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> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 6.2
  # Example on computation of extensional ([A]), coupling ([B]),
  and bending ([Dm])
  # stiffness matrices for a laminate
> restart :
  with(LinearAlgebra) :
> # Enter the number of plies
> n := 3 :
> # Define extensional, coupling, and bending stiffness matrices
> A := Matrix(3) :
  B := Matrix(3) :
  Dm := Matrix(3) :
> # Define laminate stiffness matrix
> QL := Matrix(6) :
> # Define ply surface coordinate vector in meters
```

$$h := \begin{bmatrix} -\frac{7.5}{1000} \\ -\frac{2.5}{1000} \\ \frac{2.5}{1000} \\ \frac{7.5}{1000} \end{bmatrix} :$$

```
> # Define ply angle vector in radians
```

$$\text{theta} := \begin{bmatrix} 0 \\ \frac{\text{Pi}}{6} \\ -\frac{\text{Pi}}{4} \end{bmatrix} :$$

```
> # Define Qbar array
  Qbar := Array(1 ..3, 1 ..3, 1 ..n) :
  ArrayNumElems(Qbar);
```

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> # Enter properties of the unidirectional lamina
  # From Table 2.1 for graphite/epoxy (unit = MPa)
> E1 := 181000 :
  E2 := 10300 :
  nu12 := 0.28 :
  G12 := 7170 :
> # Calculate elements of the compliance matrix for the
  unidirectional lamina
```

> $S11 := \frac{1}{E1}$:

$S12 := -\frac{nu12}{E1}$:

$S22 := \frac{1}{E2}$:

$S66 := \frac{1}{G12}$:

> **# Calculate elements of the reduced stiffness matrix for the unidirectional lamina**

> $Q11 := \frac{S22}{S11 \cdot S22 - S12^2}$:

$Q22 := \frac{S11}{S11 \cdot S22 - S12^2}$:

$Q12 := -\frac{S12}{S11 \cdot S22 - S12^2}$:

$Q66 := \frac{1}{S66}$:

> **# Calculate elements of transformed reduced stiffness matrix for each angle lamina**
> **# Unit = MPa**

> **for** i **from** 1 **by** 1 **to** n
while *true* **do**

$Qbar[1, 1, i] := Q11 \cdot (\cos(\text{theta}[i, 1]))^4 + Q22 \cdot (\sin(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2$:

$Qbar[1, 2, i] := (Q11 + Q22 - 4 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^2 \cdot (\cos(\text{theta}[i, 1]))^2 + Q12 \cdot ((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4)$:

$Qbar[1, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1])) \cdot (\cos(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\sin(\text{theta}[i, 1]))^3 \cdot \cos(\text{theta}[i, 1])$:

$Qbar[2, 2, i] := Q11 \cdot (\sin(\text{theta}[i, 1]))^4 + Q22 \cdot (\cos(\text{theta}[i, 1]))^4 + 2 \cdot (Q12 + 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2$:

$Qbar[2, 3, i] := (Q11 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1])) \cdot (\sin(\text{theta}[i, 1]))^3 - (Q22 - Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^3 \cdot \sin(\text{theta}[i, 1])$:

$Qbar[3, 3, i] := (Q11 + Q22 - 2 \cdot Q12 - 2 \cdot Q66) \cdot (\cos(\text{theta}[i, 1]))^2 \cdot (\sin(\text{theta}[i, 1]))^2 + Q66 \cdot ((\cos(\text{theta}[i, 1]))^4 + (\sin(\text{theta}[i, 1]))^4)$:

$Qbar[2, 1, i] := Qbar[1, 2, i]$:
$Qbar[3, 1, i] := Qbar[1, 3, i]$:
$Qbar[3, 2, i] := Qbar[2, 3, i]$:
end do:

> **# Calculate elements of extensional stiffness matrix [A], coupling stiffness matrix [B], and bending stiffness matrix [Dm]**
> **# Units: [A]--> MPa.m; [B]--> MPa.m^2; [Dm]--> MPa.m^3**

> **for** i **from** 1 **by** 1 **to** 3
while *true* **do**
for j **from** 1 **by** 1 **to** 3
while *true* **do**
$A[i, j] = 0$:

$B[i, j] := 0 :$
$Dm[i, j] := 0 :$
for k **from** 1 **by** 1 **to** n
while *true* **do**
$A[i, j] := A[i, j] + Qbar[i, j, k] \cdot (h[k+1, 1] - h[k, 1]) :$
$B[i, j] := B[i, j] + \frac{1}{2} \cdot Qbar[i, j, k] \cdot \left(h[k+1, 1]^2 - h[k, 1]^2\right) :$
$Dm[i, j] := Dm[i, j] + \frac{1}{3} \cdot Qbar[i, j, k] \cdot \left(h[k+1, 1]^3 - h[k, 1]^3\right) :$
end do:
end do:
end do:

> *evalf*(*A*);

$$\begin{bmatrix} 1739.240863 & 388.3864106 & 56.6337309 \\ 388.3864106 & 453.2535123 & -114.0636096 \\ 56.6337309 & -114.0636096 & 452.4825438 \end{bmatrix} \tag{2}$$

> evalf(B);

$$\begin{bmatrix} -3.128833804 & 0.9855215539 & -1.071656126 \\ 0.9855215539 & 1.157790698 & -1.071656126 \\ -1.071656126 & -1.071656126 & 0.9855215540 \end{bmatrix} \tag{3}$$

> evalf(Dm);

$$\begin{bmatrix} 0.03343203414 & 0.006460977237 & -0.005240293688 \\ 0.006460977237 & 0.009319771318 & -0.005595913147 \\ -0.005240293688 & -0.005595913147 & 0.007662779735 \end{bmatrix} \tag{4}$$

> **# Form laminate stiffness matrix QL by converting stress unit to Pa**

> **for** *i* **from** 1 **by** 1 **to** 3
while *true* **do**
for *j* **from** 1 **by** 1 **to** 3
while *true* **do**
$QL[i, j] := A[i, j] \cdot 10^6 :$
$QL[i, j+3] := B[i, j] \cdot 10^6 :$
$QL[i+3, j] := B[i, j] \cdot 10^6 :$
$QL[i+3, j+3] := Dm[i, j] \cdot 10^6 :$
end do:
end do:

> **# Form loading vector N**

$$> N := \begin{bmatrix} 1000 \\ 1000 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}:$$

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> # Find midplane strains and curvatures
```

$$> Res := evalf(\,LinearSolve(QL, N)\,);$$

$$Res := \begin{bmatrix} 3.122948352\ 10^{-7} \\ 0.000003491704604 \\ -7.598445480\ 10^{-7} \\ 0.00002971437363 \\ -0.0003285023896 \\ 0.0004101466831 \end{bmatrix} \quad \textbf{(5)}$$

```
> # Calculate global strains
```

$$> epsx := Res[1,1] + z \cdot Res[4,1];$$
$$epsy := Res[2,1] + z \cdot Res[5,1];$$
$$gammaxy := Res[3,1] + z \cdot Res[6,1];$$

$$epsx := 0.00002971437363\ z + 3.122948352\ 10^{-7}$$
$$epsy := -0.0003285023896\ z + 0.000003491704604$$
$$gammaxy := 0.0004101466831\ z - 7.598445480\ 10^{-7} \quad \textbf{(6)}$$

```
> # Calculate global strains at z = - 2.5 mm
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$$> subs\left(z = -\frac{2.5}{1000}, epsx\right);$$
$$subs\left(z = -\frac{2.5}{1000}, epsy\right);$$
$$subs\left(z = -\frac{2.5}{1000}, gammaxy\right);$$

$$2.380089011\ 10^{-7}$$
$$0.000004312960578$$
$$-0.000001785211256 \quad \textbf{(7)}$$

```
> # Calculate global stresses(on 30-degree ply) (in Pa)
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$$> sigmax := evalf(\,(Qbar[1,1,2] \cdot epsx + Qbar[1,2,2] \cdot \mathrm{epsy} + Qbar[1,3,2] \cdot gammaxy) \cdot 10^{6}\,);$$
$$sigmay := evalf(\,(Qbar[2,1,2] \cdot \mathrm{epsx} + Qbar[2,2,2] \cdot \mathrm{epsy} + Qbar[2,3,2] \cdot gammaxy) \cdot 10^{6}\,);$$
$$tauxy := evalf(\,(Qbar[3,1,2] \cdot \mathrm{epsx} + Qbar[3,2,2] \cdot \mathrm{epsy} + Qbar[3,3,2] \cdot gammaxy) \cdot 10^{6}\,);$$

$$sigmax := 1.481317925\ 10^{7}\ z + 1.063300340\ 10^{5}$$
$$sigmay := 1.421474740\ 10^{6}\ z + 77467.82379$$

(8)

$$tauxy := 1.008968412\ 10^{7}\ z + 59031.78949 \quad (8)$$

```
> # Calculate global stresses at z = -2.5 mm (on 30-degree ply) (in Pa)
```

$$> sigx := subs\left(z = -\frac{2.5}{1000}, sigmax\right);$$

$$sigy := subs\left(z = -\frac{2.5}{1000}, sigmay\right);$$

$$txy := subs\left(z = -\frac{2.5}{1000}, tauxy\right);$$

$$sigx := 69297.08588$$

$$sigy := 73914.13694$$

$$txy := 33807.57919 \quad (9)$$

```
> # Compute local stresses at z = -2.5 (on 30-degree ply) (in Pa)
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$$> c := \cos\left(\frac{\mathrm{Pi}}{6}\right):$$

$$s := \sin\left(\frac{\mathrm{Pi}}{6}\right):$$

$$> sigma1 := evalf\left(c^2 \cdot sigx + s^2 \cdot sigy + 2 \cdot s \cdot c \cdot txy \right);$$

$$sigma2 := evalf\left(s^2 \cdot sigx + c^2 \cdot sigy - 2 \cdot s \cdot c \cdot txy \right);$$

$$tau12 := evalf\left(-s \cdot c \cdot sigx + s \cdot c \cdot sigy + \left(c^2 - s^2\right) \cdot txy \right);$$

$$\sigma 1 := 99729.57108$$

$$\sigma 2 := 43481.65174$$

$$\tau 12 := 18903.03136 \quad (10)$$

```
> # Compute local strains at z = -2.5 mm
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$$> eps1 := \frac{1}{E1 \cdot 10^6} \cdot sigma1 - \frac{nu12}{E1 \cdot 10^6} \cdot sigma2;$$

$$eps2 := -\frac{nu12}{E1 \cdot 10^6} \cdot sigma1 + \frac{1}{E2 \cdot 10^6} \cdot sigma2;$$

$$gamma12 := \frac{tau12}{G12 \cdot 10^6};$$

$$eps1 := 4.837276718\ 10^{-7}$$

$$eps2 := 0.000004067241796$$

$$\gamma 12 := 0.000002636406047 \quad (11)$$

```
> # Load Nx acting on ply 1
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$$> sxx1 := \left(Qbar[1, 1, 1] \cdot epsx + Qbar[1, 2, 1] \cdot epsy + Qbar[1, 3, 1] \cdot gammaxy\right) \cdot 10^6 :$$

$$> Nx1 := int(sxx1, z = h[1, 1] .. h[2, 1]);$$

$$Nx1 := 223.2004826 \quad (12)$$

```
> # Load Nx acting on ply 2
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$$> sxx2 := \left(Qbar[1, 1, 2] \cdot epsx + Qbar[1, 2, 2] \cdot epsy + Qbar[1, 3, 2] \cdot gammaxy\right) \cdot 10^6 :$$

$$> \mathrm{Nx2} := int(sxx2, z = h[2, 1] .. h[3, 1]);$$

$$Nx2 := 531.6501700 \quad (13)$$

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> # Load Nx acting on ply 3
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$$> \mathrm{sxx3} := \left(Qbar[1, 1, 3] \cdot epsx + Qbar[1, 2, 3] \cdot epsy + Qbar[1, 3, 3] \cdot gammaxy\right) \cdot 10^6 :$$

> Nx3 := $int(sxx3, z = h[3, 1]..h[4, 1])$;

$$Nx3 := 245.1493450 \tag{14}$$

>