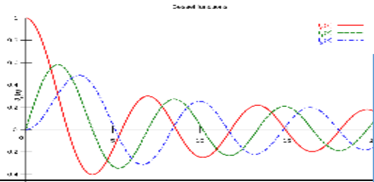


Claude-Louis Navier

10, 1785 in Dijon - August 21, 1836

Navier-Stokes

We can write the general (3D) formula in a more compact form given below – the Navier-Stokes equation

The formula is really a *mnemonic* – it contains all the physics you're likely to need in a *single* equation

The vector form given here is general (not just Cartesian)

$$\rho\left[\frac{\partial \underline{u}}{\partial t}+\underline{u}\cdot\nabla\underline{u}\right]=\eta\underline{\nabla^2 u}-\underline{\nabla P}+\Delta\rho g\hat{\underline{y}}$$

Yuk! *Inertial* term. Source of turbulence. See next slide.

Pressure gradient

Diffusion-like viscosity term. Warning: $\underline{\nabla^2 u}$ is complicated, especially in non-Cartesian geom.

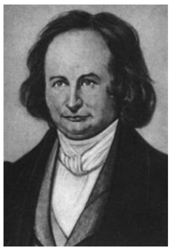
Buoyancy force (e.g. thermal or electromagnetic) $\hat{\underline{y}}$ is a unit vector

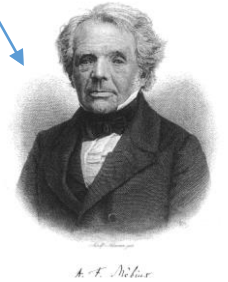
F. Nimmo EART162 Spring 10

1821 Navier "Navier-Stokes equations" for an incompressible fluid.
1821 Cauchy rigorous *A Course of Analysis* for students at the Ecole Polytechnique
1822 Fourier's prize winning essay of 1811 is published as *Analytical Theory of Heat* with Fourier analysis.
1824 Carnot the second law of thermodynamics, "Carnot cycle" in a book on steam engines
1824 Abel a six page pamphlet on impossibility of resolution of equation of order >4.
1824 Bessel "Bessel functions" for study of planetary perturbations.
1824 Steiner synthetic geometry, published in 1832.
1826 Ampère math derivation of the electrodynamic force law
1826 Crelle *Crelle's Journal* contains several papers by Abel.
1827 Jacobi elliptic functions in a letter to Legendre (Abel worked on this too).
1827 Möbius *Der barycentrische Calkul* on projective and affine geometry: homogeneous coordinates, projective transformations. (Feuerbach and Plücker introduced homogeneous coordinates too).
1828 Gauss differential geometry: "Gaussian curvature", *theorema egregrium*.
1828 Green properties of the potential function, applies to electricity and magnetism. "Green's theorem", "Green's function".
1828 Abel doubly periodic elliptic functions.
1829 Galois first work on the algebraic equations submitted to the Acad. Sci. in Paris.
1829 Lobachevsky non-euclidean (hyperbolic) geometry in the *Kazan Messenger* (in St Petersburg Acad. Sci. Ostrogradski rejected it).
1830 Poisson "Poisson's ratio" in elasticity involving stresses and strains on materials.

Jacob Steiner (1796-1863)
The greatest pure geometer since Apollonius of Perga

Carl Gustav Jacob Jacobi (1804-1851) worked on elliptic functions, dynamics, differential equations, and number theory. Used elliptic functions in number theory: for proving of Fermat's two-square theorem and Lagrange's four-square theorem, and similar results for 6 and 8 squares. New proofs of quadratic reciprocity, Jacobi symbol, higher reciprocity laws, continued fractions, Jacobi sums. The Jacobian determinant. 1841 he reintroduced the partial derivative ∂ notation of Legendre.





$$f(z)=\frac{az+b}{cz+d}$$

George Green

- July 14, 1793 - May 31, 1841
- British mathematician and physicist
- First person to try to explain a mathematical theory of electricity and magnetism
- Almost entirely self-taught!
- Published "*An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism*" in 1828.
- Entered Cambridge University as an undergraduate in 1833 at age 40.

George Green (1793 - 1841)

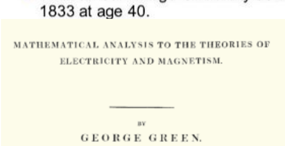
George Green was the son of a baker and left school at the tender age of 9 to follow in his father's footsteps. Even at this age he exhibitted an interest in mathematics. Being of lower social standing, he was not able to afford the costs of a university. Green instead, took upon himself the responsibility of self-education. With his basic education, he began reading and studying mathematical papers as well as other documents. In 1828 at the age of 35, he published possibly his greatest work, entitled "An Essay on the Application of Mathematical Analysis to the Theories of Electridcity and Magnetism." In this publication, he made his first attempts to apply mathematical theory to electrical phenomena. Many of its subscribers were not able to really understand the contents, importance, or significance of this work. Two years later, one of the exceptions to this, Sir Edward Bromheadm, met with George and encouraged him to publish two other recognized 'memoirs', "Mathematical Investigations Concerning the Laws of Equilibrium of Fluids Analogous to the Electric Fluid" and "On the Determination of Exterior and Interior Attractions of Ellipsoids fo Variable Densities." He also published a paper entitled "Researches on the Vibrations of Pendulum in Fluid Media."
In 1833, at the age of 40, he turned down an invitation from Cambrige University and admitted himself to Caius College. He gained recognition and went on to publish papers on wave theory dealing with the hydrodynamics of wave motion and reflection and refraction of light and sound.

circulation form

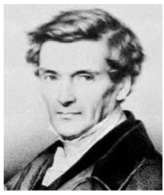
$$\iint_R \left(\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\right)dx\,dy=\oint_C \vec{F}\cdot\vec{T}\,ds$$

divergence form

$$\iint_R \left(\frac{\partial F_x}{\partial x}+\frac{\partial F_y}{\partial y}\right)dx\,dy=\oint_C \vec{F}\cdot\vec{n}\,ds$$



- Gustave Gaspard Coriolis (1792-1843).
- Using just a pen, paper and mathematics, he figured out why the wind turns, curves, and goes around in circles.
- Then named his resulting discovery the Coriolis effect



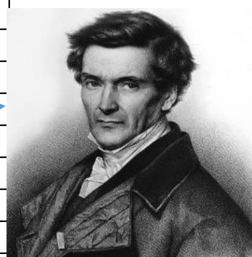
Other Green's theorems

- They are related to divergence (aka Gauss', Ostrogradsky's or Gauss-Ostrogradsky) theorem,

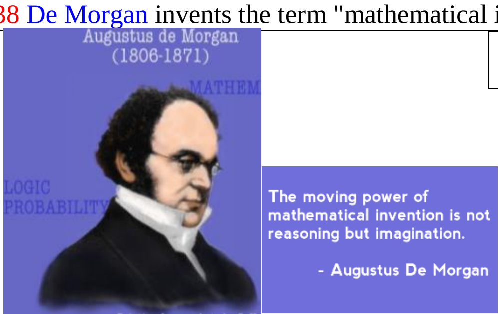
$$\oint_{\partial D}\vec{A}\cdot d\vec{\Sigma}=\int_D(\nabla\cdot\vec{A})dV$$

An engineer and physicist, He introduced the terms 'work' and 'kinetic energy' with their present scientific meaning, and studied flow on a rotating body like the Earth.

1831 Möbius introduces the "Möbius function" and the "Möbius inversion formula".
1831 Cauchy power series expansions of analytic functions of a complex variable.
1832 Steiner projective geometry based on metric considerations.
1832 János Bolyai non-Euclidean geometry in an appendix to an essay by Farkas Bolyai, his father.
1834 Hamilton *On a General Method in Dynamics*: characteristic function in dynamics.
1835 Coriolis "Coriolis force"; another work: math theory of billiards.
1836 Ostrogradski (1801-1862) rediscovers Green's theorem.
1836 Liouville math journal *Journal de Mathématiques Pures et Appliquées*.
1837 Poisson The law of large numbers, Poisson distribution for a discrete random variable.
1837 Dirichlet gives a general definition of a function.
1837 Liouville integral equations, Sturm-Liouville theory for solving such equations.
1837 Wantzel (1814-1848): duplicating a cube and trisecting an angle could not be solved
1838 De Morgan invents the term "mathematical induction" and makes the method precise.



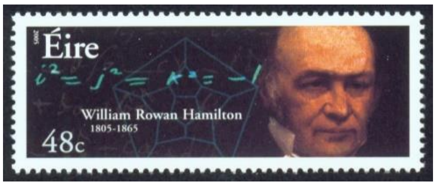
Joseph Liouville
(1809-1882, X1825) Le bicentenaire

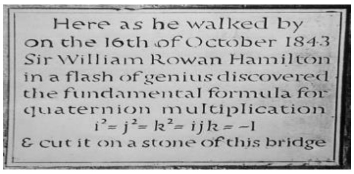


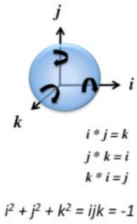
De Morgan Logician, prolific in popular writing, public education, the first president of London Math Soc.

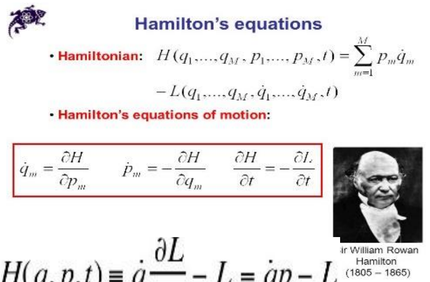
Augustus De†Morgan 1806-1871

- developed two laws of negation
- interested, like other mathematicians, in using mathematics to demonstrate logic
- furthered Boole's work of incorporating logic and mathematics
- formally stated the laws of set theory









$$H(q_1,\ldots,q_M,p_1,\ldots,p_M,t)=\sum_{m=1}^{M}p_m\dot q_m-L(q_1,\ldots,q_M,\dot q_1,\ldots,\dot q_M,t)$$

$$\dot q_m=\frac{\partial H}{\partial p_m}\qquad \dot p_m=-\frac{\partial H}{\partial q_m}\qquad \frac{\partial H}{\partial t}=-\frac{\partial L}{\partial t}$$

Lecture 11. Middle of 19th century

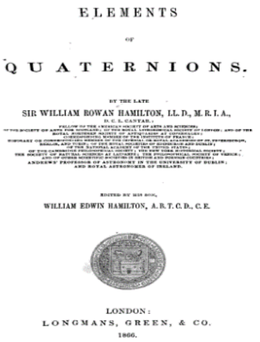
$$H(q,p,t)\equiv \dot q\frac{\partial L}{\partial \dot q}-L=\dot q p-L$$

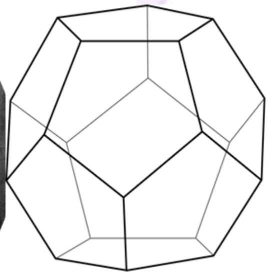
William Rowan Hamilton (1805-1865)

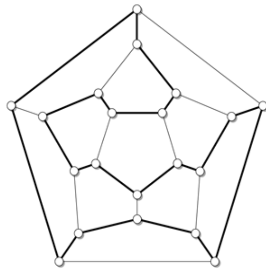
Sir William Rowan Hamilton (1805-1865) classical mechanics, algebra, graphs. 1827 optics, Hamiltonian mechanics with Hamiltonian equations, that revolutionized Newtonian Mechanics (Hamiltonian is a Legendre transformation of Lagrangian). 1835 knighted. 1843 invented quaternions, also dot and cross-product. 1856 a puzzle Icosian game: to find a Hamiltonian cycle on a wooden model of a dodecahedron. 1852: in a letter to Hamilton de Morgan states the 4-color problem (proposed by his student).

The Icosian Game

- The object of the game was to find a path that visited each of the 20 vertices exactly once.
- In honor of Hamilton and the his game, a path that uses each vertex of a graph exactly once is known as a **Hamiltonian path.**
- If the path ends at the starting vertex, it is called a **Hamiltonian circuit.**

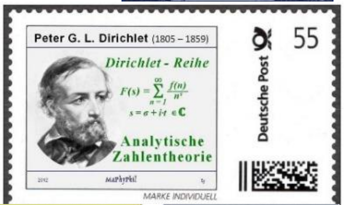

ELEMENTS
OF
QUATERNIONS.
BY THE LATE
SIR WILLIAM ROWAN HAMILTON, LL.D., M.R.I.A.,
EDITED BY HIS SON,
WILLIAM EDWIN HAMILTON, A.B.T.C.D., C.E.
LONDON:
LONGMANS, GREEN, & CO.
1866.





Johann Peter Gustav Lejeune Dirichlet (1805-1859) number theory, math analysis, PDE and math physics; Probability theory

Number Theory: 1823-24 Fermat's last theorem for the case *n*=5, brought immediate fame, (Fermat proved for *n*=4 and Euler for *n*=3), in 1825 he lectured at the French Acad. of Sci. on his proof, later proved for *n*=14. 1827-28 biquadratic reciprocity law; 1837 Dirichlet's theorem on arithmetic progressions, Dirichlet characters and L-functions. 1838-39 the first class number formula for quadratic forms, the Dirichlet unit theorem (fundamental result in algebraic number theory).



He first used the pigeonhole principle (Dirichlet Box principle) a basic counting argument, in the proof of a theorem in diophantine approximation, later named after him Dirichlet's approximation theorem.

The first statement of the Pigeonhole Principle is believed to have been made by the German Mathematician Dirichlet in 1834 under the name *Schubfachprinzip* (drawer principle).

$$f(x)=\begin{cases}1, & \text{if } x \text{ is rational}\\ 0, & \text{if } x \text{ is irrational}\end{cases}$$

Math Analysis: 1837 the modern formal definition of a function, Dirichlet function as not integrable one, Dirichlet's test for the convergence of series; 1829 memoir on the convergence of the Fourier series, introduced the Dirichlet kernel and the Dirichlet integral

$$D_m(u)=\frac{1}{2}+\cos u+\cdots+\cos mu=\frac{\sin(m+\frac{1}{2})u}{\sin\frac{u}{2}}$$

$$\int_0^\infty\frac{\sin t}{t}\,dt=\int_0^\infty\mathcal{L}\{\sin t\}(s)\,ds=\int_0^\infty\frac{1}{s^2+1}\,ds=\arctan s\Big|_0^\infty=\frac{\pi}{2},$$

Johann Peter Gustav Lejeune Dirichlet

- In the field of mathematical analysis, a **general Dirichlet series** is an infinite series that takes the form

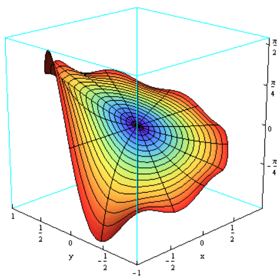
$$\sum_{n=1}^{\infty}a_n e^{-\lambda_n s},$$

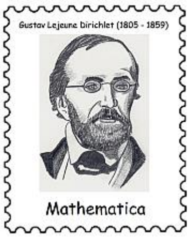
Conditions (Dirichlet conditions) on *f(t)* to yield a convergent Fourier series:

1. *f(t)* is single-valued everywhere.
2. *f(t)* has a finite number of finite discontinuities in any one period.
3. *f(t)* has a finite number of maxima and minima in any one period.
4. The integral $\int_{t_0}^{t_0+T}|f(t)|dt<\infty$ for any t_0.

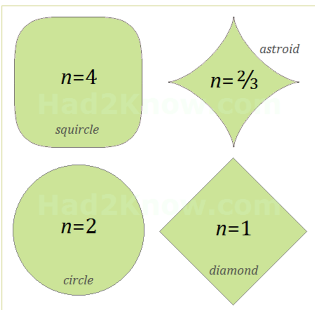
PDE and Math Physics: Dirichlet problem as the boundary value problem for PDE (for Laplace equation), unicity of the solution via the principle of minimizing the Dirichlet energy (Dirichlet principle as called later by Riemann). Math Physics applications: in potential theory, theory of heat and hydrodynamics. The condition for equilibrium is that the potential energy is minimal.

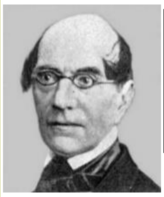
$$D[u]=\int_\Omega|\nabla u|^2\,dV$$





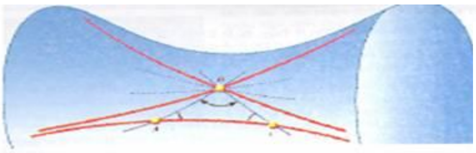
Probability theory: Dirichlet distribution and Dirichlet process: improvement of Laplace's method related to the central limit theorem limit theorems.

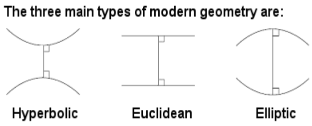




1839 Gabriel Lamé (1795-1870) proved Fermat's Last Theorem for *n* = 7. Worked also on curvilinear coordinates, PDE, Lame Curves (superellipses)

$$\left|\frac{x}{a}\right|^n+\left|\frac{y}{b}\right|^n=1$$

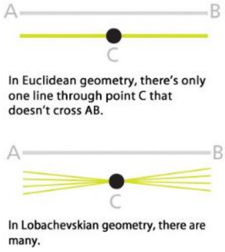




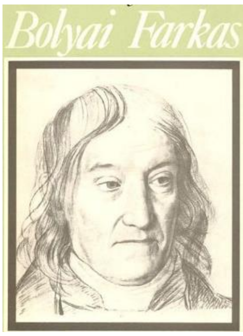
Nikolai Ivanovich Lobachevsky (1792-1856) "Copernicus of Geometry", known for revolutionary Lobachevskian geometry.

Russian (of Polish origin) born in Nizhny Novgorod, 1807-11 studied and then worked in Kazan University, taught by Martin Bartels (who was a school teacher and friend of Gauss); 1814 Lecturer, 1822 Full Professor, 1830 Rector.

1823 main work *Geometry* is completed (published much later); 1826 first report on Non-Euclidean geometry, 1829-30 published in *Kazan Messenger* (it was rejected by proceedings of St. Petersburg Acad. of Sci.; gossip: Ostrogradski said that "there is not enough of Math. Analysis"); 1835–1838 Book *New Foundations of Geometry*, and then several others. 1866 French translation of Lobachevski is published, with some of Gauss's correspondences.



János Bolyai (1802-1860) Hungarian, one of the founders of non-Euclidean geometry. 1820-23 working on a treatise, published in 1832 as an appendix to a mathematics textbook by his father, Farkas, a friend of Gauss (in earlier times he was also obsessed with Euclid's 5th Postulate and warned his son not to repeat his mistake "because it too may take all your time and deprive you of your health, peace of mind and happiness in life").

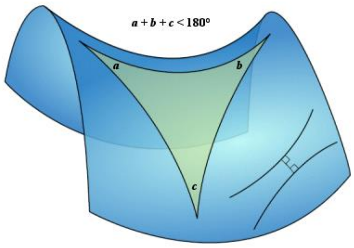


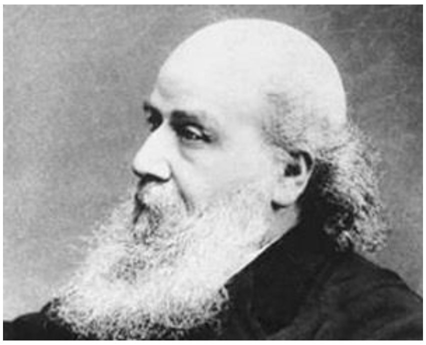
Never publishing more than the 24 pages of this Appendix, Janos left more than 20,000 pages of math when died. These now are in the Bolyai–Teleki library. Janos was also a polyglot and violin player performing in Vienna.

It was recently discovered (Notices AMS 2011, No1) that previously widely circlulating images of Janos Bolyai are in fact fake (the image on the left is a "true" computer reconstruction).

> Out of nothing I have created a strange new universe.
>
> (Janos Bolyai)

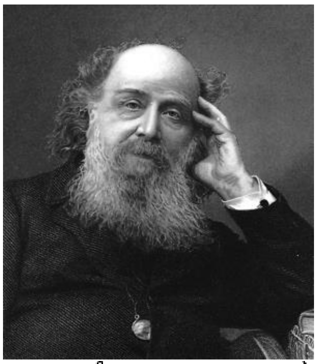
Gauss and non-euclidean geometry: 1824 a letter to Taurinus: *The assumption that the sum of the three angles of a triangle is less than 180° leads to a curious geometry, quite different from ours* [*i.e. Euclidean geometry*] *but thoroughly consistent, which I have developed to my entire satisfaction, so that I can solve every problem in it excepting the determination of a constant, which cannot be fixed a priori. .... the three angles of a triangle become as small as one wishes, if only the sides are taken large enough, yet the area of the triangle can never exceed, or even attain a certain limit, regardless of how great the sides are.* 1831 a letter to Farkas about publication of Janos: *To praise it would amount to praising myself. For the entire content of the work ... coincides almost exactly with my own meditations which have occupied my mind for the past thirty or thirty-five years*. However in another letter: *I regard this young geometer Bolyai as a genius of the first order*. Gauss appreciated Lobachevsky's works very highly, but had no personal correspondence with him and did not make public comment on his works.



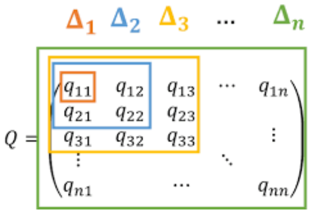


$$AX + XB = C.$$

$$\det(I_m + AB) = \det(I_n + BA),$$



James Joseph Sylvester (1814-1897) matrix theory, invariant theory, number theory, partition theory and combinatorics. Invented terms: "matrix", "graph" (in combinatorics) and "discriminant". Concept of inertia indices, the inertia law for symmetric matrices, a method to find the indices, Sylvester Matrix, whose discriminant, Resultant, vanishes if and only if two polynomials have a common root. Sylvester equation for matrices, Sylvester identity.

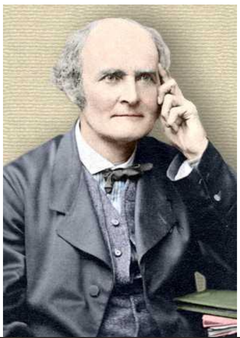


$$\Delta_1 \quad \Delta_2 \quad \Delta_3 \quad \cdots \quad \Delta_n$$

$$Q = \begin{pmatrix} q_{11} & q_{12} & q_{13} & \cdots & q_{1n} \\ q_{21} & q_{22} & q_{23} & & \vdots \\ q_{31} & q_{32} & q_{33} & & \\ \vdots & & & \ddots & \\ q_{n1} & & \cdots & & q_{nn} \end{pmatrix}$$

$$\begin{pmatrix} f_m & \cdots & f_0 & & & \\ & f_m & \cdots & f_0 & & \\ & & \ddots & & \ddots & \\ & & & f_m & \cdots & f_0 \\ g_n & \cdots & g_0 & & & \\ & g_n & \cdots & g_0 & & \\ & & \ddots & & \ddots & \\ & & & g_n & \cdots & g_0 \end{pmatrix}$$

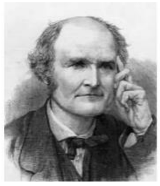
1876 prof of math at the new Johns Hopkins University; 1878 founded the *American Journal of Mathematics*. (The only other mathematical journal in the US at that time was the *Analyst*, which eventually became the *Annals of Mathematics*.)



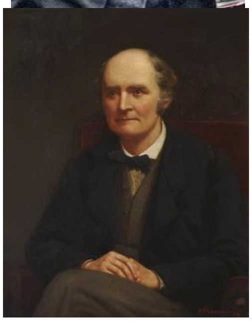
Arthur Cayley (1821-1895) found the modern British school of pure mathematics; 966 math articles (250 in 14 years as he was working as a lawer); friend of Sylvester.

Arthur Cayley
1821-1895

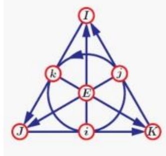
Introduced matrix multiplication



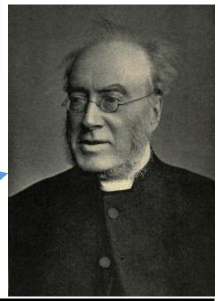
Groups: defined an abstract group in the modern way (as a set with a binary operation satisfying certain properties) Cayley Tables (for multiplication law); before by groups mathematicians had meant permutation groups (Cayley's theorem).



Octonions in 1843 John T. Graves, friend of Hamilton, mentioned them in a letter to Hamilton, but published his result in 1845, later than Cayley's article on them.

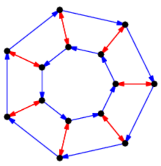


×	i	j	k	E	I	J	K
i	−1	k	−j	I	−E	−K	J
j	−k	−1	i	J	K	−E	−I
k	j	−i	−1	K	−J	I	−E
E	−I	−J	−K	−1	i	j	k
I	E	−K	J	−i	−1	−k	j
J	K	E	−I	−j	k	−1	−i
K	−J	I	E	−k	−j	i	−1



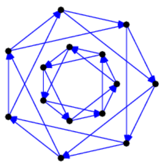
Matrices: 1857 postulated Cayley-Hamilton Theorem in a letter to Sylvester, verified for order 2 and 3, proved later by Frobenius. 1858 memoir on the algebra of matrices, multiplication, inverse matrix.

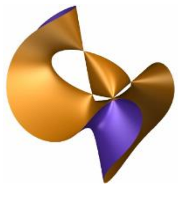
Algebraic Geometry: 1869 "*A Memoir on Cubic Surfaces*", Cayley and Salmon (1819-1904) discovered the 27 lines on a cubic surface. He founded the theory of ruled surfaces, constructed the Chow variety of all curves in projective 3-space.

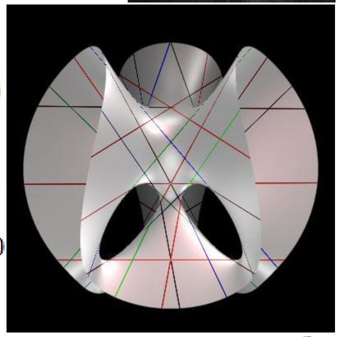


1876 *Treatise on Elliptic Functions*.

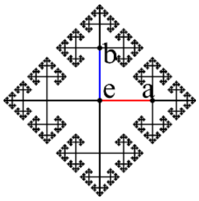
Graph Theory: Cayley graphs representing Group Law.



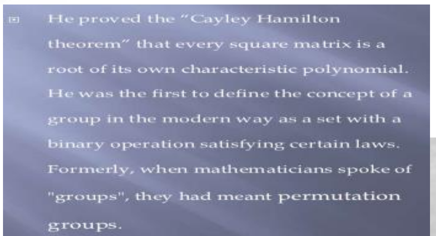




$$\frac{1}{x_0} + \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} = 0$$



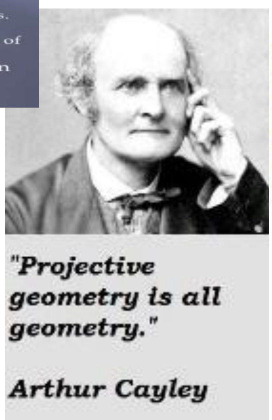
$$X^3 + Y^3 + Z^3 + W^3 = (X + Y + Z + W)^3$$



He proved the "Cayley Hamilton theorem" that every square matrix is a root of its own characteristic polynomial. He was the first to define the concept of a group in the modern way as a set with a binary operation satisfying certain laws. Formerly, when mathematicians spoke of "groups", they had meant permutation groups.

Early Life

- Arthur Cayley was born in Richmond, London, England, on 16 August 1821.
- As a child, Cayley enjoyed solving complex math problems for fun. He entered Trinity College, Cambridge, where he excelled in Greek, French, German, and Italian, as well as mathematics. He worked as a lawyer for 14 years.



ARTHUR CAYLEY

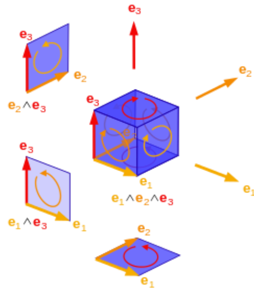
BORN: 16 August 1821 AD
DIED: 26 January 1895 AD
NATIONALITY: British
KNOWN FOR : Founder of modern British school of pure mathematics
AWARDS: Smith's Prize (1842), De-Morgan Medal (1884) and Royal Medal

CONTRIBUTION:

1.Cayley made valuable contributions to algebraic theory of curves and surfaces, graph theory,group theory, linear algebra, combinatorics and elliptic functions.

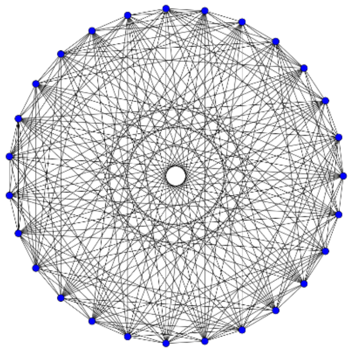
2.In geometry, however, his work was based on analytic geometry. In 1859, Cayley was the first person to realize that Euclidean geometry was a special case of projective geometry and ten years later,his projective metric helped people understand the relationship between the different types of non-Euclidean geometries.

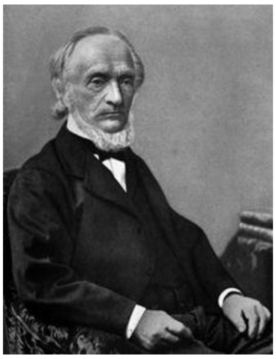
3.In 1876, Cayley published a book called 'An Elementary Treatise on Elliptic Functions'. This was the only book he wrote and it was based on his studies of the Karl Jacobi's point of view.



Hermann Günther Grassmann (1809-1877) German polymath, neohumanist, general scholar, and publisher. His math work was little noted until he was in his sixties.

First developed linear algebra and the notion of a vector space, the theory of linear independence, subspace, linear independence, span, dimension, join and meet of subspaces, projections. In 1890s Peano gave formal definitions, and only around 1920 Hermann Weyl and others rediscovered and popularized them.

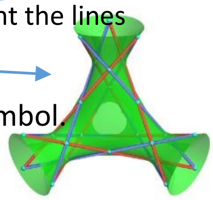


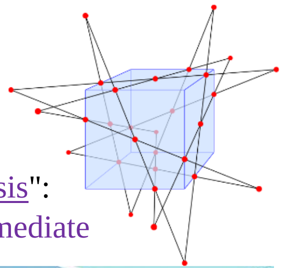


Ludwig Schläfli (1814-1895) Swiss mathematician: geometry and complex analysis (at the time called function theory) who was one of the key figures in developing the notion of higher-dimensional spaces.

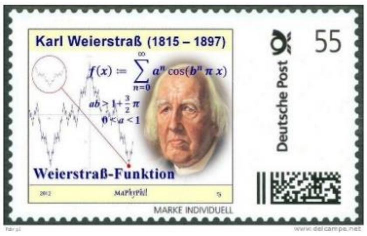
The Schläfli symbol of a regular polyhedron is {*p*,*q*} if its faces are *p*-gons, and each vertex is surrounded by *q* faces. Schläfli graph represent the lines on a cubic surface, as well as Schläfli double six.

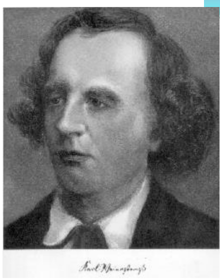
1870 got a prize for 27 lines, 36 double sixes and Schläfli symbol.

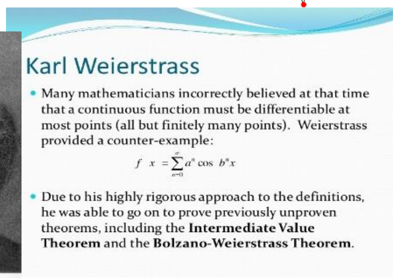




Karl Theodor Wilhelm Weierstrass (1815-1897) "father of modern analysis": formalized the definition of the continuity in ε-δ language, proved the intermediate value theorem and the Bolzano–Weierstrass theorem, corrected Cauchy on convergence, developed uniform convergence, and studied continuous functions on closed bounded intervals.







Otto Hesse 1811-1874
algebraic invariants

1841 Gauss a treatise on optics with formulae for the position and size of the image by a lens

1841 Jacobi memoir *De determinantibus functionalibus* devoted to the Jacobian.

1842 Otto Hesse introduces the "Hessian determinant" in a paper on cubics and conics.

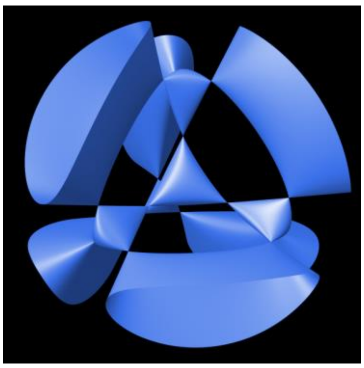
1842 Stokes *On the steady motion of incompressible fluids*.

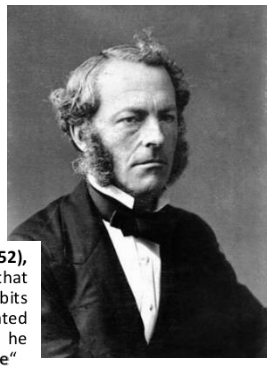
1843 Hamilton quaternions

1843 Liouville the unknown work of Galois is found; 1846 published in *Liouville's Journal*

1843 Kummer "ideal complex numbers" that lead to the development of ring theory.

$$H(f) = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x\,\partial y} \\ \frac{\partial^2 f}{\partial y\,\partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}.$$



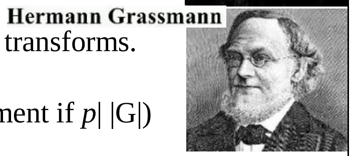


- **Sir George G. Stokes (1852),** made the first observation that the mineral 'fluorspar' exhibits fluorescence when illuminated with ultraviolet light, and he coined the word "**fluorescence**"

1843 Cayley investigates "geometry of *n* dimensions" using determinants as the major tool.

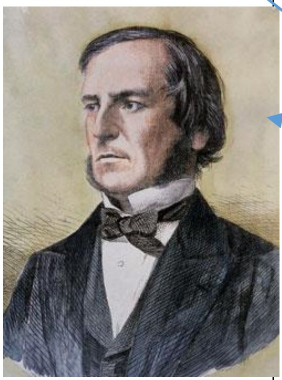
1844 Liouville finds the first transcendental numbers

1844 Grassmann an algebra based on points, lines and planes manipulated using some rules.

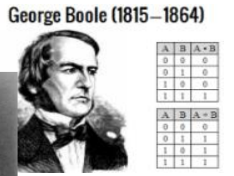


1845 Cayley *Theory of Linear Transformations* with the composition of linear transforms.

1845 Cauchy "Cauchy's theorem" of group theory (existence of an order *p* element if *p*| |G|)

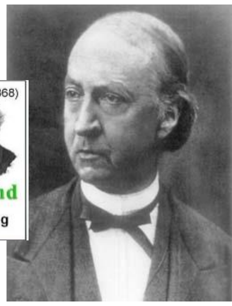


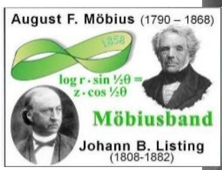
1846 Maxwell at the age of 14: *On the description of oval curves, and those having a plurality of foci*.



1847 Boole *The Mathematical Analysis of Logic* foundations of logic, binary system

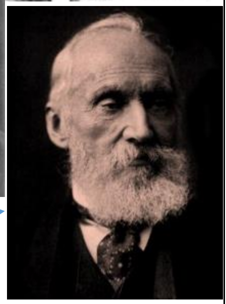
1847 De Morgan proposes two "de Morgan's laws" of set theory





1847 Listing 1808-1882 introduced term *Topology*

1847 Von Staudt 1798-1867 *Geometrie der Lage* used projective geometry (synthetic as free from any metric calculations) to provide a foundation for arithmetic



1848 Thomson (Lord Kelvin) proposes the absolute temperature scale named after him.

1849 Hermite applies Cauchy's residue techniques to doubly periodic functions.

1850 Chebyshev *On Primary Numbers* proves Bertrand's conjecture: existence of primes n<p<2n, for *n* >1

1850 Sylvester *On a New Class of Theorems* first used the word "matrix".

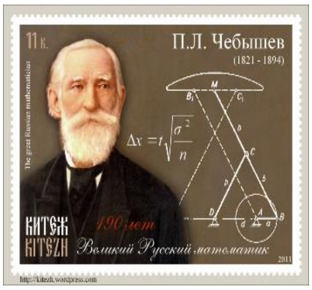
Hermitian, Skew-Hermitian, and Unitary Matrices

A square matrix $\mathbf{A} = [a_{kj}]$ is called

Hermitian	if	$\bar{\mathbf{A}}^{\mathsf{T}} = \mathbf{A}$,	that is,	$\bar{a}_{kj} = a_{jk}$
skew-Hermitian	if	$\bar{\mathbf{A}}^{\mathsf{T}} = -\mathbf{A}$,	that is,	$\bar{a}_{kj} = -a_{jk}$
unitary	if	$\bar{\mathbf{A}}^{\mathsf{T}} = \mathbf{A}^{-1}$.		

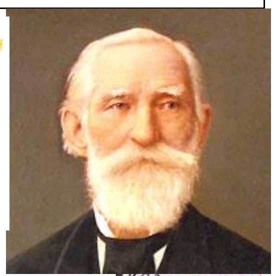
Example Hermitian Skew-Hermitian unitary

$$\mathbf{A} = \begin{pmatrix} 4 & 1-3i \\ 1+3i & 7 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 3i & 2+i \\ -2+i & -i \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} \frac{1}{2}i & \frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2}i \end{pmatrix}$$



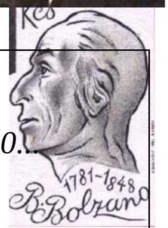
The Chebyshev's theorem states that the proportion of the values from a data set that falls within the number of standard deviations of the mean will be at least one minus one over k squared.

$$P \geq 1 - \frac{1}{k^2}$$



1851 Bolzano Paradoxes of the Infinite introduces his ideas about infinite sets.

Bolzano–Weierstrass Theorem: Every sequence of real numbers has a monotonic subsequence.

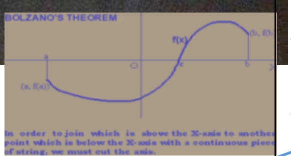


1851 Liouville transcendental "Liouville numbers", 0.11000100000000000000000100000... where there is a 1 in place n! and 0 elsewhere.

1851 Riemann's doctoral thesis contains "Riemann surfaces" and their properties.

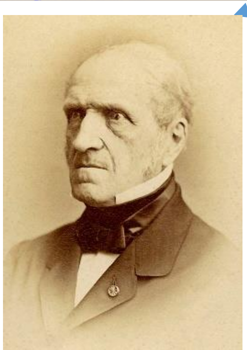
1852 Sylvester establishes the theory of algebraic invariants.

Similarity Invariants Similar matrices often have properties in common;for example,if A and B are similar matrices,then A and B have the same determinant.To se



1852 Francis Guthrie poses the Four Colour Conjecture to De Morgan.

1852 Chasles Traité de géométrie introduced and studies cross ratio, pencils and involutions.



1853 Hamilton publishes Lectures on Quaternions.

1853 Shanks gives π to 707 places (in 1944: Shanks was wrong from the 528th place).

Value of $\pi = 3\cdot$

·14159 26535 89793 23846 26433 83279 50288 41971 69399 37510 58209 74944
59230 78164 06286 20899 86280 34825 34211 70679 82148 08651 32823 06647
09384 46095 50582 23172 53594 08128 48111 74502 84102 70193 85211 05559
64462 29489 54930 38196 44288 10975 66593 34461 28475 64823 37867 83165
27120 19091 45648 56692 34603 48610 45432 66482 13393 60726 02491 41273
72458 70066 06315 58817 48815 20920 96282 92540 91715 36436 78925 90360
01133 05305 48820 46652 13841 46951 94151 16094 33057 27036 57595 91953
09218 61173 81932 61179 31051 18548 07446 23798 34749 56735 18857 52724
89122 79381 83011 94912 98336 73362 44193 66430 86021 39501 60924 48077
23094 36285 53096 62027 55693 97986 95022 24749 96206 07497 03041 23668
86199 51100 89202 38377 02131 41694 11902 98858 25446 81639 79990 46597
00081 70029 63123 77381 34208 41307 91451 18398 05709 85 &c.

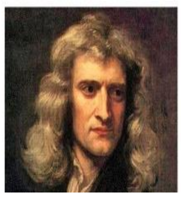
April 14, 1873.

1853: William Shanks determines π to 707 digits!

Augustus DeMorgan (1806-1871) notices a smaller number of appearances of the digit 7 in Shanks' determination.

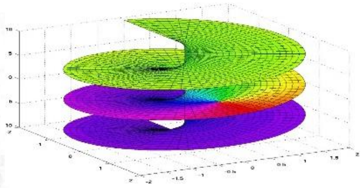
Later on it was discovered that there was an error beginning at the 528 digit of Shanks' calculation

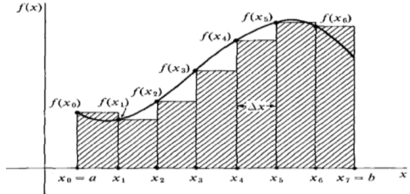
Newton's pi



Newton computed pi to 15 digits writing later

"I am ashamed to tell you to how many figures I carried these computations, having no other business at the time."





Georg Friedrich Bernhard Riemann (1826-1866) Riemann Integral, Riemann Surfaces, founder of Riemannian Geometry, stated Riemann Hypothesis.

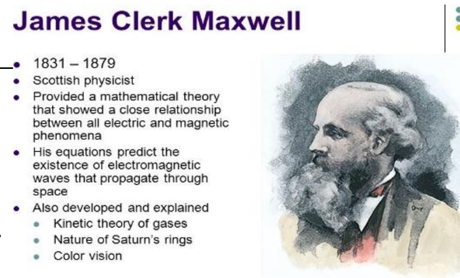
Real analysis rigorous formulation of the integral, the Riemann integral, Fourier series.

Complex analysis geometric foundation via theory of Riemann surfaces that combined Analysis with Geometry, Cauchy-Riemann equations, Riemann mapping theorem,

Analytic number theory 1859 prime-counting function, Riemann hypothesis,

Differential geometry 1854 Habilitation lecture "*On the hypotheses which underlie geometry*" at Göttingen on the Riemannian geometry that generalizes Gaussian differential geometry for surfaces. It was only published twelve years later in 1868 by Dedekind, recognized now as one of the most important works in geometry. Riemann suggested using dimensions higher than merely three or four in order to describe physical reality.

1857 an attempt to promote Riemann to extraordinary professor status at the University of Göttingen failed, but he was paied a regular salary. 1859 after Dirichlet's death, he was promoted to head the math department at Göttingen.



1854 Boole reduced logic to algebra known as Boolean algebra.

1854 Cayley defined an abstract group.

1855 Maxwell *On Faraday's lines of force* with equations for electric and magnetic fields.

1857 Riemann *Theory of abelian functions* Riemann surfaces and their topological properties, multi-valued functions as single valued over "Riemann surface", general inversion generalizing works of Abel and Jacobi.

1858 Cayley definition of a matrix, a term introduced by Sylvester in 1850, and in *A Memoir on the Theory of Matrices* he studies its properties.

1858 Möbius describes the "Möbius strip"; Listing did the same discovery in the same year.

1858 Dedekind rigorous definition of irrational numbers with "Dedekind cuts". The idea comes to him while he is thinking how to teach differential and integral calculus.

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p\ prime} \frac{1}{1-p^{-s}}$$

1859 Riemann makes a conjecture about the zeta function which involves prime numbers. The Riemann's hypothesis is perhaps the most famous unsolved problem in mathematics in the 21st century.

Of the zeros of the Riemann zeta function, defined by analytic continuation from

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \qquad \mathrm{Re}(s) > 1,$$

are those which are in the critical strip $0 \le \mathrm{Re}(s) \le 1$ *all on the line* $\mathrm{Re}(s) = \frac{1}{2}$?

1861 Weierstrass discovers a continuous curve that is not differentiable any point.

1862 Maxwell proposes that light is an electromagnetic phenomenon.

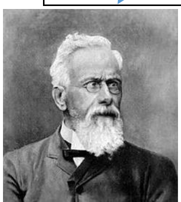
1863 Weierstrass in his lecture course: complex numbers are the only commutative algebraic extension of the reals.

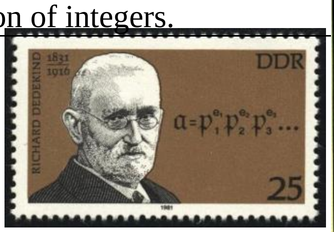
1866 Hamilton's *Elements of Quaternions* (unfinished) ~800 pages is published after death by his son.

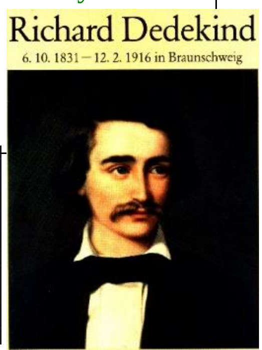
1868 Beltrami in *Essay on an Interpretation of Non-Euclidean Geometry* a model for the non-euclidean geometry

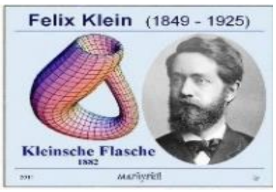
1871 Betti publishes a memoir on topology which contains the "Betti numbers".

1872 Dedekind formal construction of real numbers, a rigorous definition of integers.





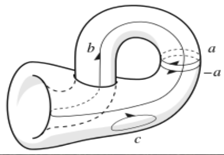




Lecture 12. End of 19th– beginning of 20th

Christian Felix Klein (1849-1925) group theory, complex analysis, non-Euclidean geometry, and on the connections between geometry and group theory. 1872 Erlangen Program classifying geometries by their underlying symmetry groups

Geometry: 1870 projective transformations of the Kummer surface, asymptotic lines on the invariant curves (joint with Sophus Lie), Klein bottle



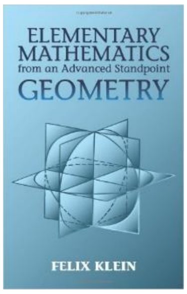
1871 *On the So-called Non-Euclidean Geometry*: Cayley-Klein metric ending the controversy of non-Euclidean geometry (but Cayley never accepted Klein's argument). 1872 *Erlangen Program*: a modern unified approach to geometry viewed as the study of the properties of a space that is invariant under a given group of transformations.

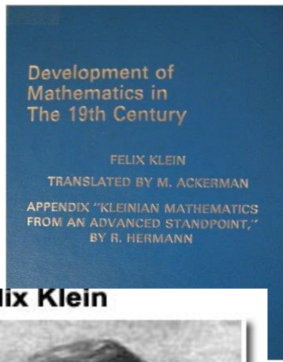
Complex analysis: idea of the modular group that moves the fundamental region of the complex plane so as to tessellate that plane. 1879 action of PSL(2,7) (of order 168) on the Klein quartic $x^3y + y^3z + z^3x = 0$. 1882 treated complex analysis in a geometric way, connecting potential theory, conformal mappings and fluid dynamics.

1884 book on the icosahedron, a theory of automorphic functions, connecting algebra and geometry. Poincaré published an outline of his theory of automorphic functions in 1881, both sought to state and prove a grand uniformization theorem. Klein succeeded in formulating such a theorem and in sketching a strategy for proving it, but while doing this work his health collapsed.

Klein summarized his work on automorphic and elliptic modular functions in a four volume treatise, written with Robert Fricke over a period of about 20 years.

1890s: Klein turned to mathematical physics. In 1894 he launched the idea of an encyclopedia of mathematics including its applications, which ran until 1935, provided an important standard reference of enduring value. Under Klein's editorship, *Mathematische Annalen* (founderd by Clebsch in 1868) became one of the very best mathematics journals in the world surpassing *Crelle's Journal* based out of the University of Berlin.

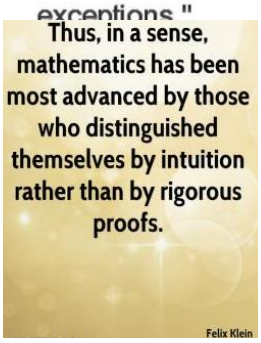


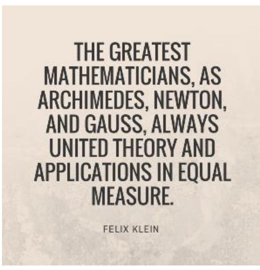


~1900, Klein took an interest in mathematical school education. In 1905, he played a decisive role in formulating a plan recommending that analytic geometry, the rudiments of differential and integral calculus, and the function concept be taught in secondary schools. This recommendation was gradually implemented in many countries around the world. In 1908, Klein was elected president of the International Commission on Mathematical Instruction at the Rome International Congress of Mathematicians. Under his guidance, the German branch of the Commission published many volumes on the teaching of mathematics at all levels in Germany.

"Everyone knows what a curve is, until he has studied enough mathematics to become confused through the countless number of possible exceptions."

- Felix Klein





Felix Klein

(25 April 1849 – 22 June 1925)

Felix Christian Klein was a German mathematician, known for his work in group theory, function theory, non-Euclidean geometry, and on the connections between geometry and group theory. His 1872 Erlangen Program, classifying geometries by their underlying symmetry groups, was a hugely influential synthesis of much of the mathematics of the day.

Transcendental numbers The name "transcendental" comes from Leibniz paper, 1682, where he proved that $\sin(x)$ is not an algebraic function of x. Euler proved irrationality of e in 1737 (published in 1744) and he was probably the first to define transcendental *numbers*.

In 1768 Lambert conjectured that e and π were both transcendental in his paper proving the number π is irrational.

In 1844 Liouville proved the existence of transcendental numbers and in 1851 gave the first decimal examples such as the Liouville constant $\sum_{k=1}^{\infty} 10^{-k!} = 0.11000100000000000000000010000\ldots$

In 1873 Charles Hermite proved that e is transcendental.

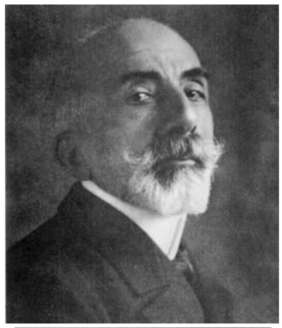
In 1874, Cantor proved that the algebraic numbers are countable and the real numbers are uncountable, and thus there are as many transcendental numbers as there are real numbers.

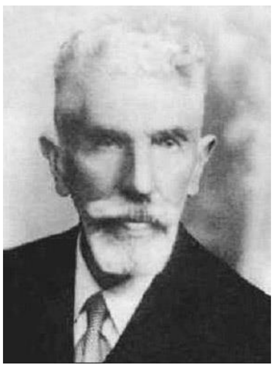
In 1882, Lindemann proved that π is transcendental. He first showed that e^a is transcendental when a is algebraic and not zero. Then, since $e^{i\pi} = -1$ is algebraic, $i\pi$ and therefore π must be transcendental. This implied impossibility of squaring the circle and trisection of angle.

1900 in Hilbert's 7th problem: If a is an algebraic number, that is not zero or one, and b is an irrational algebraic number (say $2^{\sqrt{2}}$) is a^b necessarily transcendental? This was proved in 1934. It is still unknown if e^e, π^e, 2^e, 2^π, π^π are irrational (transcendental).

Birth of Italian Algebro-Geometric School

Luigi Cremona (1830-1903) algebraic curves and algebraic surfaces, reformed advanced mathematical teaching in Italy (serving as the Vice-President of Sanate and Minister of Public Education). 1866 Memoir on Cubic surfaces awarded, Cremona group





Corrado Segre (1863-1924) algebraic curves, surface, Segre cubic treefold; teacher of Gino Fano, Beniamino Segre, Francesco Severi, Beppo Levi

Guido Castelnuovo (1865-1952) algebraic geometry, statistics and probability theory, Castelnuovo criterion

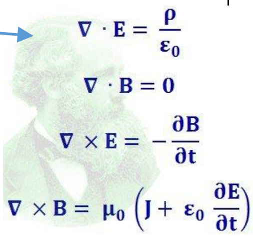
Federigo Enriques (1871-1946) classified algebraic surfaces (birationally)

1872 Heine the "Heine-Borel theorem".

1872 Sylow three "Sylow theorems" about finite groups proved for permutation groups.

1872 Klein's "*Erlanger programm*": geometry via group of transformations.

1873 Maxwell *Electricity and Magnetism* contains four "Maxwell's equations".



$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot B = 0$$

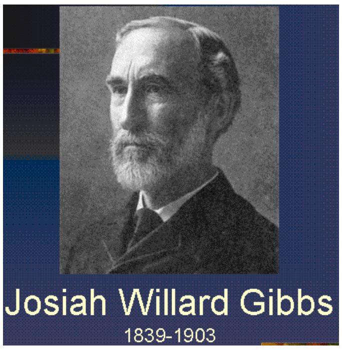
$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \mu_0 \left(J + \varepsilon_0 \frac{\partial E}{\partial t} \right)$$

1873 Hermite proves that *e* is a transcendental number.

1873 Gibbs publishes two important papers on diagrams in thermodynamics.

1874 Cantor the first paper on set theory, infinities come in "different sizes"; almost all numbers are transcendental. 1877 Cantor is surprised at his own discovery that there is a one-one correspondence between points on the interval [0, 1] and points in a square.



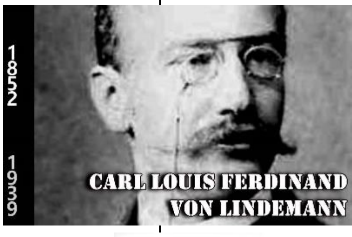
1876 Gibbs application of math to chemistry.

1878 Sylvester founds the *American Journal of Mathematics*.

1880 Poincaré on automorphic functions.

1881 Venn introduces "Venn diagrams", a useful tools in set theory.

1881 Gibbs develops vector analysis in a pamphlet written for the use of his own students. The methods will be important in Maxwell's mathematical analysis of electromagnetic waves.



1882 Lindemann π is transcendental that answers to the squaring the circle problem

1882 Mittag-Leffler founds the journal *Acta Mathematica*.

1883 Poincaré initiates studying of analytic functions of several complex variables.

1884 Volterra begins his study of integral equations.

1884 Mittag-Leffler construct meromorphic function with prescribed poles and singular parts.

1884 Frobenius proves Sylow's theorems for abstract groups.

1885 Weierstrass shows that a continuous function on a finite subinterval of the real line can be uniformly approximated arbitrarily closely by a polynomial.

1886 Peano proves that if $f(x, y)$ is continuous then ${}^{dy}/_{dx} = f(x, y)$ has a solution.

1887 Levi-Civita publishes a paper developing the calculus of tensors.

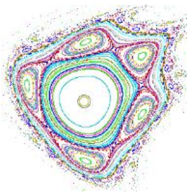
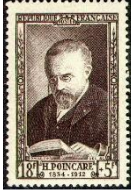
1888 Dedekind in *The Nature and Meaning of Numbers* gives "Peano axioms" (Peano in 1889)

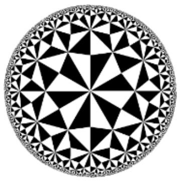
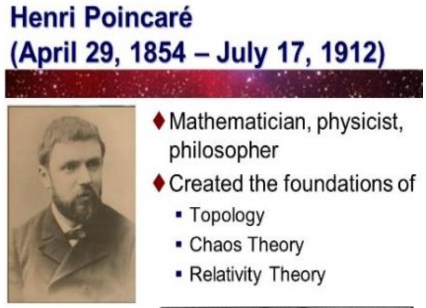
1888 Engel and Lie Theory of Transformation Groups on continuous groups of transformations.

1890 Peano discovers a space filling curve.

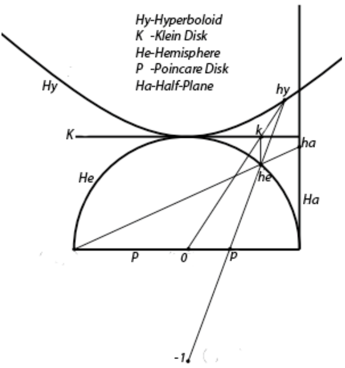
Giuseppe Peano

(27 August 1858 – 20 April 1932)

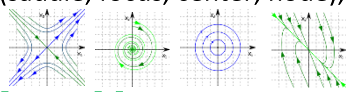
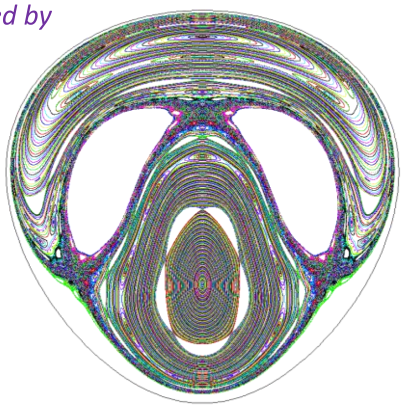
Jules Henri Poincaré 1854 –1912 *The Last Universalist*
Father of Topology, Dynamical systems, Chaos theory, Poincaré conjecture (solved in 2002–03). Philosopher of science and popularizer of math and physics.

Physics: fluid mechanics, celestial mechanics (three-body problem), optics, electricity, telegraphy, capillarity, elasticity, thermodynamics, potential theory, quantum theory, theory of relativity and physical cosmology.

Automorphic forms: *uniformization theorem*, **Poincare Disc model** of hyperbolic geometry

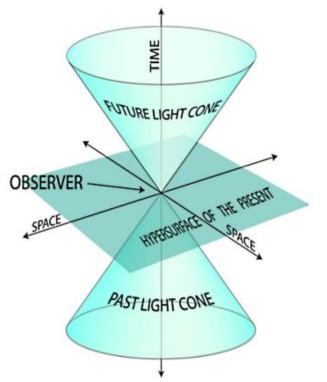
Differential equations: a series of memoirs "*On curves defined by differential equations*" (1881–1882) qualitative theory of differential equations: integral curves, classified singular points (saddle, focus, center, node), limit cycles (finiteness) and the loop index.

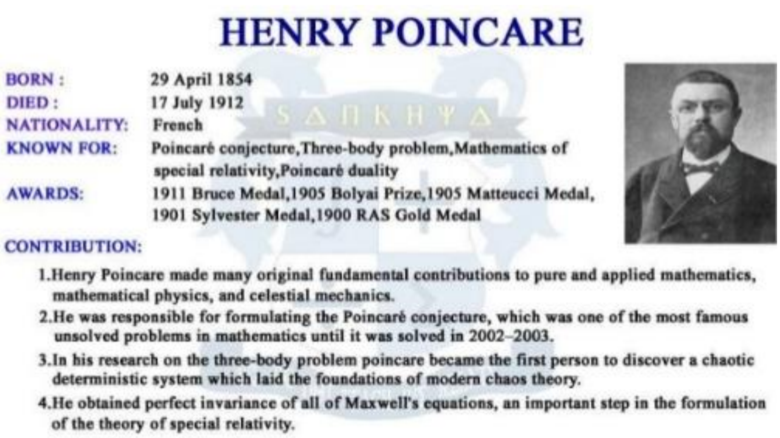
Celestian mechanics: the three-body problem "*New Methods of Celestial Mechanics*" (1892–1899), "*Lectures on Celestial Mechanics*" (1905–1910): showed that the three-body problem is not integrable (cannot be expressed in algebraic and transcendental functions through the coordinates and velocities of the bodies), the general theory of dynamical systems.

Relativity: the "local time" $t' = t - vx/c^2$ was invented by Hendrik Lorentz in 1895 to interpret to explain the failure of Michelson–Morley experiment (optical and electrical); in 1898 in The Measure of Time Poincare looked for the "deeper meaning" and postulated the constancy of the speed of light. In 1900 he discussed the *Principle of relative motion* in two papers and named it *principle of relativity* in 1904: ***no physical experiment can distinguish a state of uniform motion from a state of rest***. In 1906 he noted that a Lorentz transformation is merely a rotation in four-dimensional space about the origin preserving $x^2 + y^2 + z^2 - c^2t^2$ invariant. In 1907 Hermann Minkowski worked out the consequences of this. Poincaré found that the electromagnetic field energy behaves like a fluid with a mass density of E/c^2.

Topology 1895 *"Analysis Situs"* with 5 supplements in 1899-1904 laid foundations, definition of homotopy and homology, Betti numbers and the fundamental group. The Poincare Conjecture first was stated as a "theorem", but then a counterexample, the *Poincare sphere* was constructed.



JUST PASSING THROUGH SPACE & TIME

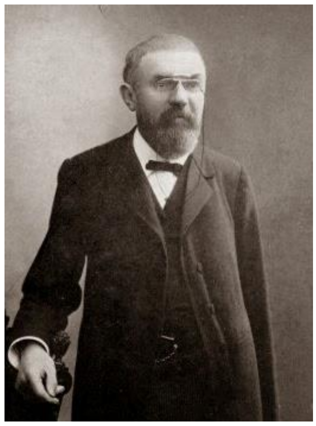


HENRY POINCARE

BORN :	29 April 1854
DIED :	17 July 1912
NATIONALITY:	French
KNOWN FOR:	Poincaré conjecture, Three-body problem, Mathematics of special relativity, Poincaré duality
AWARDS:	1911 Bruce Medal, 1905 Bolyai Prize, 1905 Matteucci Medal, 1901 Sylvester Medal, 1900 RAS Gold Medal

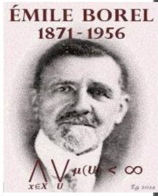
CONTRIBUTION:

1. Henry Poincare made many original fundamental contributions to pure and applied mathematics, mathematical physics, and celestial mechanics.
2. He was responsible for formulating the Poincaré conjecture, which was one of the most famous unsolved problems in mathematics until it was solved in 2002-2003.
3. In his research on the three-body problem poincare became the first person to discover a chaotic deterministic system which laid the foundations of modern chaos theory.
4. He obtained perfect invariance of all of Maxwell's equations, an important step in the formulation of the theory of special relativity.



1891 Fedorov and Schönflies classify crystallographic space groups (230 of them).

1892 Poincaré *New Methods in Celestial Mechanics* (putting in doubt the stability proofs of the solar system given by Lagrange and Laplace)



1894 Poincaré begins work on algebraic topology, 1895 publishes *Analysis situs*

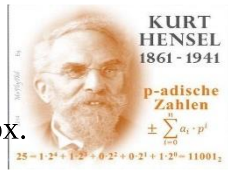
1894 Borel introduces "Borel measure".

1894 Cartan classified finite dimensional simple Lie algebras over the complex numbers.

EMILE BOREL
1871-1956

1895 Cantor publishes the first of two major surveys on transfinite arithmetic.

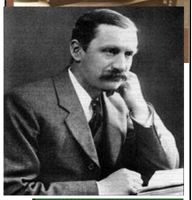
1896 Frobenius introduces group characters, in 1897 representation theory,
1898 the induced representations and the "Frobenius Reciprocity Theorem".

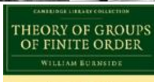


1897 Hensel invents the *p*-adic numbers.

1897 Burali-Forti is the first to discover of a set theory paradox.

1897 Burnside publishes *The Theory of Groups of Finite Order*



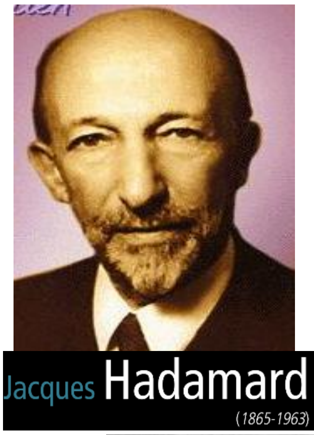


1898 Hadamard geodesics on surfaces of negative curvature, symbolic dynamics.

1899 Hilbert *Foundations of Geometry* putting geometry in a formal axiomatic setting.

1899 Lyapunov on the stability of sets of ordinary differential equations.

1900 Hilbert 23 problems at the II ICM in Paris as a challenge for the 20th century.

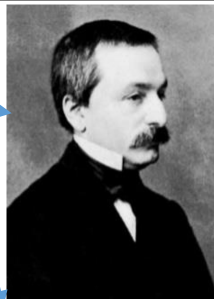


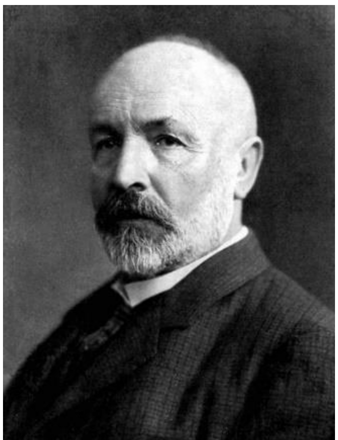
1900 Goursat in *Cours d'analyse mathematique* introduces many new analysis concepts.

1900 Fredholm integral equations

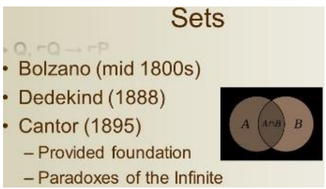
1900 Levi-Civita and Ricci-Curbastro set up the theory of tensors

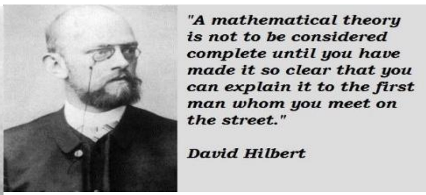
Georg Ferdinand Ludwig Philipp Cantor (1845-1918) elaborated set theory, constructed a hierarchy of infinite (well-ordered) sets, cardinal and ordinal transfinite numbers. Cantor's theory raised resistance from Leopold Kronecker (describing Cantor as a "scientific charlatan", a "renegade" and a "corrupter of youth") and Henri Poincaré (calling his ideas "grave disease infecting the discipline of mathematics") and later from Hermann Weyl and L. E. J. Brouwer, while Ludwig Wittgenstein raised philosophical objections ("utter nonsense" that is "laughable" and "wrong"). Cantor, a devout Lutheran, believed the theory had been communicated to him by God. Cantor's suffered from depression since 1884 to the end of his life because of such hostile attitude and blames.

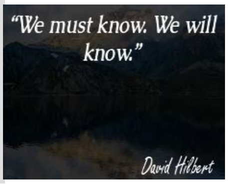




In 1904, the Royal Society awarded Cantor its Sylvester Medal, the highest honor (it was the second medal, and the first one was awarded to Minkovsky in 1901). D.Hilbert defended it by declaring: “From his paradise that Cantor with us unfolded, we hold our breath in awe; knowing, we shall not be expelled”. In 1899, Cantor discovered a paradox: what is the cardinal number of the set of all sets? It led Cantor to formulate a concept called ***limitation of size***: the collection of all sets was "too large" to be a set.







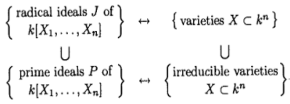
DAVID HILBERT (1862–1943)

THE GERMAN mathematician, physicist, and philosopher David Hilbert had enormous influence on mathematics at the beginning of the 20th century. In geometry, his influence has been likened to that of the ancient Greek mathematician Euclid. His work on axiomatic principles in this field was particularly significant. At the International Mathematical Congress in 1900, in Paris, France, he presented to the delegates 23 unsolved mathematical problems. These became known as Hilbert's problems, and although some have since been solved, others continue to challenge mathematicians. He is also remembered for work on logic and for his later research in physics.

David Hilbert (1862-1943) one of the most influential and universal mathematicians of the 19th and early 20th centuries.

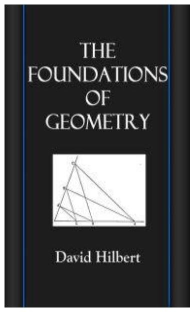
Algebraic Geometry, Theory of Invariants

Nullstellenzats (zero-locus theorem) relates geometry and algebra; 1888 *Hilbert's basis theorem* a finite set of generators for the invariants in any number of variables: existential proof raised controversy (Kronecker, etc.).



$$I(V(J)) = \sqrt{J}$$

Algebraic Number Theory 1897 fundamental treatise *Zahlbericht* ("report on numbers"). Solution to the problem of Waring, Hilbert modular forms, again "existential proof". Conjectures on class field theory, Hilbert class field and of the Hilbert symbol of local class field theory.



Classical Geometry, foundations 1899 *Foundations of Geometry* Hilbert's axioms replaced the traditional axioms of Euclid, along with the modern axiomatic method.

23 Hilbert's Problems in a talk "*The Problems of Mathematics*" on the Second International Congress of Mathematicians in Paris, 1900.

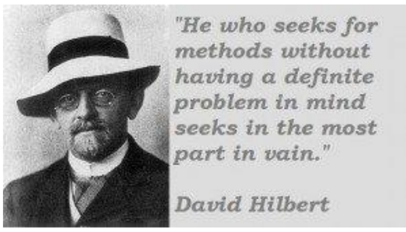
The 23 Mathematical Problems of Hilbert

1. The continuum hypothesis
2. Prove that the axioms of arithmetic are consistent.
3. Given any two polyhedra of equal volume, is it always possible to cut the first into finitely many polyhedral pieces which can be reassembled to yield the second?
4. Construct all metrics where lines are geodesics.
5. Are continuous groups automatically differential groups?
6. Mathematical treatment of the axioms of physics
7. Is a^b transcendental, for algebraic $a \neq 0,1$ and irrational algebraic b?
8. The Riemann hypothesis ("the real part of any non-trivial zero of the Riemann zeta function is ½") and other prime number problems, among them Goldbach's conjecture and the twin prime conjecture
9. Find the most general law of the reciprocity theorem in any algebraic number field.
10. Find an algorithm to determine whether a given polynomial Diophantine equation with integer coefficients has an integer solution.
11. Solving quadratic forms with algebraic numerical coefficients.
12. Extend the Kronecker–Weber theorem on abelian extensions of the rational numbers to any base number field.
13. Solve 7-th degree equation using continuous functions of two parameters.
14. Is the ring of invariants of an algebraic group acting on a polynomial ring always finitely generated?
15. Rigorous foundation of Schubert's enumerative calculus.
16. Describe relative positions of ovals originating from a real algebraic curve and as limit cycles of a polynomial vector field on the plane.
17. Express a nonnegative rational function as quotient of sums of squares.
18. (a) Is there a polyhedron which admits only an anisohedral tiling in three dimensions? (b) What is the densest sphere packing?
19. Are the solutions of regular problems in the calculus of variations always necessarily analytic?
20. Do all variational problems with certain boundary conditions have solutions?
21. Proof of the existence of linear differential equations having a prescribed monodromic group
22. Uniformization of analytic relations by means of automorphic functions
23. Further development of the calculus of variations

Famous Hilbert Talk in 1900

- Presented the 23 problems for research
- Talked about mathematics history
- Showed expectations for the future of mathematics
- Showed the necessity why to solve these problems
- Gave a challenge and hope to mathematician through the 23 problems
- Became the most influential speech in the study of mathematics

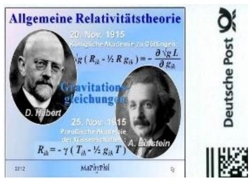
Why give this talk?



$$\langle f, g \rangle = \int_a^b f(x)g(x)\,dx$$

Functional Analysis 1909 study of dif. and integral equations; the concept of Hilbert space.

Math Physics 1912 studying kinetic gas theory, radiation theory, the molecular theory. 1915 general relativity, Einstein received an enthusiastic reception at Göttingen. "*The Foundations of Physics*": axioms of the field equations (Einstein–Hilbert action).

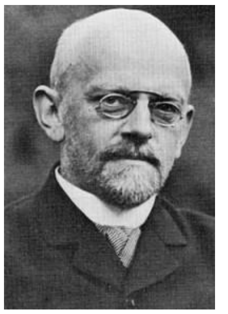


Hilbert's program of axiomatization, Math Logic, formalist school 1920 a research project in *metamathematics*: all of math follows from a correctly chosen finite system of axioms, which is provably consistent. Hilbert's *formalist school* is one of three major recipes to resolve the *crisis of foundations of Math* because of the paradoxes discovered. Hilbert is one of the founders of proof theory and mathematical logic, and one of the first to distinguish between mathematics and metamathematics.

A Brief History: Hilbert's Program

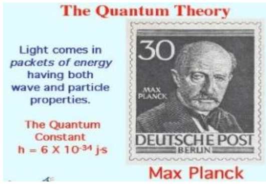
- Hilbert's program (1920's)
  1. Axiomatization of all mathematics
  2. Completeness: all true mathematical statements can be proved in the formalism
  3. Consistency: no contradiction can be obtained in the formalism of mathematics
  4. Decidability: there should be an *algorithm* for deciding the truth or falsity of any mathematical statement

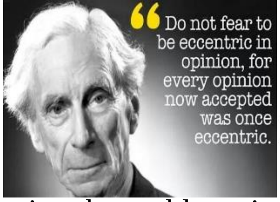
"We must know. We will know".

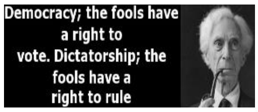


Max Planck
(1858-1947)

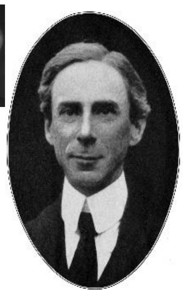
Max Karl Ernst Ludwig Planck, (April 23, 1858 – October 4, 1947) was a German physicist who discovered quantum physics, initiating a revolution in natural science and philosophy. He is regarded as the founder of quantum theory, for which he received the Nobel Prize in Physics in 1918







Bertrand Russell
1872-1970



1901 Russell discovers "Russell's paradox" indicating the problems in naive set theory.

1901 Planck proposes quantum theory.

Henri Leon Lebesgue (1875-1941)

1901 Lebesgue formulates the theory of measure. 1902 the "Lebesgue integral".

1901 Dickson *Linear groups with an exposition of the Galois field theory*.

1902 Beppo Levi states the axiom of choice for the first time.

1902 Gibbs foundational *Elementary Principles of Statistical Mechanics*

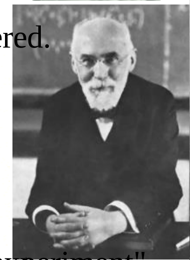
1903 Castelnuovo *Geometria analitica e proiettiva* his main work in algebraic geometry.

Guido Castelnuovo

1904 Zermelo uses the axiom of choice to prove that every set can be well ordered.

1904 Lorentz introduces the "Lorentz transformations".

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$
$$y' = y$$
$$z' = z$$
$$t' = \frac{t - \frac{v}{c^2}\cdot x}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

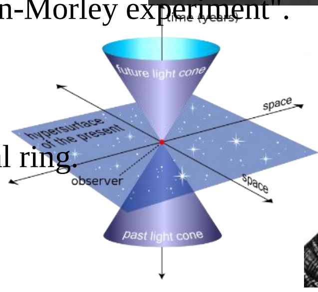


1904 Poincaré proposes the Poincaré Conjecture

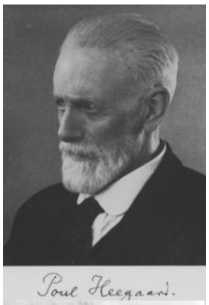
1904 Poincaré a lecture on a theory of relativity, explains "Michelson-Morley experiment".

1905 Einstein publishes the special theory of relativity.

1905 Lasker factorization of ideals into primary ones in a polynomial ring.

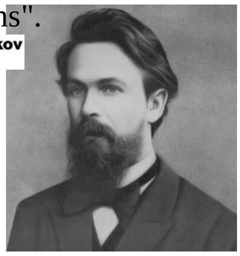


1906 Fréchet formulated the abstract notion of compactness.



1906 Markov studies random processes that are subsequently known as "Markov chains".

Andrei Andreyevich Markov
1856-1922



1906 Bateman applies Laplace transforms to integral equations.

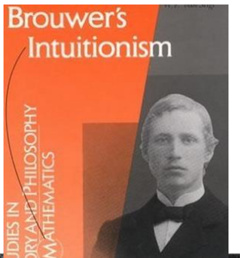
1906 Koch describes a continuous curve of infinite length and nowhere differentiable.

1907 Fréchet integral representation for functionals (proved independently by Riesz).

1907 Einstein gravitational acceleration is indistinguishable from mechanical one.

1907 Heegaard and Dehn *Analysis Situs* marks the origin of combinatorial topology.

1907 Brouwer's attacked foundations of math, beginning of the Intuitionist School.



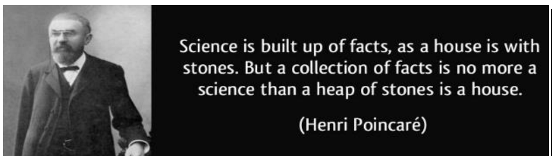
1907 Dehn the word problem and the isomorphism problem for group presentations.

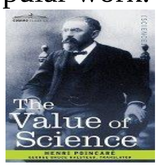
1907 Riesz "Riesz-Fischer theorem" concerning Fourier analysis on Hilbert space.

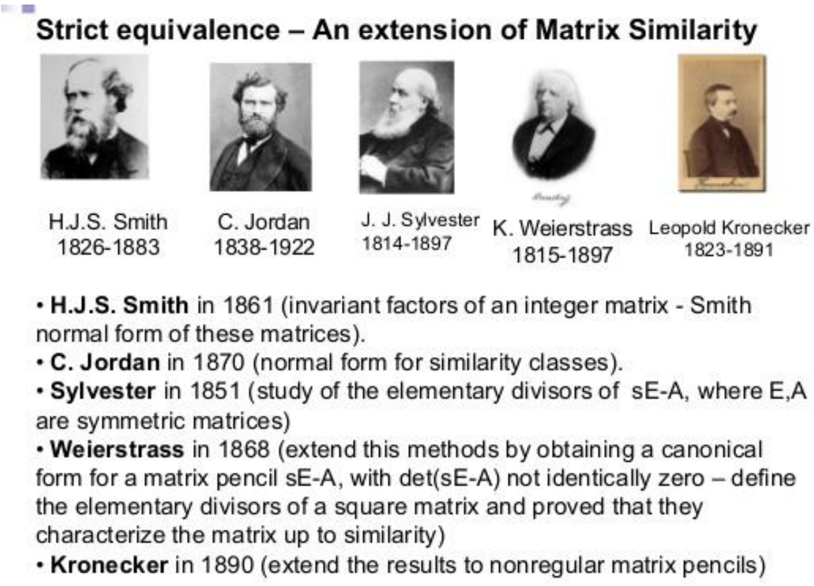
1908 Hardy-Weinberg math basis for population genetics, propagation of genetic traits

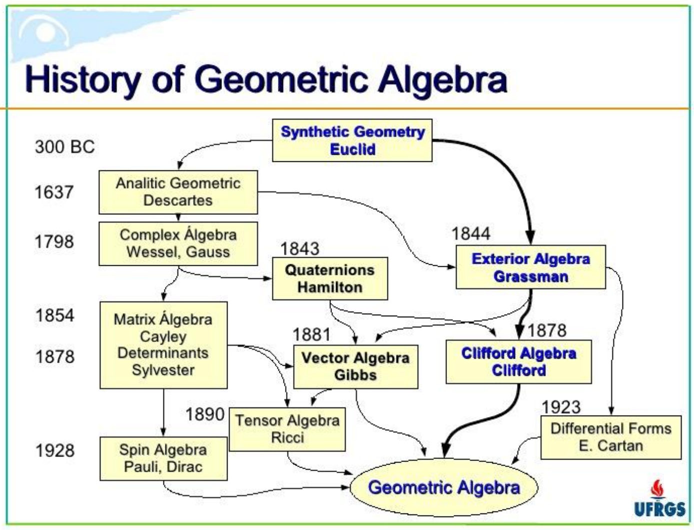
1908 Zermelo bases set theory on seven axioms to overcome the difficulties with set theory

1908 Poincaré *Science and Method*, perhaps his most famous popular work.

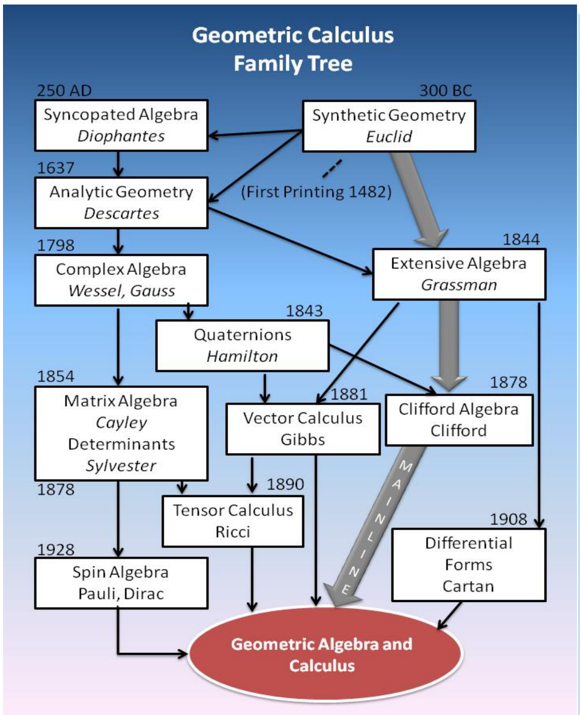


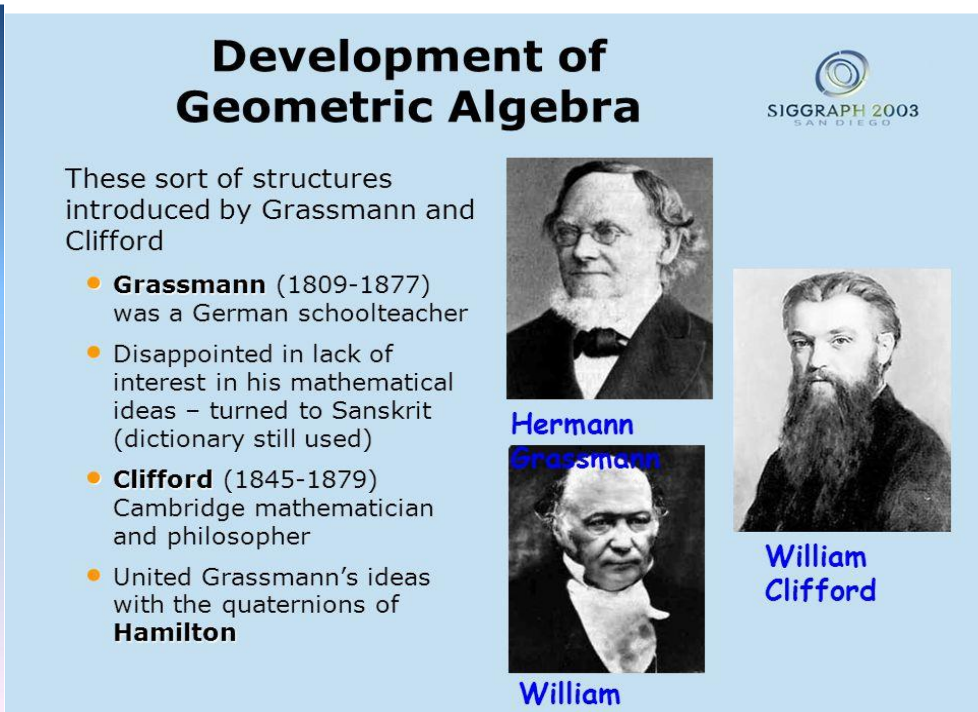






Algebraic Invariant Theory





1880 Dedekind defined rings, but the word "*ring*" was introduced by Hilbert in 1892

The concept and word "*ideal*" was introduced by Dedekind in 1876, it was motivated by "*ideal numbers*" that were defined and studied by Kummer. It was developped later by Hilbert and E.Nöther.

The first attempt for formalize arithmetics was made by Grassmann in 1860s, who introduced the successor operation with the induction axiom and derived the arithmetical properties from them. A set of axioms was proposed in 1881 by Peirece, in 1888 it was improved by Dedekind, and was finally precised by Peano in 1889, so-called Peano axioms, or Dedekind-Peano axioms.

Peano tried to formalize mathematics and worked on an Encyclopedy of Math, "Formulario Project". He introduced and used there various new notations, some of them are in common use now, like symbol ϵ (variation of ε) for inclusion.

The first International Congress of Mathematicians was held in 1897 in Zurich, and the second one in 1900 in Paris (there Hilbert in an invited lecture proposed famous 23 problems for the next era).

John von Neumann) (1903 –1957)

major contributions set theory, functional analysis, quantum mechanics, ergodic theory, continuous geometry, economics and game theory, computer science, numerical analysis, hydrodynamics and statistics, as well as many other mathematical fields.

Regarded as one of the foremost mathematicians of the 20th century

Jean Dieudonné called von Neumann "the last of the great mathematicians."