EE 521 Fall 2007
Homework 3

1. Consider a thin, linear, center-fed dipole antenna located along the z axis of a coordinate frame. The current along the antenna is assumed to be sinusoidal, i.e.,

$$I(z) = \begin{cases} I_0 \sin\left(\frac{kd}{2} - k|z|\right), & |z| < \frac{d}{2} \\ 0 & \text{elsewhere} \end{cases}.$$

   (a) Determine the multipole expansion coefficients of the electric field radiated by this antenna, i.e., expand the radiated electric field in terms of spherical vector wave functions.

   (b) Use your result for part (a) to determine the multipole expansion of the electric field radiated by a Hertzian dipole (i.e., as $d \to 0$).

2. Determine the Green's function which satisfies the partial differential equation

$$\left(\nabla^2 + k^2\right) G(x, y, z; x', y', z') = -\delta(x - x')\,\delta(y - y')\,\delta(z - z')$$

   for $-\infty < x < \infty$, $-\infty < y < \infty$, and $0 \leqslant z < \infty$, along with the boundary condition

$$\frac{\partial G}{\partial z} - jkZ_s G = 0 \quad \text{at } z = 0$$

   and the radiation condition for $e^{j\omega t}$ time dependence as $|x|, |y|, z \to \infty$. This is the Green's function for the half space $z \geqslant 0$ with an impedance boundary condition at $z = 0$ surface. Express your result as a sum of two free-space Green's functions and interpret this result by using the image concept.