Middle East Technical University Low-dimensional Topology Math 710 Spring 2005

Midterm II

(take-home)

Problem 1. *Consider knots K: a) figure-8 knot; b) torus knot $K_{2,5}$ and consider the double cover $M_K \to S^3$ branched along K.*

(1) *Present M_K as an open book;*
(2) *give a Heegaard diagram of M_K;*
(3) *give a Kirby diagram of M_K;*
(4) *determine $H_1(M_K)$.*

Problem 2. *Let $X_d \to \mathbb{C}\mathrm{P}^1 \times \mathbb{C}\mathrm{P}^1$ denotes the double covering branched along a non-singular curve of degree $(6, 2d)$ (such complex surfaces are called Horikava surfaces). Determine*

(1) $\chi(X_d)$,
(2) $b_i(X_d)$, *for all* i,
(3) $\sigma(X_d)$, $b_2^{\pm}(X_d)$,
(4) $p_g(X_d)$, $c_1^2(X)$,
(5) $h^{i,j}(X_d)$

Problem 3. *Let $C_{m,n}$ be the space of all curve of degree (m, n) in $X = \mathbb{C}\mathrm{P}^1 \times \mathbb{C}\mathrm{P}^1$.*

(1) *Determine the genus of non-singular curves $A \in C_{m,n}$*
(2) *What is the dimension of $C_{m,n}$?*
(3) *Find the dimension of the subvariety $R_{m.n} \subset C_{m,n}$ formed by rational curves.*
(4) *Which minimal number of points one need to fix in X, so that there will be only finitely many curves $A \in R_{m,n}$ passing through these points ?*

Problem 4. *Show the scheme of resolution for the singularities $\{f_i(x, y) = 0\}$, where $f_1(x, y) = x^4 + y^3$, $f_2(x, y) = (x^3 + y^2)y$, $f_3(x, y) = x^5 + y^3$. Justify that the resolution graphs for the surface singularities $\{f_i + z^2 = 0\}$, $i = 1, 2, 3$ are E_6, E_7, E_8.*

Problem 5. *Consider a plumbing 4-manifold $X = P(\Gamma)$, where Γ is the weighted tree shown below.*

(1) *minimize Γ by contraction and sketch the new graph Γ_1 that you obtain.*
(2) *What is the relation between the manifolds X and $X_1 = P(\Gamma_1)$?*
(3) *Determine the matrix of the intersection form of X_1. Is it definite ? Is it even ? (explain)*
(4) *What is the order of $H_1(\partial X_1)$? Determine this group.*
(5) *Can Γ_1 and Γ be resolution graphs of some singularities ?*
(6) *Sketch a Kirby diagram representing X and X_1.*
(7) *Assuming that X_1 is the resolution graph of a singularity, evaluate $c_1(X_1)$ on the components of the exceptional divisor E.*
(8) *Represent $c_1(X_1)$ as a linear combination of the components of E.*