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> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 4.4
  # Example on angle lamina
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> $restart:$
> $with(LinearAlgebra):$

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> # Define lamina angle
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> $s := evalf\left(\sin\left(\frac{\text{Pi}}{3.}\right)\right):$
> $c := evalf\left(\cos\left(\frac{\text{Pi}}{3.}\right)\right):$

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> # Define material properties for graphite/epoxy
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> $E1 := 181.0 \cdot 10^3:$
> $E2 := 10.3 \cdot 10^3:$
> $nu12 := 0.28:$
> $G12 := 7.17 \cdot 10^3:$

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> # Find elements of the compliance matrix ( unit=1/(MPa) )
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> $S11 := \frac{1}{E1};$
> $S12 := -\frac{nu12}{E1};$
> $S22 := \frac{1}{E2};$
> $S66 := \frac{1}{G12};$

$$5.525 \times 10^{-6}$$
$$-1.547 \times 10^{-6}$$
$$9.709 \times 10^{-5}$$
$$1.395 \times 10^{-4} \qquad \textbf{(1)}$$

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> # Find elements of the transformed compliance matrix ( unit=1/
  (MPa) )
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> $S11bar := S11 \cdot c^4 + S22 \cdot s^4 + (2 \cdot S12 + S66) \cdot c^2 \cdot s^2;$
> $S12bar := (S11 + S22 - S66) \cdot c^2 \cdot s^2 + S12 \cdot (c^4 + s^4);$
> $S16bar := (2 \cdot S11 - 2 \cdot S12 - S66) \cdot s \cdot c^3 - (2 \cdot S22 - 2 \cdot S12 - S66) \cdot s^3 \cdot c;$
> $S22bar := S11 \cdot s^4 + S22 \cdot c^4 + (2 \cdot S12 + S66) \cdot c^2 \cdot s^2;$
> $S26bar := (2 \cdot S11 - 2 \cdot S12 - S66) \cdot s^3 \cdot c - (2 \cdot S22 - 2 \cdot S12 - S66) \cdot s \cdot c^3;$
> $S66bar := 2 \cdot (2 \cdot S11 + 2 \cdot S22 - 4 \cdot S12 - S66) \cdot s^2 \cdot c^2 + S66 \cdot (c^4 + s^4);$

$$8.053 \times 10^{-5}$$
$$-7.878 \times 10^{-6}$$
$$-3.234 \times 10^{-5}$$
$$3.475 \times 10^{-5}$$
$$-4.696 \times 10^{-5}$$
$$1.141 \times 10^{-4} \qquad \textbf{(2)}$$

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> # Invert Sbar matrix to find transformed reduced stiffness matrix
  ( unit = MPa )
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$$> Sbar := \begin{bmatrix} S11bar & S12bar & S16bar \\ S12bar & S22bar & S26bar \\ S16bar & S26bar & S66bar \end{bmatrix} :$$

$Qbar := MatrixInverse(Sbar);$

$$Qbar := \begin{bmatrix} 2.365 \times 10^{4} & 3.246 \times 10^{4} & 2.005 \times 10^{4} \\ 3.246 \times 10^{4} & 1.094 \times 10^{5} & 5.419 \times 10^{4} \\ 2.005 \times 10^{4} & 5.419 \times 10^{4} & 3.674 \times 10^{4} \end{bmatrix} \quad (3)$$

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> # Define global stress vector (in MPa)
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$$> sig\_glo := \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix} :$$

```
> # Find global strains
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$> eps\_glo := Multiply(\ Sbar, sig\_glo);$

$$eps\_glo := \begin{bmatrix} 5.534 \times 10^{-5} \\ -3.078 \times 10^{-4} \\ 5.328 \times 10^{-4} \end{bmatrix} \quad (4)$$

```
> # Define transformation matrix and Reuter matrix
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$$> T := \begin{bmatrix} c^2 & s^2 & 2 \cdot s \cdot c \\ s^2 & c^2 & -2 \cdot s \cdot c \\ -s \cdot c & s \cdot c & c^2 - s^2 \end{bmatrix} :$$

$$R : = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} :$$

```
> # Find local strains
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$> eps\_loc := Multiply(\ Multiply(\ Multiply(R, T), MatrixInverse(R)), eps\_glo\ );$

$$eps\_loc := \begin{bmatrix} 1.367 \times 10^{-5} \\ -2.662 \times 10^{-4} \\ -5.809 \times 10^{-4} \end{bmatrix} \quad (5)$$

```
> # Find local stresses (in MPa)
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$> sig\_loc := Multiply(T, sig\_glo);$

(6)

$$sig\_loc := \begin{bmatrix} 1.714 \times 10^{0} \\ -2.714 \times 10^{0} \\ -4.165 \times 10^{0} \end{bmatrix} \quad (6)$$

> # Find principal stresses (in MPa)

$$> smax := evalf\left(\frac{(sig\_glo[1,1] + sig\_glo[2,1])}{2} + \mathrm{sqrt}\left(\left(\frac{(sig\_glo[1,1] - sig\_glo[2,1])}{2} \right)^2 + sig\_glo[3,1]^2 \right) \right);$$

$$smin := evalf\left(\frac{(sig\_glo[1,1] + sig\_glo[2,1])}{2} - \mathrm{sqrt}\left(\left(\frac{(sig\_glo[1,1] - sig\_glo[2,1])}{2} \right)^2 + sig\_glo[3,1]^2 \right) \right);$$

$$4.217 \times 10^{0}$$

$$-5.217 \times 10^{0} \quad (7)$$

> # Orientation of principal axes of stress (in degrees)

$$> thetap := evalf\left(\frac{\left(\frac{1}{2} \cdot \arctan\left(\frac{2 \cdot sig\_glo[3,1]}{sig\_glo[1,1] - sig\_glo[2,1]} \right) \right) \cdot 180}{\mathrm{Pi}} \right);$$

$$2.900 \times 10^{1} \quad (8)$$

> # Maximum shear stress

$$> tau\_max := \frac{(smax - smin)}{2};$$

$$4.717 \times 10^{0} \quad (9)$$

> # Direction of maximum shear stress (in degrees)

$$> thetas := evalf\left(\frac{\frac{1}{2} \cdot \arctan\left(-\frac{(sig\_glo[1,1] - sig\_glo[2,1])}{2 \cdot sig\_glo[3,1]} \right) \cdot 180}{\mathrm{Pi}} \right);$$

$$-1.600 \times 10^{1} \quad (10)$$

> # Principal strains

$$> eps\_max := evalf\left(\frac{(eps\_glo[1,1] + eps\_glo[2,1])}{2} + \mathrm{sqrt}\left(\left(\frac{(eps\_glo[1,1] - eps\_glo[2,1])}{2} \right)^2 + \left(\frac{eps\_glo[3,1]}{2} \right)^2 \right) \right);$$

$$eps\_min := evalf\left(\frac{(eps\_glo[1,1] + eps\_glo[2,1])}{2} - \mathrm{sqrt}\left(\left(\frac{(eps\_glo[1,1] - eps\_glo[2,1])}{2} \right)^2 + \left(\frac{eps\_glo[3,1]}{2} \right)^2 \right) \right);$$

$$1.961 \times 10^{-4}$$

$$-4.486 \times 10^{-4} \quad (11)$$

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> # Orientation of principal axes of strain (in degrees)
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> $thetap := evalf\left(\frac{\left(\frac{1}{2}\cdot\arctan\left(\frac{eps\_glo[3,1]}{eps\_glo[1,1]-eps\_glo[2,1]}\right)\right)\cdot 180}{\mathrm{Pi}}\right);$

$$2.786\times 10^{1} \quad \textbf{(12)}$$

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> # Maximum shear strain
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> $gamma\_max := eps\_max - eps\_min;$

$$6.448\times 10^{-4} \quad \textbf{(13)}$$

```
> # Direction of maximum shear strain (in degrees)
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> $thetas := evalf\left(\frac{\frac{1}{2}\cdot\arctan\left(-\frac{(eps\_glo[1,1]-eps\_glo[2,1])}{eps\_glo[3,1]}\right)\cdot 180}{\mathrm{Pi}}\right);$

$$-1.714\times 10^{1} \quad \textbf{(14)}$$

>