15 January 2008

EE 521 Fall 2007
Final Exam : (Open books)
Duration : 180 min.

1. Find the Green's function which satisfies the differential equation

$$\left(x\frac{d^2}{dx^2} + 2\frac{d}{dx} - \frac{2}{x} \right) g\left(x; x'\right) = \delta\left(x - x'\right)$$

over the interval $0 < x < \infty$. We require that the Green's function be finite over $0 < x < \infty$.
Hint : To solve the homogeneous equation, assume a solution of the form $y = x^{\alpha}$.

2. Let $\overline{\Gamma}\left(\mathbf{r}; \mathbf{r}'\right)$ be a solution of

$$\nabla \times \nabla \times \overline{\Gamma}\left(\mathbf{r}; \mathbf{r}'\right) - k^2 \overline{\Gamma}\left(\mathbf{r}; \mathbf{r}'\right) = \overline{\mathbf{I}}\delta\left(\mathbf{r} - \mathbf{r}'\right)$$

in a volume V. On the surface S surrounding V let $\overline{\Gamma}$ satisfy either the boundary condition

$$\mathbf{n} \times \overline{\Gamma} = 0 \quad \text{or} \quad \mathbf{n} \times \nabla \times \overline{\Gamma} = 0.$$

Show that the dyadic Green's function satisfies the symmetry condition

$$\mathbf{J}_1\left(\mathbf{r}_1\right) \cdot \overline{\Gamma}\left(\mathbf{r}_1; \mathbf{r}_2\right) \cdot \mathbf{J}_2\left(\mathbf{r}_2\right) = \mathbf{J}_2\left(\mathbf{r}_2\right) \cdot \overline{\Gamma}\left(\mathbf{r}_2; \mathbf{r}_1\right) \cdot \mathbf{J}_1\left(\mathbf{r}_1\right).$$

Hint : Use the Green's identity

$$\int_V \left[\left(\nabla \times \nabla \times \mathbf{A}\right) \cdot \mathbf{B} - \mathbf{A} \cdot \left(\nabla \times \nabla \times \mathbf{B}\right) \right] dV = - \int_S \left\{ \left[\mathbf{n} \times \left(\nabla \times \mathbf{A}\right)\right] \cdot \mathbf{B} + \left(\mathbf{n} \times \mathbf{A}\right) \cdot \left(\nabla \times \mathbf{B}\right) \right\} dS$$

with

$$\mathbf{A} = \overline{\Gamma}\left(\mathbf{r}; \mathbf{r}_1\right) \cdot \mathbf{J}_1\left(\mathbf{r}_1\right), \quad \mathbf{B} = \overline{\Gamma}\left(\mathbf{r}; \mathbf{r}_2\right) \cdot \mathbf{J}_2\left(\mathbf{r}_2\right).$$

3. The geometry of a two dimensional potential problem is defined in polar coordinates by the surfaces $\phi = 0$, $\phi = \beta$, and $\rho = a$, as indicated in the Fig. (1) below.

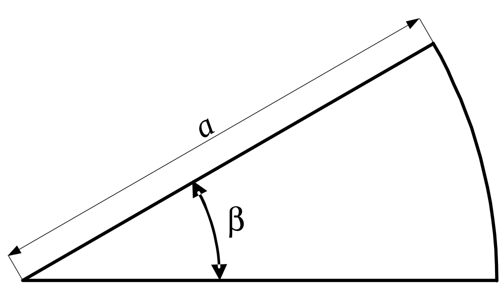

Figure 1: Geometry for Problem (3).

Using separation of variables in polar coordinates, obtain an expression for the Green's function that satisfies $G = 0$ on the boundary surfaces.

4. Consider a z-polarized plane wave propagating in the positive x direction, i.e.

$$\mathbf{E}^i = E_0 e^{-jkx} \mathbf{a}_z$$

Assume that this field is incident upon a conducting cylinder of radius a as shown in Fig. (2).

(a) Express the incident field in the form

$$E_z^i = E_0 \sum_{n=-\infty}^{\infty} a_n J_n(k\rho) e^{jn\phi},$$

i.e., determine the coefficients a_n.

Hint :

$$J_n(x) = \frac{j^n}{2\pi} \int_0^{2\pi} e^{-j(n\phi + x\cos\phi)} \, d\phi$$

(b) The total field with the conducting cylinder present is the sum of the incident and scattered fields, that is,

$$\mathbf{E}^t = \mathbf{E}^i + \mathbf{E}^s$$

Determine the scattered field $\mathbf{E}^s$. If you could not solve part (a), determine the scattered field in terms of the coefficients a_n.

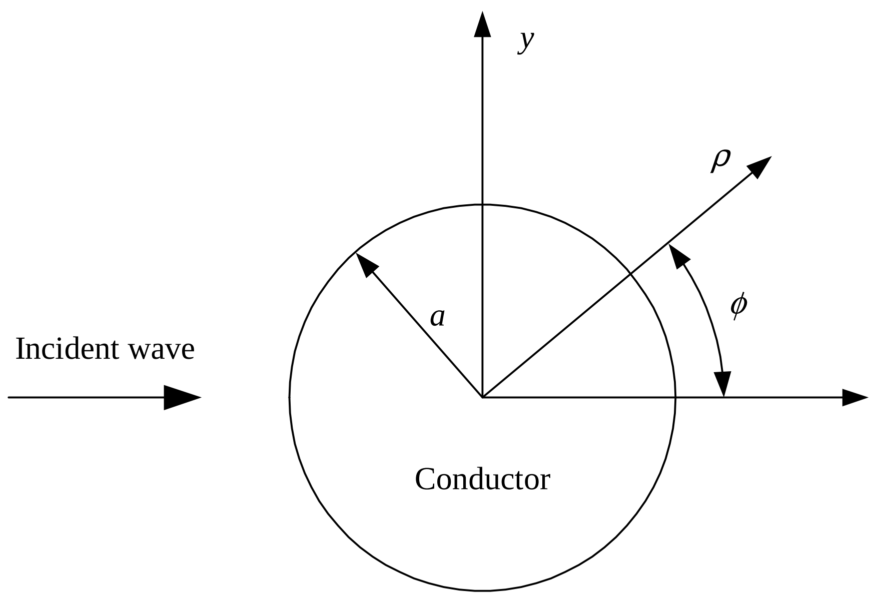

Figure 2: Geometry for Problem (4).