Midterm II

(take-home)

Problem 1. *Suppose that for a smooth mapping* $f\colon X \to Y$, $d_x f = 0$ *at all points* $x \in X$. *Prove that* f *is constant on the connected components of* X.

Problem 2. *Prove that any embedded circle in* $\mathbb{R}^n$, $n \geq 5$, *bounds an embedded disc.*

Problem 3. *Assume that* $f\colon X \to Y$ *is a submersion. Prove that any vector field,* W, *on* Y *can be lifted to* X, *that is to say, there exists a vector field* V *on* X, *such that* $d_x(V(x)) = W(f(x))$, $\forall x \in X$.

Problem 4. *Assume that* $s\colon X \to E$ *is a continuous section of a smooth Euclidian vector bundle* $p\colon E \to X$. *Prove that for any* $\epsilon > 0$ *there exists a smooth* ϵ*-approximation,* $s'\colon X \to E$, *of* s.