MIDTERM 1 (MARCH 30)

Problem 1. (20 pts) *Consider a gluing scheme for a surface F described by the words $abca^{-1}$, d^2cef, $ghfg^{-1}$.*

(1) *determine the number of vertices for such a scheme;*
(2) *determine the number of boundary components of F;*
(3) *find the Euler characteristics $\chi(F)$ and $\chi(\hat{F})$ (here and below, $\hat{F}$ denotes a closed surface obtained by filling the holes in F by discs);*
(4) *classify the surfaces F and $\hat{F}$ (i.e., recognize them as one of the surfaces F_g, $F_{g,l}$, N_k, $N_{k,l}$);*

Problem 2. (30 pts) *Consider the link L and its spanning surface F shown on the Figure.*

(1) *Justify if this surface is orientable (presenting your arguments);*
(2) *determine the Euler characteristic $\chi(F)$;*
(3) *determine the Euler characteristic $\chi(\hat{F})$;*
(4) *classify F and $\hat{F}$ as the surfaces of type F_g, $F_{g,l}$, N_k, or $N_{k,l}$.*
(5) *sketch a set of basic cycles on F;*
(6) *find the algebraic number of semi-twists for your cycles and determine which of them are disorienting (if there is any).*

Problem 3. (20 pts) *For the link L in the previous problem*

(1) *determine the linking numbers between the components of L (oriented like it is shown)*
(2) *evaluate the Whitney indices of the components of L as they are projected to the plane;*
(3) *sketch the Seifert circles;*
(4) *write the code of the Seifert circles.*

Problem 4. (10 pts) *Sketch a braid $b = \sigma_2\sigma_4\sigma_1^{-1}\sigma_3^{-1}$ with 5 strands and determine the number of components of $\hat{b}$.*

Problem 5. (10 pts) *Assume that knots K_1 and K_2 can be presented as closed braids with n and m strands respectively. Prove that $K_1\#K_2$ has a presentation as a closed braid with $n+m$ strands.*

Problem 6. (10 pts) *Assume that a triangulation of a surface $F = T^2\#T^2\#\mathbb{R}\mathrm{P}^2$ contains $2n$ triangles. What can be the number of vertices in this triangulation ?* **(Hint:** *make use of the Euler characteristic of F.*)