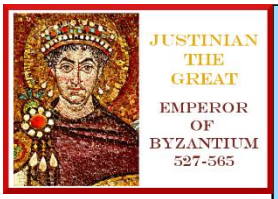
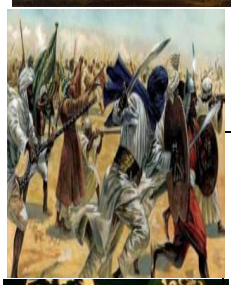
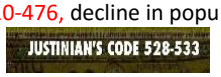




Lecture 1. Education and Math in the Medieval Europe

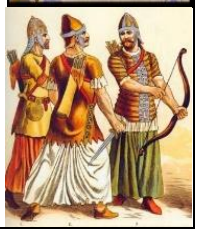


Historical context: ***Early Middle Ages*** or ***Dark Ages*** refer to the period of 5th – 10th centuries

5th century: Fall of the Western Roman Empire in 410-476, decline in population, deurbanization, loss of culture
6th century: Justinian I published Code of Civil Law 

and retakes Rome from Ostrogoths
7th century: Arab army capture new territories
8th century: Carolingian Renaissance: monastic and cathedral schools were established everywhere to train men of civil service, Palace School at Aahen (in Abbasside Caliphate: Islamic Golden Age, "House of Wisdom")

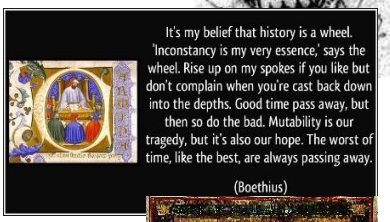
Barbarian invasions: the Viking Age 793-1066, raids of Magyars (Hungarians) 795-1001, invasions of Saracens (Arabs, Berbers, Moors and Turks) first occupation in 8th century, continued in 9th-11th.



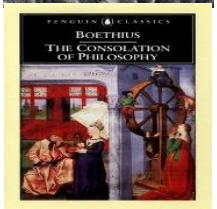
Great Schism 1054 separation between the East Orthodox and the West Catholic Churches

Boethius (480-524) noble Roman, Christian philosopher, "*last of the Romans and the first of Scholastics*"

Magister officiorum (head of government and court services) of Theodoric, king of Italy and of Goths, who later imprisoned and executed Boethius in charges of conspiracy



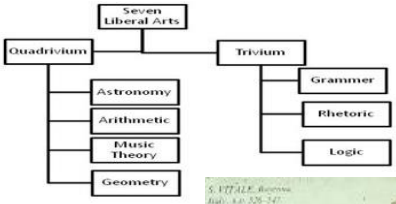
Consolation of Philosophy philosophical treatise composed in jail: on *Weal of fortune*, evil and death, etc., one of the most popular and influential works of the Middle Ages



Translated many books from Greek to Latin: philosophy (Aristotle, Plato), Math ("*Arithmetic*" of Nicomachus, geometry books, etc.) were textbooks for **quadrivium** in the Middle Ages.

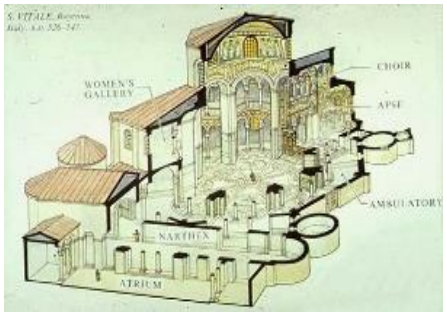
Cassiodorus (485-585) Roman statesman and writer, was *Magister officiorum* after Boethius

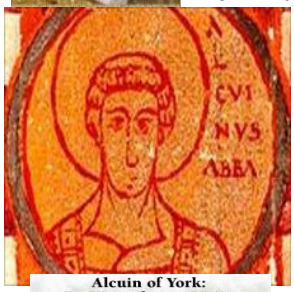
Established a *system of monastery education* (School at Vivarium) based on *7 Liberal Arts*



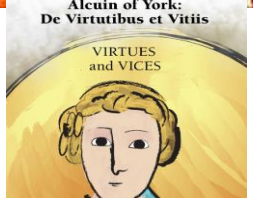
Through Early Middle Ages, schools at some monasteries and cathedrals were the only centers to get a minimal education based on ***Trivium*** and aimed for religious needs. There existed only a few places in Europe to get a higher education based on ***Quadrivium.***

Anthemius of Tralles (474-534) and **Isidorus of Miletus** mathematicians and architects hired by Emperor Justinian to construct a new church of Hagia Sophia at Constantinople in 532-37, after it was destroyed by the Nika Riot. Isidorus taught at the universities of Alexandria and Constantinople, produced the first comprehensive compilation of Archimedes' work. Arthemius studied the focal properties of conics.





Alcuin of York (735-804) made York Cathedral School one of the most important European center of leaning, with the best library in Europe, in 775 writes elementary texts in arithmetic, geometry and astronomy, in 781 appointed head of Charlemagne's Palace School at Aachen, developed Carolingian miniscule that replaced less readable unspaced capital script. Being a personal friend of Charlemagne became a teacher of his two sons.

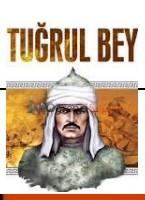




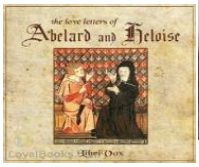


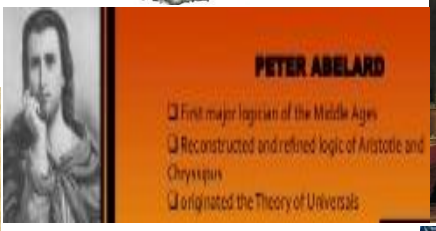
Historical context: ***High Middle Ages*** refer to the period of 11th – 13th centuries: grows of population, developments of town, raise of culture, technology, inventions (windmill, mechanical clock, transparent glass, eyeglasses, 3-field rotation, horse collar, gunpowder).

11th century: Seljuk Empire is founded by Tughril Beg. The First Crusade (of nine): Jerusalem re-taken.
1206 Genghis Khan became Khagan, Mongol Empire, 1299 Osman I establish Ottoman Empire




Peter Abelard 1079–1142 philosopher and *the first major logician* of the Middle Ages "*the keenest thinker and boldest theologian of the 12th Century*". His lectures in Paris were extremely popular and crowded. He was condemned by Church and the king of France forbade to teach on his land, so he was teaching sitting on a tree and then from a boat. His affair with and love for Héloïse d'Argenteuil has become legendary.

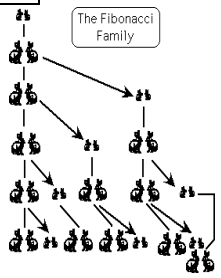





Translation of scientific works from Arabic to Latin
1142 *Adelard of Bath* three translations of Euclid's Elements, *Alchorismi* of Al-Khwārizmī, etc.
1144 *Gherard of Cremona* begins translating Ptolemy's *Almagest* and many other books
1145 *Robert of Chester* translated *Liber algebrae et almucabola*, of Al-Khwārizmī, etc.

Arabic-latin Translators: Herman of Carinthia, Robert of Ketton, Adelard of Bath, Gerard of Cremona, Michael Scot, Arnaldus de Villa Nova

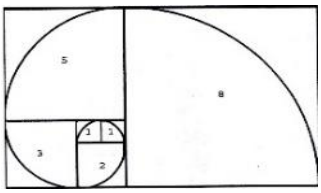
Leonardo Pisano (of Pisa) called Fibonacci (son of Bonacci family) 1170-1250



Liber Abaci (Book of Calculation) 1202: *Hindo-Arabic numerals* Modus Indorum (Method of Indians)

Many arithmetical and algebraic examples: a problem about rabbits leaded to the Fibonacci sequence 1,2,3,5,8,13,… problems about perfect numbers, Chineese remainder theorem, sum of arith/geom series

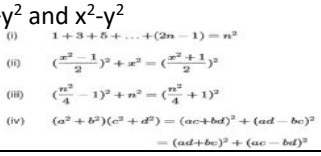
Practica geometriae 1220: geometry problems based on Euclid's Elements



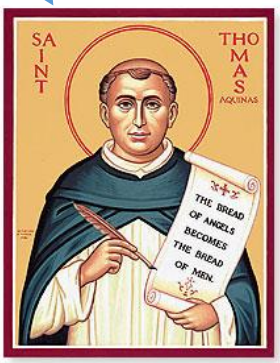
Flos 1220: approximation to a root of $10x+2x^2+x^3=20$ (asked by Johannes of Palermo, problem from Omar Khayyam's book)

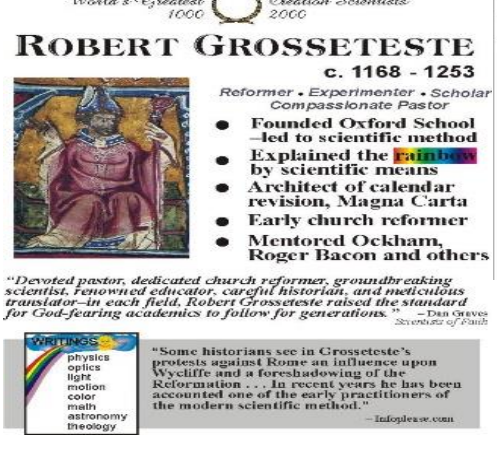
$$1^0\ 22^I\ 7^{II}\ 42^{III}\ 33^{IV}\ 4^V\ 40^{VI}$$
$$\left(1+\frac{22}{60}+\frac{7}{3{,}600}+\frac{42}{216{,}000}+\cdots\right)$$

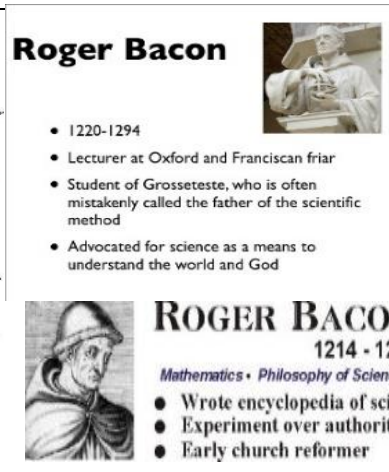
Liber Quadratorium 1225 the most impressive work: 1+3+5+…; on Pythagorean triples; both x^2+y^2 and x^2-y^2 cannot be squares at the same time; x^4-y^4 cannot be a square; $(a^2+b^2)(c^2+d^2)=(ab+cd)^2+(ad-bc)^2$

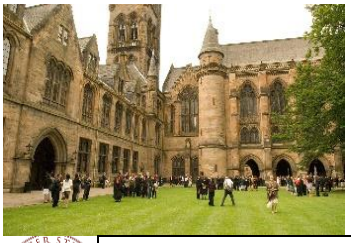


1225 *Jordanus de Nemore (Nemorarius):* **astronomy; in math an early algebraic notation with letters is used**
1230 *John of Holliwood* (*Johannes de Sacrobosco*): arithmetic, astronomy and calendar reform
1260 *Campanus of Navarra* (chaplain to Pope): astronomy, Latin edition of Euclid's *Elements* that became standard for 200 years
1274 *Thomas Aquinas'* work "Summa Theologica" is published







Universities "*universitas magistorum et scholarium*" (guild of professors and students) self-governed bodies, evolved from cathedrals and monastic schools and having "academic freedoms", legal rights in towns granted by a special decrees of kings

1088 University of Bologna, specialization in Law, paid and governed by students, who were usually of a senior age
1117 University of Oxford: paid by the king, grew since 1167, after Henry II banned English students from attending the University of Paris,
1150 University of Paris (Sorbonne): paid by Church, governed by teachers, students start at age 13-14; model for other Universities
1209 University of Cambridge formed by academic who escaped from Oxford after a quarrel with the town administration

Lecture 2. Culture and Math in 14th-15th centuries

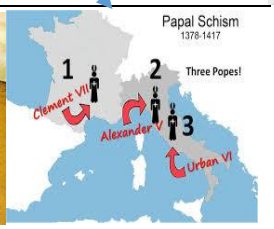


Late Middle Ages 1300-1450: many disasters, population of France dropped 50% in 1300-1350, recovered till 1450
Climate change: *Medieval Warm Period* (950-1250) transformed into the ***Little Ice Age*** (1350-1850)
1315-1317 ***Great Famine***: heavy rains and cool at summer, no crops, no bread (available "only for kings")
1337-1453 ***100 Years' War*** in France; ***popular revolts*** in Flanders (1323-28), France (1356-58), England (1381)
1347-1350 Plague ***Black Death*** killed 35 million (1/3 of population) in Europe, epidemy repeated every 10 years
Religious Wars: *Reformation*, ***Great Papal Schism*** 1378-1418 (three rival Popes, each supported by some states)



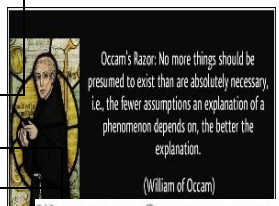
Petrarca


Giotto
Dante



Italian Renaissance started as a cultural movement in Florence in 14th century

Dante Alighieri (1265–1321) poet, the author of *Divine Comedy*, "the Father of the Italian language".
Francesco Petrarca (1304–1374) scholar and poet, a model for lyrical poetry and "Italian style", "Father of Humanism"
Giovanni Boccaccio (1313–1375) writer, poet, humanist, the author of *The Decameron* and *On Famous Women*
Giotto di Bondone (1267–1337) painter and architect, broke with "Byzantine style" and initiated "Renaissance style"



Math Ideas in 14th century

1321 Levi ben Gerson (Gersonides): *Book of Numbers* on arithmetical operations, permutations and combinations, 1342 *De sinibus, chordis et arcubus* (*On Sines, Chords and Arcs*): proves the sine theorem
~~1364~~ *Nicole Oresme Latitudes of Forms* an early version of coordinate system that may be influences Descartes

Nicole Oresme (1320-1382) *most original thinker* of XIV: philosopher, economist, astronomer, mechanic and math

Introduced an idea of graph of function, named coordinates *lattitudo* and *longitude*; for the graph of speed as function of time realized that the distance is the area; introduced an average speed.

Proved divergence of a harmonic series; used fractional powers of numbers.

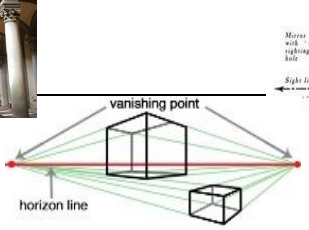
Discussed possibility of rotation of Earth; incommensurability of lengths of day/month/year in a book of 1382 *Le Livre du ciel et du monde* (*The Book of Heaven and Earth*) he translated it from Latin to French by request of king Charles V


Leonardo


Botticelli


Lorenzo de' Medici

Quattrocento: Italian Renaissance of 15th cent. under patronage of Lorenzo de Medici 1449–1492, Lord of Florence
Leonardo da Vinci 1452–1519 polymath, a greatest painter, "Universal Genius", "Renaissance Man"
Sandro Botticelli, 1445–1510 painter



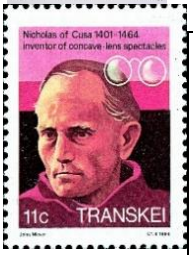
Theory of linear perspective representation of 3-dimensional objects based on existence of a single vanishing point to which all parallel lines in a plane, other than the plane of the canvas, converge. Understanding of the relation between the actual length of an object and its length in the picture depending on its distance behind the plane of the canvas.

1415 Florentine artist and architect Filippo Brunelleschi (1377–1446) discovered the laws of perspective;

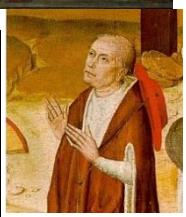
1434 the first general treatise *Della Pictura* on the *laws of perspective* by Florentine architect Leone Alberti (1404–1472) who worked also on maps (geometrical mapping) and had a book on *cryptography*.

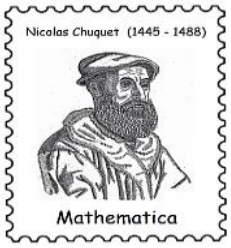
Piero della Francesca (1420-1492) Florentine artist, among "many math books" the survived ones: *Abacus treatise*, *Short book on the five regular solids* and *On perspective for painting*.

Piero della Francesca



1450 Nicholas of Cusa studies geometry and logic. He studied the infinitely large and the infinitely small, considered the circle as the limit of regular polygons. Claimed the orbits of planets to be elliptic.





1470 Chuquet writes *Triparty en la science des nombres*, the first French algebra book. Zero and negative exponents are considered. Rationals/irrationals are discussed. Theory of equations: allowed negative coefficients. Introduced words "billion", "trillion", "quadrillion".

Chuquet wrote an important text Triparty en la science des nombres. This is the earliest French algebra book.

The Triparty en la science des nombres (1484) covers arithmetic and algebra. It was not printed however until 1880 so was of little influence. The first part deals with arithmetic and includes work on fractions, progressions, perfect numbers, proportion etc. In this work negative numbers, used as coefficients, exponents and solutions, appear for the first time. Zero is used and his rules for arithmetical operations includes zero and negative numbers. He also uses $x0 = 1$ for any number x.
The sections on equations cover quadratic equations where he discusses two solutions.

1472 Peurbach publishes *Theoricae Novae Planetarum* (*New Theory of the Planets*). He uses Ptolemy's epicycle theory of the planets but believes they are controlled by the sun.

1474 Regiomontanus (Johannes Muller) *Ephemerides*, astronomical tables for the years 1475 to 1506 AD, and proposes a method for calculating longitude by using the moon. 1475 *De triangulis planis et sphaericis* (*Concerning Plane and Spherical Triangles*), studies spherical trigonometry to apply it to astronomy.

Johannes Gutenberg

Invention of printing: 1439, Gutenberg's Bible was printed in 1455

1482 the first mathematics book printed: Campanus of Novara's edition of Euclid's *Elements*.

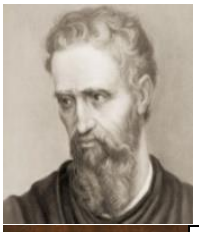
Mathematical Symbols "+", "-" and "=": 1202 Fibonacci: "plus", "minus", 1556 Tartalia: p., m.

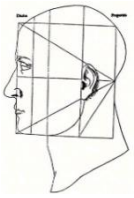
Sign + was first used in 1360 by N.Oresme, in 1489 J.Widman writes an arithmetic book in German which contains both signs + and - . They were also used in 1514 by V.Hoecke, and in 1542 by R.Recorde, who also used sign "=" in 1557. In 1631 William Oughtred (1575-1660) used symbols "×", "/", "||", sin, cos, and introduced logarithmic scale.

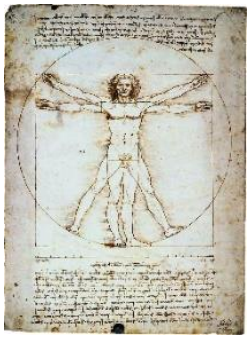
In 1484 Chucke: used $R^2.14.\overline{p.}\,R^2.180$ for $\sqrt{14+\sqrt{180}}$

In 1646 Vieta, used $B \text{ in } \overline{D \text{ quad.}} + B \text{ in } D$ for $B(D^2 + BD)$.

+ / -	plus / minus	Johannes Widmann (1489) / Luca Pacioli (1494) / Giel Vander Hoecke (1514)
x	multiplication	William Oughtred (1631 or 1618)
. (dot)	multiplication	Gottfried Leibniz (1698) / Johann Bernoulli (1694)
÷	division	Johann Rahn (1659) / John Pell (1668)
=	equals	Robert Recorde (1557) / William Oughtred (1618)
< / >	less than / greater than	Thomas Harriot (1631)
x^2	exponent	Nicolas Chuquet (1484) / Pierre Hérigone (1634) / René Descartes (1637)
$\sqrt{}$ $\sqrt[3]{}$	radical (root)	Christoff Rudolff (1525) / Albert Girard (1629) / René Descartes (1637)
$\frac{a}{b}$	fraction bar	Fibonacci (1202)
.	decimal point	Francesco Pellos (1492) / G.A. Magini (1592) / Bartholomaeus Pitiscus (1595) / John Napier (1617)







Cultural context: High Renaissance 1490-1527 (since death of Lorenzo Medici till sack of Rome by Charles V)

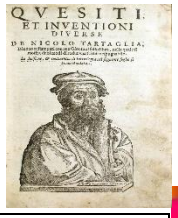
Late works of Leonardo da Vinci and Boticelli
Michelangelo Buonarroti 1475–1564 sculptor, painter, architect, poet, and engineer; "Renaissance man"
Raphael 1483–1520 painter and architect

1492 Christopher Columbus discovers "West Indies", 1499-1500 Amerigo Vespucci, 1519 Ferdinand Magellan

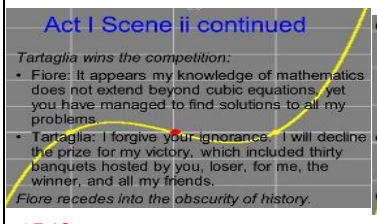
1494 Luca Pacioli (1445-1509) Franciscan friar, publishes *Summa de arithmetica, geometria, proportioni et proportionalita* which is a review of the whole of mathematics covering arithmetic, trigonometry, algebra, tables of moneys, weights and measures, games of chance, *double-entry book-keeping* and a summary of Euclid's geometry. Another book of Pacioli *De divina proportione* completed in 1496-98 was illustrated by his friend, Leonardo da Vinci. Pacioli was accused in plagiarism, because the content was mainly taken from the works of Piero della Francesca, without giving credits.

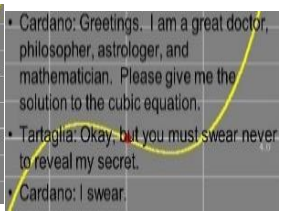
Lecture 3. Math in 16th century

Algebra: solution of the cubic equation



1515 Del Ferro 1465-1626 discovered and kept in secret a formula to solve cubic equations.
1535 Tartaglia contest with Fiore, solved the cubic equation independently, told to Cardano.

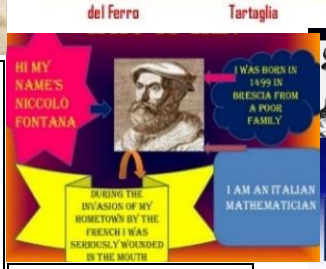


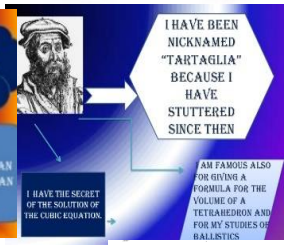


I promise never to tell

When Gerolamo Cardano (1501-1576) of Milan heard of Tartaglia's feat, he began a concerted effort to convince Tartaglia to show him the method.

Finally, on March 25, 1539, after Cardano swore a solemn oath never to publish the method, Tartaglia gave him the method, encrypted in a cipher.





Cardano's inventions: combination lock, shaft (joint), gimbal (for compass or gyroscope)

1540 Ferrari 1522-1565 secretary/student of Cardano, solved quartic equations.

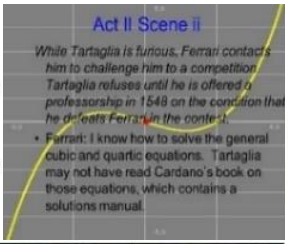
1545 Cardano 1501-1576 in *Ars Magna* gave the formula that solves any cubic equation based on Tartaglia's work and the formula for quartics discovered by Ferrari.

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

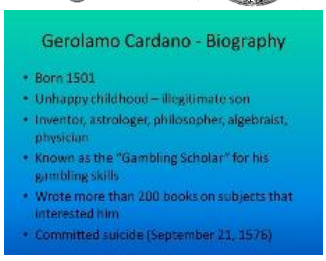
I didn't publish *your* secret ...

The *Ars Magna* showed how to solve
- the depressed cubic
- the general cubic (by first depressing it)
- another important equation (coming soon)

Tartaglia was not happy.
Remember, Cardano swore a solemn oath.
Cardano said: "Hey, I didn't publish your method; I published del Ferro's. Ok, ok, there's similarities, but I kept my word."



Girolamo Cardano
(1501-1576)



1572 Bombelli 1526-1572 in *Algebra* gave the rules for calculating with complex numbers. Diophantus problems.

Bombelli

4.7.1. RAFAEL BOBBELLI (1526-1572)

•Rafael Bombelli (baptised on 20 January 1526; died 1572[a]) was an Italian mathematician.
•Born in Bologna, he is the author of a treatise on algebra and is a central figure in the understanding of imaginary numbers.
•He was the one who finally managed to address the problem with imaginary numbers. In his 1572 book, L'Algebra, Bombelli solved equations using the method of del Ferro/Tartaglia. He introduced the rhetoric that preceded the representative symbols +i and -i and described how they both worked.

François Viète
1540-1603

1591 Viète writes *In artem analyticam isagoge* (*Introduction to the analytical art*), using letters as symbols for quantities, both known and unknown. He uses vowels for the unknowns and consonants for known quantities. Descartes, later, introduces the use of letters *x*, *y* ... at the end of the alphabet for unknowns.

$$\frac{2}{\pi} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2+\sqrt{2}}}{2} \cdot \frac{\sqrt{2+\sqrt{2+\sqrt{2}}}}{2} \cdots$$

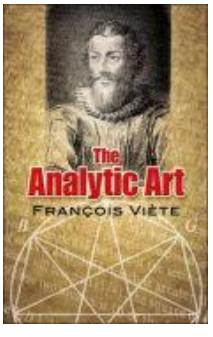
$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0 = 0$$

$$a_n \neq 0$$

$$x_1 + x_2 + x_3 + \ldots + x_n = \frac{a_{n-1}}{a_n}$$

$$x_1 \cdot x_2 + x_1 \cdot x_3 + \ldots + x_{n-1} \cdot x_n = \frac{a_{n-2}}{a_n}$$

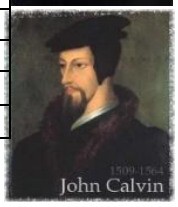
$$x_1 \cdot x_2 \cdot x_3 \cdot \ldots \cdot x_n = (-1)^n \frac{a_0}{a_n}$$





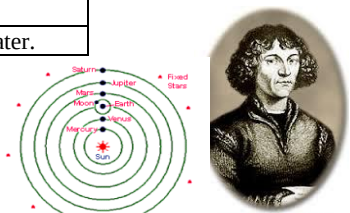




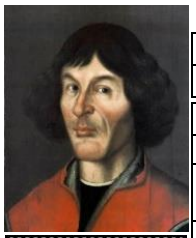
Cultural context: Renaissance in Europe (beyond Italy) started in 16th century
Hieronymus Bosch 1450-1516 and **Pieter Bruegel** the Elder 1525–1569: Netherlandish painters
Erasmus of Rotterdam 1466–1536 Dutch Renaissance humanist, Catholic priest, social critic, teacher, and theologian
Albrecht Dürer 1471–1528: painter, printmaker and theorist of the German Renaissance
Thomas More, 1478–1535 English lawyer, social philosopher, writer, statesman and humanist, in 1516 *Utopia* is published
Protestant Reformation: 1517 Martin Luther *The Ninety-Five Theses*, 1518 Zwingli, 1533 Calvin, 1547 Church of England
Counter-Reformation: *Spanish Inquisition* since 1478, in 1572 *St. Bartholomew's Day massacre*, since 1540 Jesuit order

Arithmetic, Number theory and discrete Math
1536 Hudalrichus Regius finds the fifth perfect number (previously known were 6, 28, 496 and 8128). The number $2^{12}(2^{13} - 1) = 33550336$ is the first perfect number to be discovered since ancient times. 1544 Stifel publishes *Arithmetica integra* which contains binomial coefficients and the notation +, -, √.
1555 J. Scheybl gives the sixth perfect number $2^{16}(2^{17} - 1) = 8589869056$ but his work remains unknown until 1977.
1563 G.Cardan writes his book *Liber de Ludo Aleae* on games of chance but it would not be published until 1663.
1575 Maurolico publishes *Arithmeticorum libri duo* which contains examples of inductive proofs.
1603 P.Cataldi found the 6th and 7th perfect numbers, with p=17,19. The next one was found only by Euler 130 years later.



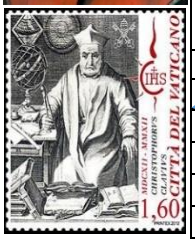
Copernicus' Universe

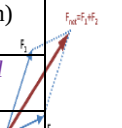
Nicolaus Copernicus (1473-1543)



Geometry
1551 Recorde translates and abridges Euclid's *Elements* as *The Pathewaie to Knowledge*.

Application of Math in Astronomy and mechanics
1541 Rheticus publishes his trigonometric tables and the trigonometrical parts of Copernicus's work.
1543 Copernicus publishes *De revolutionibus orbium coelestium* (*On the Revolutions of the Celestial Spheres*): Sun (not the Earth) is at rest in the centre of the Universe. *Copernican Revolution* was the beginning of *Scientific Revolution* of 15th-16th centuries.
1571 Viète 1540-1603 *Canon Mathematicus* a math introduction to his astronomy treatise: trigonometry with tables. In *Analytical Art* invented word "coefficient" and notation: variables-consonants and coefficients vowels, he knew the binomial formula.
1586 Stevin publishes *De Beghinselen der Weeghconst* containing the theorem of the triangle (parallelogram) of forces.
1595 Clavius writes *Novi calendarii romani apologia* that helped Pope Gregory XIII to introduce what is now called the Gregorian calendar.





Real Numbers
1585 Stevin publishes *De Thiende* in which he presents an elementary and thorough account of decimal fractions.
1590 Cataldi uses continued fractions in finding square roots;
1593 Van Roomen calculates π to 16 decimal places.

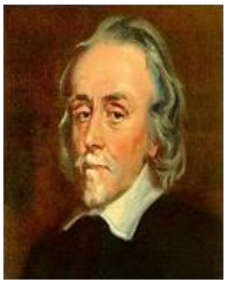
$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \ddots}}}}}$$

Scientific Revolution of 15th-16th centuries (after Copernicus)

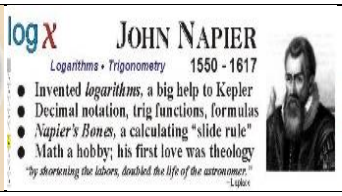
Galileo Galilei 1564-1642 "father of science": astronomer, physicist, engineer, philosopher, and mathematician, famous for defending heliocentrism, "the universe … is written in language of math", considered coordinate system and parabola as the ideal trajectory 1589 falling unequal-weight-balls from Leaning Tower of Pisa; 1610 telescope observations in *Sidereus Nuncius* (*Message from the stars*): Jupiter satellites, Venus phases, rings of Saturn, Sun spots, Milky Way; 1623 writes about Copernicus, 1632 new book

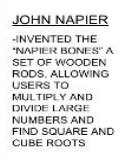
1600 William Gilbert "*De Magnete*" "Father of electricity and magnetism"
Francis Bacon 1561–1626 philosopher, statesman, scientist, jurist, orator, writer, "father of *empiricism*" (method of scientific revolution), inductive methodology, improvement of mankind, reforms in Law, 1627 "New Atlantis" utopian novel, knighted in 1603
1609 Johannes Kepler 1571–1630: *Astronomia nova* (*New Astronomy*); the orbits are elliptic, word "focus", 1596 3 laws of planetary motion, 5 platonic solids and distances
1628 William Harvey: circulary rout of blood
1661 Robert Boyle: *"Sceptical Chymist"* principles of modern chemistry
1665 Robert Hooke: *"Micrographia"* microscopic observation of cellular structure, word "cell" is introduced; 1660 law of elasticity



Lecture 4. Math in the 17th century





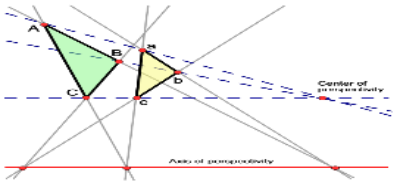
John Napier 1550-1617: invention of logarithms in 1614, Table of Logarithms; 1624: 14-digit log table

Thomas Hariote 1560-1621: recognized negative and imaginary roots, (x-a)(x-b)… has no other roots

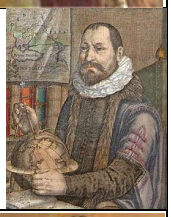
Marin Mersenne 1588-1648: theologian, philosopher, mathematician, music theorist, "father of acoustics".[1] Mersenne, "the center of the world of science and mathematics during the first half of the 1600s." In 1635 he set up the informal *Académie Parisienne* which had nearly 140 correspondents including astronomers, philosophers and mathematicians and was the precursor of the Académie des sciences established by Colbert in 1666; numbers $M_n = 2^n - 1$ are called Mersenne numbers: Mersenne primes (M_2=3, M_3=7, M_5=31, M_7 =127, but not M_{11}=2047=23×89) are related to the perfect numbers.

$$2^{ab}-1=(2^a-1)\cdot\left(1+2^a+2^{2a}+2^{3a}+\cdots+2^{(b-1)a}\right)$$
$$=(2^b-1)\cdot\left(1+2^b+2^{2b}+2^{3b}+\cdots+2^{(a-1)b}\right).$$

Girard Desargues 1591–1661: mathematician and engineer, a founders of projective geometry. Desargues' theorem



Albert Girard 1595-1632: "A new discovery of Algebra" 1629 the Fund. Theorem of Algebra, symmetric Functions, sum of powers of roots: A, A2-2B, A3-3AB+3C, etc., stated "sum of two squares" theorem of Fermat, 1626 abbreviations sin, cos, tan; the area of a spherical triangle.



Bonaventura Cavalieri 1598–1647: optics, motion, Cavalieri's principle in geometry anticipated infinitesimal and integral calculus calculus, introduced logarithms in Italy

Evangelista Toricelli 1608–1647: physicist and mathematician, inventor of the barometer, "method of Indivisibles"

Henry Oldenburg 1619-1677: German theologian, diplomat, a natural philosopher, the creator of scientific peer review, one of the foremost intelligencers of Europe of 17th century, created the first scientific journal in 1665 " Philosophical Transactions of the Royal Society"

John Wallis 1616- 1703 Math: infinitesimal calculus, chief cryptographer for Parliament and the Royal court, introduced the symbol ∞ for infinity and 1/∞ for an infinitesimal.

Christiaan Huygens 1629-1695: Dutch mathematician, astronomer, physicist (mechanics, optics), probabilist and horologist

Isaac Barrow 1630-1677: teacher of Newton, infinitesimal calculus, fundamental theorem of calculus (slope/area)

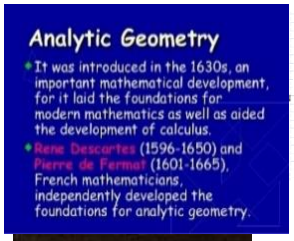
Robert Hooke 1635-1703 natural philosopher, architect and polymath, "England's Leonardo"; Law of elasticity,, etc.

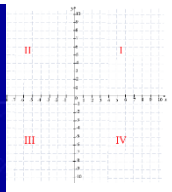
Isaac Barrow

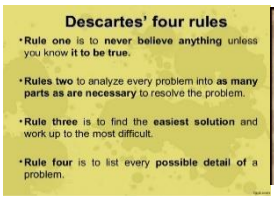
$$\frac{\pi}{2}=\frac{2}{1}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{4}{5}\cdot\frac{6}{5}\cdot\frac{6}{7}\cdots$$

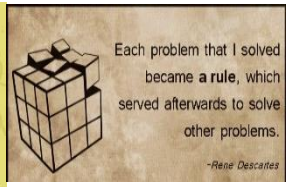
Math Subjects

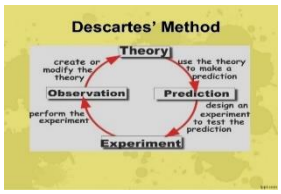
Analytic Geometry 1637 Descartes *La Geometrie* and Pierre de Fermat Synthetic Geometry Pascal and Desargues
Probability Theory: Cardan's "Liber de Ludo Aleae", letters of Pascal to Fermat
Number Theory: Mersenne, Fermat, Early Calculus: Fermat, Cavalieri
Calculus 1687 Newton (1642-1724) *Principia Mathematica* and Gottfried Leibnitz (1646-1716)
Symbolic Algebra, Symbolic Logic: Leibnitz (in modern form: George Boole 200 year later)











$$f(x) = 6x^5 - 6x^3 + 10x + 5$$

$$f(-x) = 6(-x)^5 - 6(-x)^3 + 10(-x) + 5$$

$$f(-x) = -6x^5 + 6x^3 - 10x + 5$$

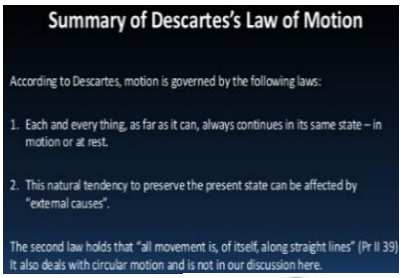
− + − + signs change three times

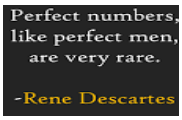
1 2 3

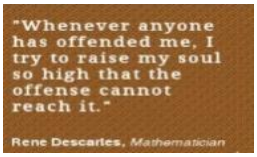
this means there are 3 or 1 possible negative real solutions

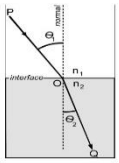
Lecture 5. Descartes, Fermat and Pascal

René Descartes (Renatus Cartesius, adj. "Cartesian") 1596-1650: philosopher, mathematician, and scientist, "the father of modern western philosophy", founder of rationalism, " the father of analytical geometry", giving a path to discovery of Calculus, one of the key figures in the scientific revolution. He spent about 20 years of his life in the Dutch Republic, died right after invitation to Sweeden. In 1633, Galileo was condemned by the Catholic Church, and Descartes abandoned plans to publish *Treatise on the World*, his work of the previous four years. In 1637 he published only a part of this work in three essays: *Les Météores* (The Meteors), *La Dioptrique* (Dioptrics) and *La Géométrie* (Geometry), preceded by an introduction, his famous *Discours de la méthode* (Discourse on the Method). *Principles of Philosophy:* published in *1647*.

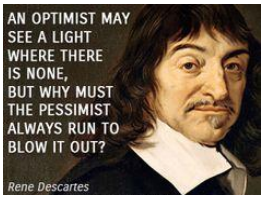


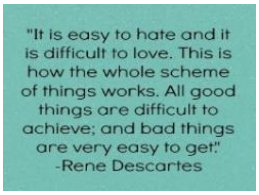






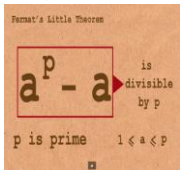
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$



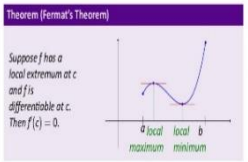


Pierre de Fermat 1601-1665: lawyer at the *Parliament* of Toulouse, France, and a mathematician: Analytic Geometry, Calculus, Number theory, Probability, Optics. Fermat's manuscript in analytic geometry ("*Introduction to Plane and Solid Loci*") published in 1679 was circulated already in 1636, one year before the publication of Descartes' *La géométrie*. Developed Calculus: maxima and minima of functions, tangents to curves, area, center of mass, least action, etc. He evaluated the integral of power functions using an ingenious trick, reducing it to the sum of geometric series (used later by Newton, and then Leibniz for the fundamental theorem of calculus).

Pierre de Fermat

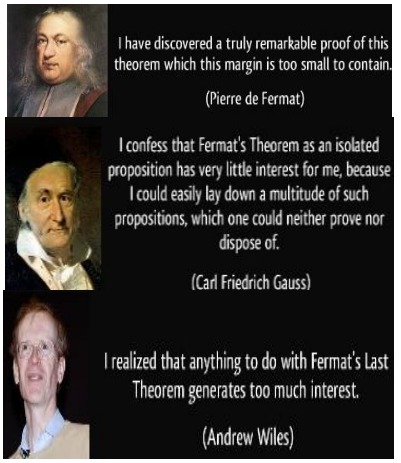


Fermat's Theorem



In number theory, Fermat studied Pell's equation, perfect numbers, amicable numbers and what would later become Fermat numbers. While researching perfect numbers he discovered the little theorem. He invented a factorization method—Fermat's factorization method—as well as the proof technique of infinite descent, which he used to prove Fermat's right triangle theorem which includes as a corollary Fermat's Last Theorem for the case $n = 4$. Fermat developed the two-square theorem, and the polygonal number theorem, which states that each number is a sum of three triangular numbers, four square numbers, five pentagonal numbers, etc.

$$x^n + y^n \neq z^n \quad n > 2$$



Fermat Numbers

Pierre de Fermat (1601 – 1665) conjectured that each number of the form $2^{2^n}+1$, now referred to as a ***Fermat number***, was prime for each natural number n.

Fermat's theorem on sums of two squares states that any odd prime number will be the sum of two square numbers if, and only if, it leaves a remainder of 1 when divided by 4.

For example:

13 = 9 + 4

= 3^2 + 2^2

This works however big the prime number is:

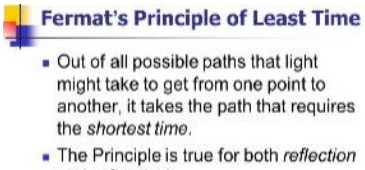
$5 = 1^2 + 2^2$ $269 = 10^2 + 13^2$

$29 = 2^2 + 5^2$ $79{,}601 = 200^2 + 199^2$

$41 = 4^2 + 5^2$ $2{,}369{,}929 = 1{,}100^2 + 1{,}077^2$

$89 = 5^2 + 8^2$ $201{,}743{,}929 = 10{,}035^2 + 10{,}052^2$

In 1654, Fermat and Pascal laied foundations of probability theory. Fermat made the first ever rigorous probability calculation to answer a question he was asked by a professional gambler: why if he bet on rolling at least one six in four throws of a die he won in the long term, whereas betting on throwing at least one double-six in 24 throws of two dice resulted in his losing.



Fermat's principle of least time or Fermat's principle (enunciated by Hero of Alexandria in the 1st century CE) leaded to the principle of least action in physics.

Blaise Pascal 1623-1662 mathematician, physicist, inventor, writer, philosopher. Study of fluids: the concepts of pressure and vacuum clarified after the work of Torricelli (many disputes before acceptance). Treatise on projective geometry at the age of 16. Probability theory: joint with Fermat (in letters). Economic and social sciences. At age 19 constructed a mechanical calculator capable of addition and subtraction, called Pascal's calculator or the Pascaline.

1654: works on philosophy and theology, *Lettres provinciales* and the *Pensées*, Math: on the arithmetical triangle, and in 1658-1659 on the cycloid and its use in calculating the volume of solids.Pascal had poor health and died right after his 39th birthday.

1
1 1
1 2 1
1 3 3 1
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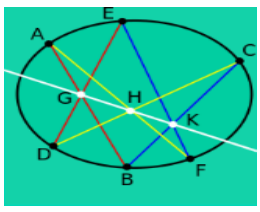
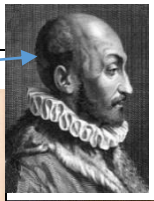


Agendas of meetings included
- Reports: oral or written
- Discussions
- Experiments: Robert Hooke first Demonstrator of Experiments

Lecture 6. End of 17th century.

The first Academies of Science are established



1560-1578 Academia Secretorum Naturae *(Accademia dei Segreti)* founded in Naples by Giambattista della Porta, a noted polymath. The society met at his home. Closed by the Inquisition.

1603 Accademia dei Lincei founded in Rome by Federico Cesi, the son of the Duke of Acquasparta (at the age of 18 with 3 other friends, 8 years older). Later in 1611 Galileo joined. After death of Cesi in 1630 Academy survived only till 1651, but in the 1870s was revived to become the national academy of Italy.

1662 The Royal Society of London granted with a Royal Charter; this Society appeared from "The Philosophical Society of Oxford" that existed since 1640s, inspired by ideas of Fransis Bacon's "New Atlantis". Lord Brouncker becomes its first President, H.Oldenburg secretary and R.Hooke Curator of Experiments.

1666 French Académie des Sciences founded by Louis XIV (suggestion of Colbert). It arose from Mersenne's group of 1630-40s, Montmor's salon (including Desargues, Gassendi and Huygens), "Académie Montmor" 1657-1664. Since 1699 "Royal", issue annual proceedings. A system of prizes since 1721 had a major impact on math and other sciences.

Progress in Math (except Calculus)

1653 Pascal, *Treatise on the Arithmetical Triangle* on "Pascal's triangle". 1654 hydrostatics, Pascal's law of pressure.
1654 Fermat and Pascal, five letters on probability, math expectation. In 1657 summary by Huygens, in 1671 by De Witt.
1655 Brouncker, continued fraction expansion of $^4/_\pi$.
1656 Huygens patents the first pendulum clock. 1660 Hooke the law of elasticity. 1660 Viviani measures the velocity of sound.
1657 Frenicle de Bessy *Solutio duorm problematum* ... solved some of Fermat's challenges.
1661 Van Schooten publishes the final volume of *Geometria a Renato Des Cartes*, on analytic geometry, with appendices by three of his disciples, de Witt, Hudde, and Heuraet.
1662 Graunt and Petty, "*Natural and Political Observations ...*", one of the first statistics books.
1668 Pell: a table of factors of all integers up to 100000.
1669 Wren: a hyperboloid of revolution is a ruled surface.
1672 Mohr: all Euclidean constructions can be carried out with compasses alone.
1678 Giovanni Ceva "Ceva's theorem"
1673 Leibniz demonstrates his arithmometer to the Royal Society. 1679: binary arithmetic (published in 1701)
1683 Seki Kowa: determinants, integer solutions of $ax - by = 1$ for integer a, b.

Progress in Calculus and its applications (including geometric analysis)
1651 Nicolaus Mercator gives the series expansion of $\log(1 + x)$.
1656 Wallis *Arithmetica infinitorum:* interpolation methods to evaluate integrals.
1657 Neile found the arc length of the cubical parabola $y^3 = a\,x^2$. 1658 Wren finds the length of an arc of the cycloid.
1657 Jan Hudde's rule about double roots
1660 De Sluze: spirals, inflection points, geometric means, curves called the "pearls of Sluze".
1663-69 Barrow is the first Lucasian Prof. of Math at Cambridge. In 1669 Newton is appointed.
1665 Newton: binomial theorem; working on the calculus after Barrow published *Lectiones Geometricae* in 1670
1667 James Gregory foundations for the infinitesimal geometry. 1668 the first calculus textbook.
1669 Wallis *Mechanica*: a math study of mechanics. In 1685 Newton's binomial theorem; popularize Harriot.
1671 James Gregory discovers Taylor's Theorem, his series expansion for $\arctan(x)$ gives a series for $^\pi/_4$.
1672 Mengoli: infinite series, an infinite product expansion for $\pi/2$.
1673 Huygens: evolutes and involutes of curves, in particular, for the cycloid and the parabola.
1675 Leibniz modern notation for an integral, 1676-77 differential rules: for producs, quotients and compositions
1684 Leibniz differential calculus in *New methods on maxima, minima and tangents*
1687 Newton, *The Mathematical Principles of Natural Philosophy:* *motion, gravity, orbits, of comets, the tides, etc.*

1690 Jacob Bernoulli word "integral" for the area under a curve, 1691 “polar coordinates”, 1692 Leibniz: "coordinate".
1690 Rolle, the theory of equations, 1691 Rolle's theorem (a method due to Hudde).
1694 Johann Bernoulli "L'Hôpital's rule", 1696 the problem of the brachristochrone.

James Gregory (1638-1675) infinite series representations trigonometric functions and their inverse, like arctan $x = x - x^3/3 + x^5/5 - x^7/7 + ...$

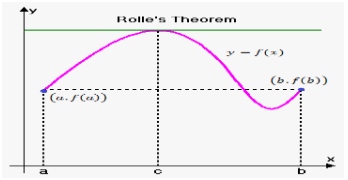
1668 *Geometriae Pars Universalis* the first published statement and proof of the fundamental theorem of the calculus (stated from a geometric point of view, and only for a special class of the curves considered by later versions of the theorem), acknowledged by Isaac Barrow

Johannes Hudde (1628-1704) mayor of Amsterdam between 1672 – 1703, a mathematician and governor of the Dutch East India Company

$$a_0b_0x^n + a_1b_1x^{n-1} + \cdots + a_{n-1}b_{n-1}x + a_nb_n = 0.$$

Newton and Leibniz mention Hudde many times and used some of his ideas in their own work on infinitesimal calculus. Hudde’s rule is a criterion for double roots.

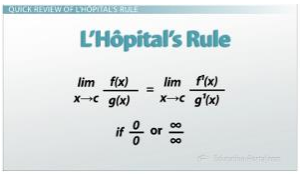
Michel Rolle (1652-1719) French mathematician best known for Rolle's theorem (1691); invented notation $\sqrt[n]{x}$ and introduced in France sign “=”; also the co-inventor in Europe of Gaussian elimination (1690).



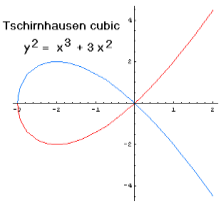
- wanted to be a military man, but poor eyesight forced him into math
- did some math on his own (solved the “brachistocrone problem”)
- paid a stipend to Johann Bernoulli, who proved this theorem and named it after him!

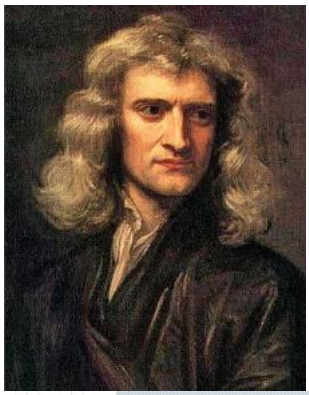
Guillaume François Antoine, Marquis de L’Hôpital (1661–1704)

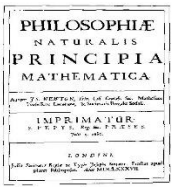
Guillaume, Marquis de l'Hôpital (1661-1704) l'Hôpital's rule for calculating limits involving indeterminate forms 0/0 and ∞/∞ did not originate with l'Hôpital but appeared in his book *Analyse des Infiniment Petits pour l'Intelligence des Lignes Courbes*, a first systematic exposition of differential calculus that became a model for subsequent treatments of calculus.

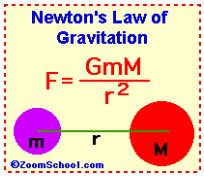


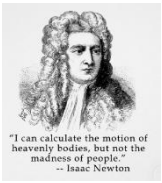
Ehrenfried Walther von Tschirnhaus (1651-1708) mathematician, physicist, physician, and philosopher. He invented the Tschirnhaus transformation; that eliminates terms x^{n-1} and x^{n-2} in equation of order n; the inventor of European porcelain

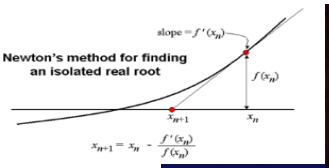




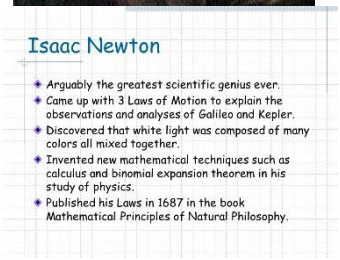




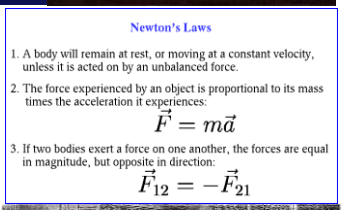




Sir Isaac Newton 1642-1726 "natural philosopher" (physicist and mathematician), one of the most influential scientists of all time and a key figure in the scientific revolution. Book *Philosophiæ Naturalis Principia Mathematica*, 1687, laid the foundations for classical mechanics: laws of motion, universal gravitation with derivation of Kepler's laws of planetary motion, with the trajectories of comets, the tides, the precession of the equinoxes, and other phenomena, Newton removed the last doubts about the validity of the heliocentric model of the Solar System. Contributed to optics: built the first reflecting telescope, a theory of colour. Studied the speed of sound, stated an empirical law of cooling.

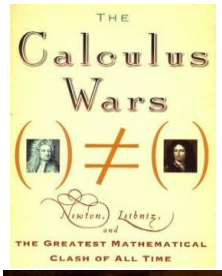


Math: credited with Leibniz for foundation of calculus: power series, the binomial theorem (with non-integer exponents), approximating the roots of a function, classification of the plane cubic curves.

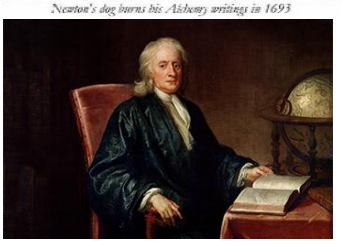


Newton's dog burns his Alchemy writings in 1693

Beyond the sciences, Newton dedicated much of his time to the biblical chronology (hermeneutics) and alchemy. Supposed that his biblical and occult studies were more important than scientific ones. Found big amont of mercury in the body after death (possibly because of alchemical experiments)/

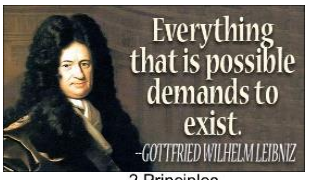


1661-65 studied at Cambridge, 1665-66 work at home on calculus: power series, binomial theorem (1665-66 Great Plague of London, 100000~20% of population died, Cambridge closed, 1665 Great Fire of London), calculus manuscript written in 1666 was not published until 1693, and in the full account only in 1704



1667 Fellow of Trinity College (required to be a priest, Newton wanted to avoid because of non-orthodox believe) 1669 Lucasian Chair after Barrow quitted (avoided to be a priest by a special order by Charles V), 1684 "*On the motion of bodies in orbits*", 1687 *Principia*, 1690 religious tracts, 1701 retied from Cambridge, 1705 knited (second scientist after Francis Bacon), 1711 priority dispute with Leibniz

Gottfried Wilhelm (von) Leibniz 1646-1716 polymath and philosopher, developed calculus (together with Newton), Leibniz's notation are in use nowadays. Inventor of mechanical calculators: added multiplication and division to Pascal's calculator, invented the Leibniz wheel, used in the arithmometer, the first mass-produced mechanical calculator. Worked on the binary number system, anticipated modern logic.



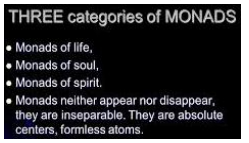
In philosophy, rationalist (along with René Descartes and Baruch Spinoza), Leibniz's optimism: "our Universe is the best possible one that God could have created".

2 Principles

- Principle of the Identity of Indiscernibles: If two objects have the same properties then they are identical with one another.
- Principle of Sufficient Reason: There is a reason why things are exactly as they are and not some other way.

$$\frac{\pi}{4} = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}.$$

In physics and technology anticipated some recent notions in philosophy, probability theory, biology, medicine, geology, psychology, linguistics, and computer science. His works on philosophy, politics, law, ethics, theology, history, and philology were scattered in various learned journals, in tens of thousands of letters, and in unpublished manuscripts (in several languages, but primarily in Latin, French, and German). There is no complete collected writings.



1660 entered Lepzig Univ. 1662 Bac, 1662 Ms. Phil., 1665 Bac in Law, 1666 Thesis in Phil. and Law, Book "*On the Art of Combinations*", worked as salaried alchemist, diplomatic essays "*To find a reason to appoint a German prince to the Polish Throne*", "*To organize a union of German states against France*", "*To involve France into a war with Turkey*", 1672 invited to Paris to discuss his "Egyptian plan" by French governement, met there Huygens and started studying Math, 1673 visited London to show his arithmometer (which he developped since 1670), 1674 invited by Duke of Brunswick, started developping Calculus, finished in 1677, but published only in 1682 in Acta Eruditorum (first scientific journal in Geman lands that he created), 1700 Berlin Academy of Science is created upon the advice of Leiniz and appointed him president (1701 Kingdom of Prussia is created), 1711 projects with Peter I, 1712 Vienna Court of Gabsburgs 1716: died out of favor: nobody from any Court and no Society (Royal or Berlin Acad. Sci) attended, grave went unmarked for 50 years.