HOMEWORK II (due to march 15)
KNOTS AND LINKS

Please, give your solution for each problem on a separate page. Do not forget to write your name and student's number on every page that you submit

Problem 1. *Sketch a torus knot $K^+_{4,1}$ and link $K^+_{4,2}$.*

(1) *Expose a deformation showing that $K^+_{4,1}$ is an unknot.*
(2) *Determine the linking number between the two components of $K^+_{4,2}$.*
(3) *Show that the components of $K^+_{4,2}$ can be permuted by a deformation.*
(4) *Show that there is no deformation of $K^+_{4,2}$ to itself which preserves the orientation of one component and reverses the orientation of the other one.*

Problem 2. *Sketch a knot diagram for $K = K^+_{3,4}$*

(1) *Sketch the mirror image $\overline{K}$ and the inverse $-K$ and determine which code $K^{\pm}_{m,n}$ describes them.*
(2) *Is K a 3-colorable knot ?*
(3) *Sketch an unknot with the same projection as K.*
(4) *Show that $U(K) \leqslant 4$, that is specify 4 crossings which should be modified to get a knot diagram of a unknot.*

Problem 3. *Sketch a diagram of $K = K^+_{2,2}(K^-_{2,3})$, that is a right $(2,2)$-cable link, whose core is a left trefoil. Determine the linking number between its components.*

Problem 4. *Sketch the positive Whitehead double $K = W^+(K^+_{2,3} \# K^-_{2,3})$ marking the positive crossings on its diagram.*

Problem 5. *Prove that for any knots K_1 and K_2*

(1) $c(K_1 \# K_2) \leqslant c(K_1) + c(K_2)$,
(2) $u(K_1 \# K_2) \leqslant u(K_1) + u(K_2)$.