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> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 6.4
  # Example on residual stresses in the [0/90] laminate
> restart :
  with(LinearAlgebra) :
> # Enter the number of plies
> n := 2 :
> # Define extensional, coupling, and bending stiffness matrices
> A := Matrix(3) :
  B := Matrix(3) :
  Dm := Matrix(3) :
> # Define fictitious thermal force [NT] and thermal moment [MT]
  vectors
> NT := Matrix(3, 1) :
  MT := Matrix(3, 1) :
> # Define laminate stiffness matrix QL and laminate load vector
  Load
> QL := Matrix(6, 6) :
  Load := Matrix(6, 1) :
> # Define ply surface coordinate vector in meters
```

$$h := \begin{bmatrix} -\frac{5}{1000} \\ 0 \\ \frac{5}{1000} \end{bmatrix} :$$

```
> # Define ply angle vector in radians
```

$$\text{theta} := \begin{bmatrix} 0 \\ \frac{\text{Pi}}{2} \end{bmatrix} :$$

```
> # Enter uniform temperature change delta_T in degrees celsius
> delta_T := -75 :
> # Define Qbar array
  Qbar := Array(1 ..3, 1 ..3, 1 ..n) :
  ArrayNumElems(Qbar);
                                   18                                  (1)
> # Define thermal expansion coefficient array
> alpha := Array(1 ..3, 1 ..1, 1 ..n) :
  ArrayNumElems(alpha);
                                   6                                   (2)
> # Enter mechanical properties of the unidirectional
  graphite/epoxy lamina
  # From Table 2.1 for graphite/epoxy (unit = MPa)
> E1 := 181000 :
  E2 := 10300 :
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$nu12 := 0.28:$
$G12 := 7170:$

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> # Enter thermal expansion coefficients of the unidirectional
graphite/epoxy lamina
# From Table 2.1 for graphite/epoxy (unit = 1/ (degrees celsius)
)
```

> $alpha1 := 0.02 \cdot (10)^{-6}:$
$alpha2 := 22.5 \cdot (10)^{-6}:$

```
> # Calculate elements of the compliance matrix for the
unidirectional lamina
```

> $S11 := \frac{1}{E1}:$

$S12 := -\frac{nu12}{E1}:$

$S22 := \frac{1}{E2}:$

$S66 := \frac{1}{G12}:$

```
> # Calculate elements of the reduced stiffness matrix for the
unidirectional lamina
```

> $Q11 := \frac{S22}{S11 \cdot S22 - S12^2}:$

$Q22 := \frac{S11}{S11 \cdot S22 - S12^2}:$

$Q12 := -\frac{S12}{S11 \cdot S22 - S12^2}:$

$Q66 := \frac{1}{S66}:$

```
> # Calculate elements of transformed reduced stiffness matrix for
each angle lamina
# Unit = MPa
```

> **for** *i* **from** 1 **by** 1 **to** n
while *true* **do**

$Qbar[1,1,i] := Q11 \cdot (\cos(\text{theta}[i,1]))^4 + Q22 \cdot (\sin(\text{theta}[i,1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i,1]))^2 \cdot (\sin(\text{theta}[i,1]))^2:$

$Qbar[1,2,i] := (Q11 + Q22 - 4 \cdot Q66) \cdot (\sin(\text{theta}[i,1]))^2 \cdot (\cos(\text{theta}[i,1]))^2 + Q12 \cdot \left((\cos(\text{theta}[i,1]))^4 + (\sin(\text{theta}[i,1]))^4\right):$

$Qbar[1,3,i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i,1])) \cdot (\cos(\text{theta}[i,1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i,1]))^3 \cdot \cos(\text{theta}[i,1]):$

$Qbar[2,2,i] := Q11 \cdot (\sin(\text{theta}[i,1]))^4 + Q22 \cdot (\cos(\text{theta}[i,1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i,1]))^2 \cdot (\sin(\text{theta}[i,1]))^2:$

$Qbar[2,3,i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i,1])) \cdot (\sin(\text{theta}[i,1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i,1]))^3 \cdot \sin(\text{theta}[i,1]):$

$Qbar[3,3,i] := (Q11 + Q22 - 2 \cdot Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i,1]))^2 \cdot (\sin(\text{theta}[i,1]))^2 + Q66 \cdot \left((\cos(\text{theta}[i,1]))^4 + (\sin(\text{theta}[i,1]))^4\right):$

$Qbar[2,1,i] := Qbar[1,2,i]:$
$Qbar[3,1,i] := Qbar[1,3,i]:$

```
Qbar[3, 2, i] := Qbar[2, 3, i] :
end do:
```

```
> # Calculate elements of thermal expansion coefficient vector for
each angle lamina
# Unit = degrees celsius
```

> for i from 1 by 1 to n
> while *true* do
> $\mathrm{alpha}[1,1,i] := alpha1\cdot(\cos(\mathrm{theta}[i,1]))^2 + alpha2\cdot(\sin(\mathrm{theta}[i,1]))^2$:
> $\mathrm{alpha}[2,1,i] := alpha1\cdot(\sin(\mathrm{theta}[i,1]))^2 + alpha2\cdot(\cos(\mathrm{theta}[i,1]))^2$:
> $\mathrm{alpha}[3,1,i] := 2\cdot(alpha1 - alpha2)\cdot\sin(\mathrm{theta}[i,1])\cdot\cos(\mathrm{theta}[i,1])$:
> end do:

```
> # Calculate elements of extensional stiffness matrix [A],
coupling stiffness matrix [B], and bending stiffness matrix [Dm]
# Units: [A]--> MPa.m; [B]--> MPa.m^2; [Dm]--> MPa.m^3
```

> for i from 1 by 1 to 3
> while *true* do
> for j from 1 by 1 to 3
> while *true* do
> $A[i,j]=0$:
> $B[i,j] := 0$:
> $Dm[i,j] := 0$:
> for k from 1 by 1 to n
> while *true* do
> $A[i,j] := A[i,j] + Qbar[i,j,k]\cdot(h[k+1,1] - h[k,1])$:
> $B[i,j] := B[i,j] + \frac{1}{2}\cdot Qbar[i,j,k]\cdot\left(h[k+1,1]^2 - h[k,1]^2\right)$:
> $Dm[i,j] := Dm[i,j] + \frac{1}{3}\cdot Qbar[i,j,k]\cdot\left(h[k+1,1]^3 - h[k,1]^3\right)$:
> end do:
> end do:
> end do:

> *evalf*(A);

$$\begin{bmatrix} 960.7864876 & 28.96924444 & 0. \\ 28.96924444 & 960.7864876 & 0. \\ 0. & 0. & 71.70000000 \end{bmatrix} \quad (3)$$

> evalf(B);

$$\begin{bmatrix} -2.143312251 & 0. & 0. \\ 0. & 2.143312251 & 0. \\ 0. & 0. & 0. \end{bmatrix} \quad (4)$$

> evalf(Dm);

$$\begin{bmatrix} 0.008006554064 & 0.0002414103704 & 0. \\ 0.0002414103704 & 0.008006554064 & 0. \\ 0. & 0. & 0.0005975000000 \end{bmatrix} \quad (5)$$

```
> # Form laminate stiffness matrix QL by converting stress unit to
Pa
```

```
> for i from 1 by 1 to 3
  while true do
  for j from 1 by 1 to 3
  while true do
  QL[i,j] := A[i,j]·10^6 :
  QL[i,j+3] := B[i,j]·10^6 :
  QL[i+3,j] := B[i,j]·10^6 :
  QL[i+3,j+3] := Dm[i,j]·10^6 :
  end do:
  end do:
> # Form fictitious thermal force [NT] and moment [MT]vectors
  # [NT] in Pa.m; [MT] in Pa.m^2
> for i from 1 by 1 to 3
  while true do
   NT[i, 1] = 0 :
   MT[i, 1] := 0 :
   for k from 1 by 1 to n
   while true do
  NT[i, 1] := NT[i, 1] + (Qbar[i, 1, k]·alpha[1, 1, k] + Qbar[i, 2, k]·alpha[2, 1, k] + Qbar[i,
     3, k]·alpha[3, 1, k])·(h[k+1, 1] − h[k, 1])·delta_T :
   MT[i, 1] := MT[i, 1] + 1/2·(Qbar[i, 1, k]·alpha[1, 1, k] + Qbar[i, 2, k]·alpha[2, 1, k]
      + Qbar[i, 3, k]·alpha[3, 1, k])·((h[k+1, 1])^2 − (h[k, 1])^2)·delta_T :
  end do:
  NT[i, 1] := NT[i, 1]·10^6 :
  MT[i, 1] := MT[i, 1]·10^6 :
  end do:
> NT;
```

$$\begin{bmatrix} -1.131238248\ 10^{5} \\ -1.131238248\ 10^{5} \\ 0. \end{bmatrix} \qquad (6)$$

```
> MT;
```

$$\begin{bmatrix} -153.7776442 \\ 153.7776442 \\ 0. \end{bmatrix} \qquad (7)$$

```
> # Form laminate load vector Load
> for i from 1 by 1 to 3
  while true do
  Load[i, 1] := NT[i, 1] :
  Load[i+3, 1] := MT[i, 1] :
  end do:
> # Find strains and curvatures
> Res := LinearSolve(QL, Load)
```

$$Res := \begin{bmatrix} -0.000390715152415603 \\ -0.000390715152415603 \\ 0. \\ -0.127647633596354 \\ 0.127647633596354 \\ 0. \end{bmatrix} \quad \textbf{(8)}$$

```
> # Determine total strains in the laminate
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$> eps\_x := Res[1, 1] + z \cdot Res[4, 1]:$
$eps\_y := Res[2, 1] + z \cdot Res[5, 1]:$
$gamma\_xy := Res[3, 1] + z \cdot Res[6, 1]:$

```
> # Determine mechanical strains in the 90-degree lamina
```

$> eps\_xM := eps\_x - \mathrm{alpha}[1, 1, 2] \cdot delta\_T:$
$eps\_yM := eps\_y - \mathrm{alpha}[2, 1, 2] \cdot delta\_T:$
$gamma\_xyM := gamma\_xy - \mathrm{alpha}[3, 1, 2] \cdot delta\_T:$

```
> # Determine stresses in the 90-degree lamina (z in m; stresses in
MPa)
```

$> sigma\_x := Qbar[1, 1, 2] \cdot eps\_xM + Qbar[1, 2, 2] \cdot eps\_yM + Qbar[1, 3, 2] \cdot gamma\_xyM:$
$sigma\_y := Qbar[2, 1, 2] \cdot eps\_xM + Qbar[2, 2, 2] \cdot eps\_yM + Qbar[2, 3, 2] \cdot gamma\_xyM:$
$tau\_xy := Qbar[3, 1, 2] \cdot eps\_xM + Qbar[3, 2, 2] \cdot eps\_yM + Qbar[3, 3, 2] \cdot gamma\_xyM:$

```
> # Compute stresses at the bottom surface of the 90-degree lamina
(in MPa)
```

$> subs\left(z = \frac{5}{1000}, sigma\_x\right);$

$$7.53482933919543 \quad \textbf{(9)}$$

$> subs\left(z = \frac{5}{1000}, sigma\_y\right);$

$$47.1829180211511 \quad \textbf{(10)}$$

$> subs\left(z = \frac{5}{1000}, tau\_xy\right);$

$$0. \quad \textbf{(11)}$$

>