M E T U
Department of Mathematics

Group	BASIC LINEAR ALGEBRA Midterm 1	List No.

Code : *Math 260*
Acad. Year : *2007-2008*
Semester : *Fall*
Coordinator : *S.F, S.O, A.S.*

Date : *October 31.2007*
Time : *17:40*
Duration : *90 minutes*

Last Name :
Name : Student No. :
Department : Section :
Signature :

4 QUESTIONS ON 4 PAGES
TOTAL 100 POINTS

1	2	3	4				

Question 1 (10+10=20 points)

Consider the following matrix:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 2 \\ 0 & 2 & 7 & 0 & 0 & 3 & 0 & -1 \\ 0 & 0 & 1 & 3 & 0 & 4 & -1 & 1 \\ 0 & 0 & 0 & 3 & 8 & 0 & 1 & -6 \\ 0 & 0 & 2 & 6 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 4 & -5 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 7 \end{pmatrix}$$

a) Find an upper triangular matrix which is row equivalent to A.

b) Find the determinant of $2A$.

Question 2 (15+10 points)

a)Find the inverse of the following matrix using elementary row operations.
(Warning: There is no partial credit for this question, so make sure that your operations are right and your final answer is correct)

$$\begin{pmatrix} 0 & 1 & -2 \\ 1 & 3 & -4 \\ -3 & -7 & 10 \end{pmatrix}$$

b) Suppose that A and B are 3×4 matrices such that B can be obtained from A by the following elementary row operations:
(i) subtract three times the first row from the third row
(ii) exchange second and third rows
(iii) multiply the third row by 5
Find the elementary matrices corresponding to those row operations and write the matrix B as a product of A and those elementary matrices.

Question 3 (15+15=30 points)

Consider the following system of linear equations:

$$\begin{aligned} x - 2ky + 3z &= 0 \\ y + kz &= 1 \\ x - ky + 4z &= 1 \end{aligned}$$

a) Find the values of k for which this system has no solutions; infinitely many solutions; unique solution.

b) Find all solutions of the system

$$\begin{aligned} x + 3y + 3z + 2t &= 1 \\ 2x + 6y + 9z + 5t &= 5 \\ 3z + t &= 6 \end{aligned}$$

Question 4 (10+15=25 points)

a) Suppose that A is a 4×3 matrix whose first and third columns are equal. Show that

$$A \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

b) Suppose that B is a 4×3 matrix such that $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$ are solutions of $BX = \begin{pmatrix} 1 \\ 4 \\ 1 \\ 1 \end{pmatrix}$. Find the second and third columns of B.