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> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 7.4.4
  # Example on design of a laminate
> restart :
  with(LinearAlgebra) :
  Digits := 24 :
> # Enter the number of plies
> n := 100 :
> # Define extensional, coupling, and bending stiffness matrices
> A := Matrix(3) :
  B := Matrix(3) :
  Dm := Matrix(3) :
> # Define laminate stiffness matrix
> QL := Matrix(6) :
> # Define ply surface coordinate vector in inches
> h := Matrix(n + 1, 1) :
> h[1, 1] := -50·0.0049213 :
  for i from 2 by 1 to n + 1
  while true do
  h[i, 1] := h[i - 1, 1] + 0.0049213 :
  end do:
> # Define ply angle vector in radians
> theta := Matrix(n, 1) :
> for i from 1 by 1 to n
  while true do
  theta[i, 1] := 0 :
  end do:
> # Define Qbar array
  Qbar := Array(1 ..3, 1 ..3, 1 ..n) :
  ArrayNumElems(Qbar);
```

$$900 \tag{1}$$

```
> # Enter properties of the unidirectional lamina
  # From Table 2.2 for graphite/epoxy (unit = Msi)
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$>$ $E1_c := 26.25 :$

$E2_c := 1.49 :$

$v12_c := 0.28 :$

$G12_c := 1.040 :$

```
> # Calculate elements of the compliance matrix for the
  graphite/epoxy unidirectional lamina
```

$>$ $S11_c := \dfrac{1}{E1_c} :$

$S12_c := -\dfrac{v12_c}{E1_c} :$

$$S22_c := \frac{1}{E2_c} :$$

$$S66_c := \frac{1}{G12_c} :$$

```
> # Calculate elements of the reduced stiffness matrix for the
  graphite/epoxy unidirectional lamina
```

$$> Q11_c := \frac{S22_c}{S11_c \cdot S22_c - S12_c^2} :$$

$$Q22_c := \frac{S11_c}{S11_c \cdot S22_c - S12_c^2} :$$

$$Q12_c := -\frac{S12_c}{S11_c \cdot S22_c - S12_c^2} :$$

$$Q66_c := \frac{1}{S66_c} :$$

```
> # Enter properties of the unidirectional lamina
  # From Table 2.2 for glass/epoxy (unit = Msi)
```

$$> E1_g := 5.6 :$$

$$E2_g := 1.2 :$$

$$v12_g := 0.26 :$$

$$G12_g := 0.6 :$$

```
> # Calculate elements of the compliance matrix for the glass/epoxy
  unidirectional lamina
```

$$> S11_g := \frac{1}{E1_g} :$$

$$S12_g := -\frac{v12_g}{E1_g} :$$

$$S22_g := \frac{1}{E2_g} :$$

$$S66_g := \frac{1}{G12_g} :$$

```
> # Calculate elements of the reduced stiffness matrix for the
  glass/epoxy unidirectional lamina
```

$$> Q11_g := \frac{S22_g}{S11_g \cdot S22_g - S12_g^2} :$$

$$Q22_g := \frac{S11_g}{S11_g \cdot S22_g - S12_g^2} :$$

$$Q12_g := -\frac{S12_g}{S11_g \cdot S22_g - S12_g^2} :$$

$$Q66_g := \frac{1}{S66_g} :$$

```
> # Calculate elements of transformed reduced stiffness matrix for
  each angle lamina
  # Unit = Msi
```

> **for** i **from** 1 **by** 1 **to** n
while *true* **do**
if $i < 41$ **or** $i > 60$ **then**

$Qbar[1,1,i] := Q11_c \cdot (\cos(\text{theta}[i,1]))^4 + Q22_c \cdot (\sin(\text{theta}[i,1]))^4 + 2 \cdot (Q12_c + 2 \cdot Q66_c) \cdot (\cos(\text{theta}[i,1]))^2 \cdot (\sin(\text{theta}[i,1]))^2 :$

$Qbar[1,2,i] := (Q11_c + Q22_c - 4 \cdot Q66_c) \cdot (\sin(\text{theta}[i,1]))^2 \cdot (\cos(\text{theta}[i,1]))^2 + Q12_c \cdot ((\cos(\text{theta}[i,1]))^4 + (\sin(\text{theta}[i,1]))^4) :$

$Qbar[1,3,i] := (Q11_c - Q12_c - 2 \cdot Q66_c) \cdot (\sin(\text{theta}[i,1])) \cdot (\cos(\text{theta}[i,1]))^3 - (Q22_c - Q12_c - 2 \cdot Q66_c) \cdot (\sin(\text{theta}[i,1]))^3 \cdot \cos(\text{theta}[i,1]) :$

$Qbar[2,2,i] := Q11_c \cdot (\sin(\text{theta}[i,1]))^4 + Q22_c \cdot (\cos(\text{theta}[i,1]))^4 + 2 \cdot (Q12_c + 2 \cdot Q66_c) \cdot (\cos(\text{theta}[i,1]))^2 \cdot (\sin(\text{theta}[i,1]))^2 :$

$Qbar[2,3,i] := (Q11_c - Q12_c - 2 \cdot Q66_c) \cdot (\cos(\text{theta}[i,1])) \cdot (\sin(\text{theta}[i,1]))^3 - (Q22_c - Q12_c - 2 \cdot Q66_c) \cdot (\cos(\text{theta}[i,1]))^3 \cdot \sin(\text{theta}[i,1]) :$

$Qbar[3,3,i] := (Q11_c + Q22_c - 2 \cdot Q12_c - 2 \cdot Q66_c) \cdot (\cos(\text{theta}[i,1]))^2 \cdot (\sin(\text{theta}[i,1]))^2 + Q66_c \cdot ((\cos(\text{theta}[i,1]))^4 + (\sin(\text{theta}[i,1]))^4) :$

$Qbar[2,1,i] := Qbar[1,2,i] :$
$Qbar[3,1,i] := Qbar[1,3,i] :$
$Qbar[3,2,i] := Qbar[2,3,i] :$

else

$Qbar[1,1,i] := Q11_g \cdot (\cos(\text{theta}[i,1]))^4 + Q22_g \cdot (\sin(\text{theta}[i,1]))^4 + 2 \cdot (Q12_g + 2 \cdot Q66_g) \cdot (\cos(\text{theta}[i,1]))^2 \cdot (\sin(\text{theta}[i,1]))^2 :$

$Qbar[1,2,i] := (Q11_g + Q22_g - 4 \cdot Q66_g) \cdot (\sin(\text{theta}[i,1]))^2 \cdot (\cos(\text{theta}[i,1]))^2 + Q12_g \cdot ((\cos(\text{theta}[i,1]))^4 + (\sin(\text{theta}[i,1]))^4) :$

$Qbar[1,3,i] := (Q11_g - Q12_g - 2 \cdot Q66_g) \cdot (\sin(\text{theta}[i,1])) \cdot (\cos(\text{theta}[i,1]))^3 - (Q22_g - Q12_g - 2 \cdot Q66_g) \cdot (\sin(\text{theta}[i,1]))^3 \cdot \cos(\text{theta}[i,1]) :$

$Qbar[2,2,i] := Q11_g \cdot (\sin(\text{theta}[i,1]))^4 + Q22_g \cdot (\cos(\text{theta}[i,1]))^4 + 2 \cdot (Q12_g + 2 \cdot Q66_g) \cdot (\cos(\text{theta}[i,1]))^2 \cdot (\sin(\text{theta}[i,1]))^2 :$

$Qbar[2,3,i] := (Q11_g - Q12_g - 2 \cdot Q66_g) \cdot (\cos(\text{theta}[i,1])) \cdot (\sin(\text{theta}[i,1]))^3 - (Q22_g - Q12_g - 2 \cdot Q66_g) \cdot (\cos(\text{theta}[i,1]))^3 \cdot \sin(\text{theta}[i,1]) :$

$Qbar[3,3,i] := (Q11_g + Q22_g - 2 \cdot Q12_g - 2 \cdot Q66_g) \cdot (\cos(\text{theta}[i,1]))^2 \cdot (\sin(\text{theta}[i,1]))^2 + Q66_g \cdot ((\cos(\text{theta}[i,1]))^4 + (\sin(\text{theta}[i,1]))^4) :$

$Qbar[2,1,i] := Qbar[1,2,i] :$
$Qbar[3,1,i] := Qbar[1,3,i] :$
$Qbar[3,2,i] := Qbar[2,3,i] :$

end if:

```
 end do:
> # Calculate elements of extensional stiffness matrix [A],
  coupling stiffness matrix [B], and bending stiffness matrix [Dm]
  # Units: [A]--> Msi.in; [B]--> Msi.in^2; [Dm]--> Msi.in^3
> for i from 1 by 1 to 3
   while true do
  for j from 1 by 1 to 3
   while true do
    A[i,j]=0 :
    B[i,j] := 0 :
    Dm[i,j] := 0 :
    for k from 1 by 1 to n
    while true do
    A[i,j] := A[i,j] + Qbar[i,j,k]·(h[k+1,1] - h[k,1]) :
    B[i,j] := B[i,j] + 1/2·Qbar[i,j,k]·(h[k+1,1]^2 - h[k,1]^2) :
  Dm[i,j] := Dm[i,j] + 1/3·Qbar[i,j,k]·(h[k+1,1]^3 - h[k,1]^3) :
    end do:
   end do:
   end do:
> evalf( A );
```

$$[[10.94021378275916375 77543, 0.19614781635940993 1642728, 0.],\ [0.19614781635940993 1642728, 0.70908843509502205 1280866, 0.],\ [0., 0., 0.46850776000000000 000000]] \tag{2}$$

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> evalf( B );
```

$$\begin{bmatrix} 0. & 3.1\ 10^{-26} & 0. \\ 3.1\ 10^{-26} & -1.3\ 10^{-25} & 0. \\ 0. & 0. & 0. \end{bmatrix} \tag{3}$$

```
> evalf( Dm );
```

$$[[0.26024977497711504 8492556, 0.00415421622987462 298168355, 0.],\ [0.00415421622987462 298168355, 0.01484339749794917 65231051, 0.],\ [0., 0., 0.01029483050931913 82133336]] \tag{4}$$

```
> # Form laminate stiffness matrix QL
> for i from 1 by 1 to 3
   while true do
   for j from 1 by 1 to 3
   while true do
   QL[i,j] := A[i,j] :
   QL[i,j+3] := B[i,j] :
   QL[i+3,j] := B[i,j] :
   QL[i+3,j+3] := Dm[i,j] :
   end do:
   end do:
> # Form loading vector N
```

> $N := \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3250\cdot(10)^{-6} \\ 0 \\ 0 \end{bmatrix};$

$$N := \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{13}{4000} \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

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> # Find midplane strains and curvatures
```

> $Res := evalf(\,LinearSolve(QL, N)\,);$

$$Res := \begin{bmatrix} 3.14759693084734420145451\ 10^{-29} \\ -1.20073852938493623205765\ 10^{-27} \\ 0. \\ 0.0125440423255924564682719 \\ -0.00351069653860648893107346 \\ 0. \end{bmatrix} \quad (6)$$

```
> # Find strains
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> $eps_x := z\cdot Res[4, 1];$
$eps_y := z\cdot Res[5, 1];$

$$eps_x := 0.0125440423255924564682719\ z$$
$$eps_y := -0.00351069653860648893107346\ z \quad (7)$$

```
> # Stresses in any of the graphite/epoxy lamina
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> $\sigma_{xc} := Qbar[1, 1, 1]\cdot eps_x + Qbar[1, 2, 1]\cdot eps_y;$
$\sigma_{yc} := Qbar[2, 1, 1]\cdot eps_x + Qbar[2, 2, 1]\cdot eps_y;$
$\sigma_{xyc} := Qbar[3, 1, 1]\cdot eps_x + Qbar[3, 2, 1]\cdot eps_y;$

$$\sigma_{xc} := 0.329281796348887417302078\ z$$
$$\sigma_{yc} := 0.00000244750744798217835492\ z$$
$$\sigma_{xyc} := 0. \quad (8)$$

```
> # Find strength ratios at z=h[101,1]
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$$> \frac{\left(\frac{217.56}{2 \cdot 1000}\right)}{subs\left(z = h[101, 1], \sigma_{xc}\right)};$$

$$1.342553149134818703739601 \qquad \textbf{(9)}$$

$$> \frac{\left(\frac{5.802}{2 \cdot 1000}\right)}{subs\left(z = h[101, 1], \sigma_{yc}\right)};$$

$$4816.96911968401158447657 \qquad \textbf{(10)}$$

```
> # Find strength ratios at z=h[1,1]
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$$> \frac{\left(\frac{-217.56}{2 \cdot 1000}\right)}{subs\left(z = h[1, 1], \sigma_{xc}\right)};$$

$$1.342553149134818703739601 \qquad \textbf{(11)}$$

$$> \frac{\left(\frac{-35.68}{2 \cdot 1000}\right)}{subs\left(z = h[1, 1], \sigma_{yc}\right)};$$

$$29622.4505671019533495560 \qquad \textbf{(12)}$$

```
> # Stresses in any of the glass/epoxy lamina
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$$> \sigma_{xg} := Qbar[1, 1, 41] \cdot eps_x + Qbar[1, 2, 41] \cdot eps_y;$$

$$\sigma_{yg} := Qbar[2, 1, 41] \cdot eps_x + Qbar[2, 2, 41] \cdot eps_y;$$

$$\sigma_{xyg} := Qbar[3, 1, 41] \cdot eps_x + Qbar[3, 2, 41] \cdot eps_y;$$

$$\sigma_{xg} := 0.07016772938319481079215 98\, z$$

$$\sigma_{yg} := -0.000303490923549790116010 68\, z$$

$$\sigma_{xyg} := 0. \qquad \textbf{(13)}$$

```
> # Find strength ratios at z=h[61,1]
```

$$> \frac{\left(\frac{154.03}{2 \cdot 1000}\right)}{subs\left(z = h[61, 1], \sigma_{xg}\right)};$$

$$22.3027315250340074609415 \qquad \textbf{(14)}$$

$$> \frac{\left(\frac{-17.12}{2 \cdot 1000}\right)}{subs\left(z = h[61, 1], \sigma_{yg}\right)};$$

$$573.123501135525279979067 \qquad \textbf{(15)}$$

```
> # Find strength ratios at z=h[41,1]
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$$> \frac{\left(\frac{-88.47}{2 \cdot 1000}\right)}{subs\left(z = h[41, 1], \sigma_{xg}\right)};$$

$$12.8099893398672897492014 \qquad \textbf{(16)}$$

> $\dfrac{\left(\dfrac{4.496}{2 \cdot 1000}\right)}{subs\left(z = h[41, 1], \sigma_{yg}\right)};$

$$150.5118727281145828730 07 \quad (17)$$

```
> # Therefore, strength ratio is 1.3426
> # Calculate total cost
```

> $cost := 80 \cdot 10 + 20 \cdot 4;$

$$cost := 880 \quad (18)$$

>