```
> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 7.5
  # Example on maximum in-plane shear stiffness
> restart :
  with(LinearAlgebra) :
> # Enter the number of plies
> n := 4 :
> # Define extensional, coupling, and bending stiffness matrices
> A := Matrix(3) :
  B := Matrix(3) :
  Dm := Matrix(3) :
> # Define ply surface coordinate vector in meters
  # h is denoted by h0
```

$$> h := \begin{bmatrix} -\frac{h0}{2} \\ -\frac{h0}{4} \\ 0 \\ \frac{h0}{4} \\ \frac{h0}{2} \end{bmatrix};$$

$$h := \begin{bmatrix} -\frac{1}{2}\,h0 \\ -\frac{1}{4}\,h0 \\ 0 \\ \frac{1}{4}\,h0 \\ \frac{1}{2}\,h0 \end{bmatrix} \qquad (1)$$

```
> # Define ply angle vector in radians
```

$$> \theta := \begin{bmatrix} the \\ -the \\ -the \\ the \end{bmatrix};$$

(2)

$$\theta := \begin{bmatrix} the \\ -the \\ -the \\ the \end{bmatrix} \quad (2)$$

```
> # Define Qbar array
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$Qbar := Array(1..3, 1..3, 1..n):$
$ArrayNumElems(Qbar);$

$$36 \quad (3)$$

```
> # Enter properties of the unidirectional lamina
  # From Table 2.1 for graphite/epoxy (unit = MPa)
```

> $E1 := 181000:$
$E2 := 10300:$
$nu12 := 0.28:$
$G12 := 7170:$

```
> # Calculate elements of the compliance matrix for the
  unidirectional lamina
```

> $S11 := \frac{1}{E1}:$

$S12 := -\frac{nu12}{E1}:$

$S22 := \frac{1}{E2}:$

$S66 := \frac{1}{G12}:$

```
> # Calculate elements of the reduced stiffness matrix for the
  unidirectional lamina
```

> $Q11 := \frac{S22}{S11 \cdot S22 - S12^2}:$

$Q22 := \frac{S11}{S11 \cdot S22 - S12^2}:$

$Q12 := -\frac{S12}{S11 \cdot S22 - S12^2}:$

$Q66 := \frac{1}{S66}:$

```
> # Calculate elements of transformed reduced stiffness matrix for
  each angle lamina
  # Unit = MPa
```

> **for** i **from** 1 **by** 1 **to** n
while *true* **do**

$Qbar[1, 1, i] := Q11 \cdot (\cos(\text{theta}[i, 1]))^4 + Q22 \cdot (\sin(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2:$

$Qbar[1, 2, i] := (Q11 + Q22 - 4 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^2 \cdot (\cos(\text{theta}[i, 1]))^2 + Q12 \cdot ((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4):$

$Qbar[1, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1])) \cdot (\cos(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^3 \cdot \cos(\text{theta}[i, 1]):$

$Qbar[2, 2, i] := Q11 \cdot (\sin(\text{theta}[i, 1]))^4 + Q22 \cdot (\cos(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2 :$

$Qbar[2, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1])) \cdot (\sin(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^3 \cdot \sin(\text{theta}[i, 1]) :$

$Qbar[3, 3, i] := (Q11 + Q22 - 2 \cdot Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2 + Q66 \cdot \left((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4 \right) :$

$Qbar[2, 1, i] := Qbar[1, 2, i] :$

$Qbar[3, 1, i] := Qbar[1, 3, i] :$

$Qbar[3, 2, i] := Qbar[2, 3, i] :$

end do:

> # Calculate elements of extensional stiffness matrix [A], coupling stiffness matrix [B], and bending stiffness matrix [Dm]
> # Units: [A]--> MPa.m; [B]--> MPa.m^2; [Dm]--> MPa.m^3

> **for** *i* **from** 1 **by** 1 **to** 3
while *true* **do**
for *j* **from** 1 **by** 1 **to** 3
while *true* **do**

$A[i, j] = 0 :$

$B[i, j] := 0 :$

$Dm[i, j] := 0 :$

for k **from** 1 **by** 1 **to** n
while *true* **do**

$A[i, j] := A[i, j] + Qbar[i, j, k] \cdot (h[k + 1, 1] - h[k, 1]) :$

$B[i, j] := B[i, j] + \frac{1}{2} \cdot Qbar[i, j, k] \cdot \left(h[k + 1, 1]^2 - h[k, 1]^2\right) :$

$Dm[i, j] := Dm[i, j] + \frac{1}{3} \cdot Qbar[i, j, k] \cdot \left(h[k + 1, 1]^3 - h[k, 1]^3\right) :$

end do:
end do:
end do:

> # Astar (in 1/(MPa.m)) matrix

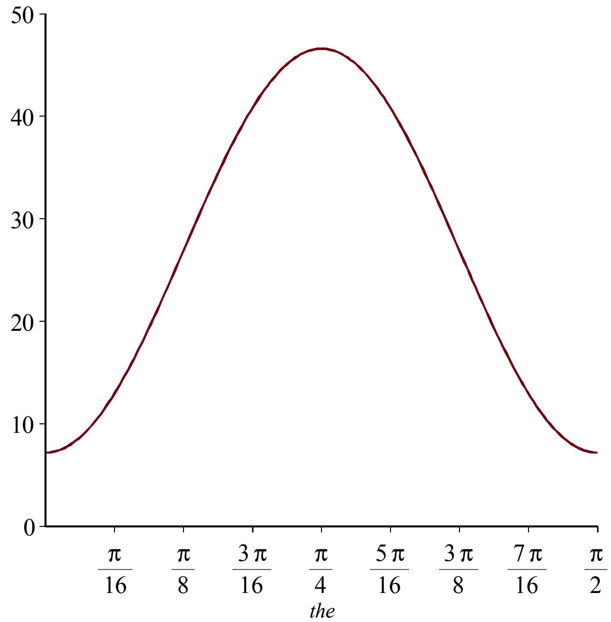
> $Astar := MatrixInverse(A) :$

> # In-plane shear modulus in GPa

> $Gxy := \frac{1}{Astar[3, 3] \cdot h0} \cdot \frac{1}{1000};$

$$Gxy := \frac{1}{1000} \left(6.713467205\ 10^{12} \sin(the)^{12} + 1.814231637\ 10^{14} \cos(the)^2 \sin(the)^{10} + 5.223014410\ 10^{14} \cos(the)^4 \sin(the)^8 + 6.951834900\ 10^{14} \cos(the)^6 \sin(the)^6 + 5.223014405\ 10^{14} \cos(the)^8 \sin(the)^4 + 1.814231636\ 10^{14} \cos(the)^{10} \sin(the)^2 + 6.713467205\ 10^{12} \cos(the)^{12}\right) \Big/ \left(9.36327365\ 10^8 \sin(the)^8 + 2.838619439\ 10^9 \cos(the)^2 \sin(the)^6 + 3.804584155\ 10^9 \cos(the)^4 \sin(the)^4 + 2.838619439\ 10^9 \cos(the)^6 \sin(the)^2 + 9.36327365\ 10^8 \cos(the)^8\right) \quad \textbf{(4)}$$

> $plot\left(Gxy, the = 0 .. \frac{\text{Pi}}{2}\right);$



> $eq := diff(Gxy, the)$:
> $fsolve\left(eq = 0, the = 0\,..\,\frac{\text{Pi}}{2}\right)$;

$$0.7853981636 \quad \textbf{(5)}$$

> $theta\_max := evalf\left(\frac{0.7853981636 \cdot 180}{\text{Pi}}\right)$;

$$theta\_max := 44.99999998 \quad \textbf{(6)}$$

> $evalf\left(\frac{subs\left(the = \frac{\text{Pi}}{4}, Gxy\right)}{subs(the = 0, Gxy)}\right)$;

$$6.498028194 \quad \textbf{(7)}$$

>