```
> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 7.1
  # Example on failure loads of a laminate
> restart :
  with(LinearAlgebra) :
> # Enter the number of plies
> n := 3 :
> # Define extensional, coupling, and bending stiffness matrices
> A := Matrix(3) :
  B := Matrix(3) :
  Dm := Matrix(3) :
> # Define ply surface coordinate vector in meters
```

$$h := \begin{bmatrix} -\frac{7.5}{1000} \\ -\frac{2.5}{1000} \\ \frac{2.5}{1000} \\ \frac{7.5}{1000} \end{bmatrix} :$$

```
> # Define ply angle vector in radians
```

$$\text{theta} := \begin{bmatrix} 0 \\ \frac{\text{Pi}}{2} \\ 0 \end{bmatrix} :$$

```
> # Define Qbar array
  Qbar := Array(1..3, 1..3, 1..n) :
  ArrayNumElems(Qbar);
```

27 **(1)**

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> # Enter properties of the unidirectional lamina
  # From Table 2.1 for graphite/epoxy (unit = MPa)
> E1 := 181000 :
  E2 := 10300 :
  nu12 := 0.28 :
  G12 := 7170 :
> # Calculate elements of the compliance matrix for the
  unidirectional lamina
```

$$S11 := \frac{1}{E1} :$$

$$S12 := -\frac{nu12}{E1} :$$

$$S22 := \frac{1}{E2} :$$

$S66 := \frac{1}{G12}$:

> # Calculate elements of the reduced stiffness matrix for the unidirectional lamina

> $Q11 := \frac{S22}{S11 \cdot S22 - S12^2}$:
$Q22 := \frac{S11}{S11 \cdot S22 - S12^2}$:
$Q12 := -\frac{S12}{S11 \cdot S22 - S12^2}$:
$Q66 := \frac{1}{S66}$:

> # Calculate elements of transformed reduced stiffness matrix for each angle lamina
Unit = MPa

> **for** i **from** 1 **by** 1 **to** n
while *true* **do**
$Qbar[1, 1, i] := Q11 \cdot (\cos(\text{theta}[i, 1]))^4 + Q22 \cdot (\sin(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2$:
$Qbar[1, 2, i] := (Q11 + Q22 - 4 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^2 \cdot (\cos(\text{theta}[i, 1]))^2 + Q12 \cdot ((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4)$:
$Qbar[1, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1])) \cdot (\cos(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^3 \cdot \cos(\text{theta}[i, 1])$:
$Qbar[2, 2, i] := Q11 \cdot (\sin(\text{theta}[i, 1]))^4 + Q22 \cdot (\cos(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2$:
$Qbar[2, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1])) \cdot (\sin(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^3 \cdot \sin(\text{theta}[i, 1])$:
$Qbar[3, 3, i] := (Q11 + Q22 - 2 \cdot Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2 + Q66 \cdot ((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4)$:
$Qbar[2, 1, i] := Qbar[1, 2, i]$:
$Qbar[3, 1, i] := Qbar[1, 3, i]$:
$Qbar[3, 2, i] := Qbar[2, 3, i]$:
end do:

> # Calculate elements of extensional stiffness matrix [A], coupling stiffness matrix [B], and bending stiffness matrix [Dm]
Units: [A]--> MPa.m; [B]--> MPa.m^2; [Dm]--> MPa.m^3

> **for** i **from** 1 **by** 1 **to** 3
while *true* **do**
for j **from** 1 **by** 1 **to** 3
while *true* **do**
$A[i, j] = 0$:
$B[i, j] := 0$:
$Dm[i, j] := 0$:
for k **from** 1 **by** 1 **to** n
while *true* **do**
$A[i, j] := A[i, j] + Qbar[i, j, k] \cdot (h[k + 1, 1] - h[k, 1])$:

$B[i,j] := B[i,j] + \frac{1}{2} \cdot Qbar[i,j,k] \cdot \left(h[k+1,1]^2 - h[k,1]^2\right)$:

$Dm[i,j] := Dm[i,j] + \frac{1}{3} \cdot Qbar[i,j,k] \cdot \left(h[k+1,1]^3 - h[k,1]^3\right)$:

end do:
end do:
end do:

\> *evalf*(*A*);

$$\begin{bmatrix} 1869.842182 & 43.45386666 & 0. \\ 43.45386666 & 1012.517281 & 0. \\ 0. & 0. & 107.5500000 \end{bmatrix} \quad \textbf{(2)}$$

\> evalf(B);

$$\begin{bmatrix} 0. & 0. & 0. \\ 0. & 0. & 0. \\ 0. & 0. & 0. \end{bmatrix} \quad \textbf{(3)}$$

\> evalf(Dm);

$$\begin{bmatrix} 0.04934828925 & 0.0008147599997 & 0. \\ 0.0008147599997 & 0.004695950685 & 0. \\ 0. & 0. & 0.002016562500 \end{bmatrix} \quad \textbf{(4)}$$

\> *Astar* := *MatrixInverse*(*A*);

$$Astar := \begin{bmatrix} 5.353 \times 10^{-4} & -2.297 \times 10^{-5} & 0.000 \times 10^{0} \\ -2.297 \times 10^{-5} & 9.886 \times 10^{-4} & 0.000 \times 10^{0} \\ 0.000 \times 10^{0} & 0.000 \times 10^{0} & 9.298 \times 10^{-3} \end{bmatrix} \quad \textbf{(5)}$$

\> **# Define load vector**

\> $N := \begin{bmatrix} Nx \\ 0 \\ 0 \end{bmatrix}$;

$$N := \begin{bmatrix} Nx \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(6)}$$

\> **# Find strains (Note that strains in this case are equal to mid-plane strains)**

\> *Eps* := *Multiply*(*Astar*, *N*);

$$Eps := \begin{bmatrix} 5.3534 \times 10^{-4}\, Nx \\ -2.2975 \times 10^{-5}\, Nx \\ 0.0000 \times 10^{0} \end{bmatrix} \quad \textbf{(7)}$$

```
> # Calculate global stresses in the 0-degree ply (n=1)
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> $sigmax1 := Qbar[1, 1, 1] \cdot Eps[1, 1] + Qbar[1, 2, 1] \cdot Eps[2, 1] + Qbar[1, 3, 1] \cdot Eps[3, 1];$
> $sigmay1 := Qbar[2, 1, 1] \cdot Eps[1, 1] + Qbar[2, 2, 1] \cdot Eps[2, 1] + Qbar[2, 3, 1] \cdot Eps[3, 1];$
> $tauxy1 := Qbar[3, 1, 1] \cdot Eps[1, 1] + Qbar[3, 2, 1] \cdot Eps[2, 1] + Qbar[3, 3, 1] \cdot Eps[3, 1];$

$$sigmax1 := 97.26393020\ Nx$$

$$sigmay1 := 1.313132561\ Nx$$

$$tauxy1 := 0. \qquad (8)$$

```
> # Calculate global stresses in the 90-degree ply (n=2)
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> $sigmax2 := Qbar[1, 1, 2] \cdot Eps[1, 1] + Qbar[1, 2, 2] \cdot Eps[2, 1] + Qbar[1, 3, 2] \cdot Eps[3, 1];$
> $sigmay2 := Qbar[2, 1, 2] \cdot Eps[1, 1] + Qbar[2, 2, 2] \cdot Eps[2, 1] + Qbar[2, 3, 2] \cdot Eps[3, 1];$
> $tauxy2 := Qbar[3, 1, 2] \cdot Eps[1, 1] + Qbar[3, 2, 2] \cdot Eps[2, 1] + Qbar[3, 3, 2] \cdot Eps[3, 1];$

$$sigmax2 := 5.472139550\ Nx$$

$$sigmay2 := -2.626265123\ Nx$$

$$tauxy2 := 0. \qquad (9)$$

```
> # Calculate global stresses in the 90-degree ply (n=3)
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> $sigmax3 := Qbar[1, 1, 3] \cdot Eps[1, 1] + Qbar[1, 2, 3] \cdot Eps[2, 1] + Qbar[1, 3, 3] \cdot Eps[3, 1];$
> $sigmay3 := Qbar[2, 1, 3] \cdot Eps[1, 1] + Qbar[2, 2, 3] \cdot Eps[2, 1] + Qbar[2, 3, 3] \cdot Eps[3, 1];$
> $tauxy3 := Qbar[3, 1, 3] \cdot Eps[1, 1] + Qbar[3, 2, 3] \cdot Eps[2, 1] + Qbar[3, 3, 3] \cdot Eps[3, 1];$

$$sigmax3 := 97.26393020\ Nx$$

$$sigmay3 := 1.313132561\ Nx$$

$$tauxy3 := 0. \qquad (10)$$

```
> # Calculate local stresses in the 0-degree ply (n=1)
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> $sigma11 := (\cos(\mathrm{theta}[1, 1]))^2 \cdot sigmax1 + (\sin(\mathrm{theta}[1, 1]))^2 \cdot sigmay1 + 2 \cdot \sin(\mathrm{theta}[1, 1]) \cdot \cos(\mathrm{theta}[1, 1]) \cdot tauxy1;$
> $sigma21 := (\sin(\mathrm{theta}[1, 1]))^2 \cdot sigmax1 + (\cos(\mathrm{theta}[1, 1]))^2 \cdot \mathrm{sigmay1} - 2 \cdot \sin(\mathrm{theta}[1, 1]) \cdot \cos(\mathrm{theta}[1, 1]) \cdot tauxy1;$
> $tau121 := -\sin(\mathrm{theta}[1, 1]) \cdot \cos(\mathrm{theta}[1, 1]) \cdot \mathrm{sigmax1} + \sin(\mathrm{theta}[1, 1]) \cdot \cos(\mathrm{theta}[1, 1]) \cdot \mathrm{sigmay1} + \left((\cos(\mathrm{theta}[1, 1]))^2 - (\sin(\mathrm{theta}[1, 1]))^2\right) \cdot \cos(\mathrm{theta}[1, 1]) \cdot tauxy1;$

$$\sigma 11 := 97.26393020\ Nx$$

$$\sigma 21 := 1.313132561\ Nx$$

$$\tau 121 := 0. \qquad (11)$$

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> # Calculate local stresses in the 90-degree ply (n=2)
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> $sigma12 := (\cos(\mathrm{theta}[2, 1]))^2 \cdot sigmax2 + (\sin(\mathrm{theta}[2, 1]))^2 \cdot sigmay2 + 2 \cdot \sin(\mathrm{theta}[2, 1]) \cdot \cos(\mathrm{theta}[2, 1]) \cdot tauxy2;$
> $sigma22 := (\sin(\mathrm{theta}[2, 1]))^2 \cdot sigmax2 + (\cos(\mathrm{theta}[2, 1]))^2 \cdot \mathrm{sigmay2} - 2 \cdot \sin(\mathrm{theta}[2, 1]) \cdot \cos(\mathrm{theta}[2, 1]) \cdot tauxy2;$
> $tau122 := -\sin(\mathrm{theta}[2, 1]) \cdot \cos(\mathrm{theta}[2, 1]) \cdot \mathrm{sigmax2} + \sin(\mathrm{theta}[2, 1]) \cdot \cos(\mathrm{theta}[2, 1]) \cdot \mathrm{sigmay2} + \left((\cos(\mathrm{theta}[2, 1]))^2 - (\sin(\mathrm{theta}[2, 1]))^2\right) \cdot \cos(\mathrm{theta}[2, 1]) \cdot tauxy2;$

$$\sigma 12 := -2.626265123\ Nx$$

$$\sigma 22 := 5.472139550\ Nx$$

$$\tau 122 := 0. \qquad (12)$$

```
> # Calculate local stresses in the 0-degree ply (n=3)
```

> $sigma13 := (\cos(\mathrm{theta}[3, 1]))^2 \cdot sigmax3 + (\sin(\mathrm{theta}[3, 1]))^2 \cdot sigmay3 + 2 \cdot \sin(\mathrm{theta}[3, 1]) \cdot \cos(\mathrm{theta}[3, 1]) \cdot tauxy3;$

$sigma23 := (\sin(\text{theta}[3, 1]))^2 \cdot sigmax3 + (\cos(\text{theta}[3, 1]))^2 \cdot \text{sigmay3} - 2 \cdot \sin(\text{theta}[3, 1]) \cdot \cos(\text{theta}[3, 1]) \cdot tauxy3;$

$tau123 := -\sin(\text{theta}[3, 1]) \cdot \cos(\text{theta}[3, 1]) \cdot \text{sigmax3} + \sin(\text{theta}[3, 1]) \cdot \cos(\text{theta}[3, 1]) \cdot \text{sigmay3} + ((\cos(\text{theta}[3, 1]))^2 - (\sin(\text{theta}[3, 1]))^2) \cdot \cos(\text{theta}[3, 1]) \cdot tauxy3;$

$$\sigma 13 := 97.26393020\ Nx$$

$$\sigma 23 := 1.313132561\ Nx$$

$$\tau 123 := 0. \tag{13}$$

```
> # Input strength values for graphite/epoxy lamina
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$> sigma1Tult := 1500 :$
$sigma1Cult := 1500 :$
$sigma2Tult := 40 :$
$sigma2Cult := 246 :$
$tau12ult := 68 :$

```
> # Determine coefficients of Tsai-Wu failure criterion
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$> H1 := \frac{1}{sigma1Tult} - \frac{1}{sigma1Cult} :$

$H2 := \frac{1}{sigma2Tult} - \frac{1}{sigma2Cult} :$

$H11 := \frac{1}{sigma1Tult \cdot sigma1Cult} :$

$H22 := \frac{1}{sigma2Tult \cdot sigma2Cult} :$

$H12 := -\frac{1}{2} \cdot \text{sqrt}\left(\frac{1}{sigma1Tult \cdot sigma1Cult \cdot sigma2Tult \cdot sigma2Cult}\right) :$

```
> # Apply Tsai-Wu failure criterion for the 0-degree ply (n=1)
```

$> eq1 := H1 \cdot sigma11 + H2 \cdot sigma21 + H11 \cdot \sigma 11^2 + H22 \cdot \sigma 21^2 + H66 \cdot \tau 121^2 + 2 \cdot H12 \cdot sigma11 \cdot sigma21;$
$solve(eq1 = 1, Nx);$

$$eq1 := 0.02749037679\ Nx + 0.004379800866\ Nx^2 - 0.00003461258368\ \sqrt{615}\ Nx^2$$

$$13.39441731,\ -21.20099715 \tag{14}$$

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> # Apply Tsai-Wu failure criterion for the 90-degree ply (n=2)
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$> eq2 := H1 \cdot \sigma 12 + H2 \cdot \sigma 22 + H11 \cdot \sigma 12^2 + H22 \cdot \sigma 22^2 + H66 \cdot \tau 122^2 + 2 \cdot H12 \cdot \sigma 12 \cdot \sigma 22;$
$solve(eq2 = 1, Nx);$

$$eq2 := 0.1145590190\ Nx + 0.003046186515\ Nx^2 + 0.000003894658332\ \sqrt{615}\ Nx^2$$

$$7.276559802,\ -43.72815701 \tag{15}$$

```
> # Calculate mid-plane strain at this level of loading
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$> Nx := 7.276559802 :$
$epsx0 := Eps[1, 1];$

$$epsx0 := 0.003895421991 \tag{16}$$

>