EE 521 Fall 2007
Homework 2: Separation of Variables

1. Use the separation of variables technique to find the Green's function that satisfies two dimensional scalar wave equation

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}+k^2\right]G\left(\rho,\phi;\rho',\phi'\right)=-\frac{1}{\rho}\delta\left(\rho-\rho'\right)\delta\left(\phi-\phi'\right)$$

   for $a \leq \rho < \infty$, $-\pi \leq \phi \leq \pi$ along with the conditions

   (a) $G(a, \phi; \rho\prime, \phi\prime) = 0$

   (b) $G(\rho, \phi; \rho\prime, \phi\prime) = G(\rho, \phi + 2\pi l; \rho\prime, \phi\prime)$

   together with the radiation condition for $e^{j\omega t}$ time dependence. Use a solution where the singularities of G_ϕ are enclosed.

2. Obtain the 2-D free space Green's function in polar coordinates by letting $a \to 0$ in your solution. Also simplify your solution by moving the origin to the source point at (ρ', ϕ').

3. The expression you obtained in (2) must be a function of $k = \omega/c$. By taking the inverse Fourier transform of your result obtain the time domain expression for the Green's function. Basing on this result what can you say about dispersionless communication in 2-D space?

Remark 1 *You can find the integral required in (3) from integral tables.*