M E T U
Mathematics Department

Math 260 – Basic Linear Algebra Midterm II		
2001–2002 **Spring Semester** **Friday, 26.4.2002** **17:40–19:10** **90 minutes**	Name : Last Name : Signature :	Student Number : Section :
	4 questions on 5 pages.	

Q1 15	Q2 15	Q3 15	Q4 15		**Total** **60**
				SHOW ALL YOUR WORK.	

(15 pts.) Q1. Let $\mathbf{P}_3$ denote the space of polynomials of degree not greater than 3 and and W denote the subset formed by polynomials, p, in $\mathbf{P}_3$, such that $p(2) = 0$.

(a) Show that W is a subspace of $\mathbf{P}_3$.

(b) Find a basis for W.

(c) Determine the dimension of W.

(15 pts.) Q2. Consider triples $S = \{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \}$, $T = \{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \}$ of vectors in $\mathbf{R^3}$.

(a) Justify that S and T are bases in $\mathbf{R^3}$.

(b) Find the coordinate matrix $[w]_T$ for vector $w = \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix}$.

(c) Find the transition matrix P from basis T to basis S.

(15 pts.) Q3. Assume that $\mathbf{B} = \{v_1, v_2, v_3\}$ is a basis in some inner product space V, such that $(v_1, v_2) = (v_2, v_3) = (v_3, v_1) = 0$, $(v_1, v_1) = 2$, and $(v_2, v_2) = (v_3, v_3) = 1$. Consider a pair of vectors, $S = \{u_1, u_2\}$, $u_1, u_2 \in V$ whose coordinate matrices are

$$[u_1]_{\mathbf{B}} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad [u_2]_{\mathbf{B}} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

(a) Justify that u_1 and u_2 are linearly independent. Is S an orthogonal basis for the subspace $\mathbf{Span}(S)$?

(b) Normalize the vectors u_1 and u_2.

(c) Find a basis for the orthogonal complement $S^{\perp}$ and complete the set S to a basis of V.

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(15 pts.) Q4. For a $n \times n$ matrix A, let C_A denote the column space of A and S_A the solution space of the matrix equation $AX = 0$ for $n \times 1$ matrices X. Show that

(a) if $M = AB$, then $S_B \subset S_M$;

(b) if $M = AB$, then $C_M \subset C_A$;

(c) if $A^2 = 0$, then $C_A \subset S_A$.