Middle East Technical University Spring 2002 422 Elementary Geometric Topology Instructor: S. Finashin

# HOMEWORK IV (DUE TO APRIL 22) PRESENTATION OF SURFACES BY GRAPHS. VECTOR FIELDS ON SURFACES.

Please, give your solution for each problem on a separate page. Do not forget to write your name and student's number on every page that you submit

**Problem 1.** *For the graph shown below*

(1) *sketch the surface that it represents and determine the number of its boundary components;*
(2) *argue why this surface, $F$, is orientable;*
(3) *choose basic cycles for $F$ and determine their intersection number in $F$;*
(4) *determine the Euler characteristic and classify surface $F$;*
(5) *determine the Seifert matrix with your choice of basic cycles;*
(6) *determine the Arf invariant (if it is well-defined) for $F$.*

**Problem 2.** *Let $F$ denote a Klein bottle with a hole.*

(1) *Choose a suitable gluing scheme for $F$ and sketch a gradient-like vector field outward normal to the boundary of $F$.*
(2) *Identify the type of the singularities of this vector field and determine their indices.*
(3) *Sketch a spine of $F$ determined by your vector field.*