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> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 7.6
  # Example on design of a laminate considering thermal expansion
  coefficient
> restart :
  with(LinearAlgebra) :
> # Enter the number of plies
> n := 15 :
> # Define extensional stiffness matrix
> A := Matrix(3) :
> # Define fictitious thermal force [NT] vector
> NT := Matrix(3, 1) :
> # Define ply surface coordinate vector in meters
  # Total thickness is h0
> h := Matrix(n + 1, 1) :
  h[1, 1] := - h0/2 :
  for i from 2 by 1 to n + 1
  while true do
  h[i, 1] := h[i - 1, 1] + h0/n :
  end do:
> # Define ply angle vector in radians
> theta := Matrix(n, 1) :
  for i from 1 by 1 to n
  while true do
  if i = (n - 1)/2 + 1 then
  theta[i, 1] := Pi/2 :
  else
  theta[i, 1] := 0 :
  end if
  end do:

> # Enter uniform temperature change delta_T in degrees celsius
> delta_T := dT :
> # Define Qbar array
  Qbar := Array(1 ..3, 1 ..3, 1 ..n) :
  ArrayNumElems(Qbar);
                                   135                              (1)
> # Define thermal expansion coefficient array
> alpha := Array(1 ..3, 1 ..1, 1 ..n) :
  ArrayNumElems(alpha);
                                   45                               (2)
> # Enter mechanical properties of the unidirectional
  graphite/epoxy lamina
```

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# From Table 2.1 for graphite/epoxy (unit = MPa)
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$> E1 := 181000 :$

$E2 := 10300 :$

$nu12 := 0.28 :$

$G12 := 7170 :$

```
> # Enter thermal expansion coefficients of the unidirectional
graphite/epoxy lamina
# From Table 2.1 for graphite/epoxy (unit = 1/ (degrees celsius)
)
```

$> alpha1 := -0.3 \cdot (10)^{-6} :$

$alpha2 := 22.5 \cdot (10)^{-6} :$

```
> # Calculate elements of the compliance matrix for the
unidirectional lamina
```

$> S11 := \frac{1}{E1} :$

$S12 := -\frac{nu12}{E1} :$

$S22 := \frac{1}{E2} :$

$S66 := \frac{1}{G12} :$

```
> # Calculate elements of the reduced stiffness matrix for the
unidirectional lamina
```

$> Q11 := \frac{S22}{S11 \cdot S22 - S12^2} :$

$Q22 := \frac{S11}{S11 \cdot S22 - S12^2} :$

$Q12 := -\frac{S12}{S11 \cdot S22 - S12^2} :$

$Q66 := \frac{1}{S66} :$

```
> # Calculate elements of transformed reduced stiffness matrix for
each angle lamina
# Unit = MPa
```

> **for** *i* **from** 1 **by** 1 **to** n
while *true* **do**

$Qbar[1, 1, i] := Q11 \cdot (\cos(\text{theta}[i, 1]))^4 + Q22 \cdot (\sin(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2 :$

$Qbar[1, 2, i] := (Q11 + Q22 - 4 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^2 \cdot (\cos(\text{theta}[i, 1]))^2 + Q12 \cdot \left((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4 \right) :$

$Qbar[1, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1])) \cdot (\cos(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^3 \cdot \cos(\text{theta}[i, 1]) :$

$Qbar[2, 2, i] := Q11 \cdot (\sin(\text{theta}[i, 1]))^4 + Q22 \cdot (\cos(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2 :$

$Qbar[2, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1])) \cdot (\sin(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^3 \cdot \sin(\text{theta}[i, 1]) :$

$Qbar[3, 3, i] := (Q11 + Q22 - 2 \cdot Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2$

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      + Q66·( (cos(theta[i, 1]))^4 + (sin(theta[i, 1]))^4) :
   Qbar[2, 1, i] := Qbar[1, 2, i] :
   Qbar[3, 1, i] := Qbar[1, 3, i] :
   Qbar[3, 2, i] := Qbar[2, 3, i] :
   end do:
```

```
> # Calculate elements of thermal expansion coefficient vector for
  each angle lamina
  # Unit = degrees celsius
```

```
> for i from 1 by 1 to n
   while true do
   alpha[1, 1, i] := alpha1·(cos(theta[i, 1]))^2 + alpha2·(sin(theta[i, 1]))^2 :
   alpha[2, 1, i] := alpha1·(sin(theta[i, 1]))^2 + alpha2·(cos(theta[i, 1]))^2 :
   alpha[3, 1, i] := 2·(alpha1 − alpha2)·sin(theta[i, 1])·cos(theta[i, 1]) :
   end do:
```

```
> # Calculate elements of extensional stiffness matrix [A]
  # Unit: [A]--> MPa.m
```

```
> for i from 1 by 1 to 3
   while true do
  for j from 1 by 1 to 3
   while true do
    A[i, j] = 0 :
     for k from 1 by 1 to n
    while true do
    A[i, j] := A[i, j] + Qbar[i, j, k]·(h[k + 1, 1] − h[k, 1]) :
    end do:
   end do:
   end do:
```

```
> evalf( A );
```

$$\begin{bmatrix} 1.703801402\ 10^{5}\ h0 & 2896.924442\ h0 & 0. \\ 2896.924442\ h0 & 21777.15744\ h0 & 0. \\ 0. & 0. & 7170.000000\ h0 \end{bmatrix} \qquad (3)$$

```
> # Form fictitious thermal force [NT] vector
  # [NT] in MPa.m
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```
> for i from 1 by 1 to 3
   while true do
    NT[i, 1] = 0 :
     for k from 1 by 1 to n
    while true do
   NT[i, 1] := NT[i, 1] + (Qbar[i, 1, k]·alpha[1, 1, k] + Qbar[i, 2, k]·alpha[2, 1, k] + Qbar[i,
      3, k]·alpha[3, 1, k])·(h[k + 1, 1] − h[k, 1])·delta_T :
    end do:
   end do:
```

```
> NT;
```

$$\begin{bmatrix} 0.02538959407\ h0\ dT \\ 0.2171673584\ h0\ dT \\ 0. \end{bmatrix} \qquad (4)$$

```
> # Extensional compliance matrix
```

> $Astar := MatrixInverse(\ A\ );$

$$Astar := \begin{bmatrix} \frac{0.000005882533763}{h0} & -\frac{7.825289362\ 10^{-7}}{h0} & 0. \\ -\frac{7.825289362\ 10^{-7}}{h0} & \frac{0.00004602377193}{h0} & 0. \\ 0. & 0. & \frac{0.0001394700140}{h0} \end{bmatrix} \quad \textbf{(5)}$$

> **# Strain vector**

> $Res := Multiply(\ Astar, NT\ );$

$$Res := \begin{bmatrix} -2.05845976\ 10^{-8}\ dT \\ 0.000009974992882\ dT \\ 0. \end{bmatrix} \quad \textbf{(6)}$$

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