = 43 + 48.5 + G_r - 583$$

or $G_r = 318.5$ *dB. If a parabolic antenna with an aperture efficiency of* 0.75 *is used this would mean an antenna radius of*

$$G_r = 0.75\left(\frac{\pi d}{3.6}\right)^2 = 10^{31.8} \Rightarrow d = 1.05 \times 10^{14}\,\mathrm{m} = 702\ AU.$$

which appears to be quite impractical (to give an idea, the distance of earth to Pluto is only 32.5 *AU). However, it must be kept in mind that the receiver sensitivity can be improved by signal integration.*