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> # Prof. Dr. Serkan Dağ
  # ME 451 Introduction to Composite Structures
> # File 4.3
  # Elements of Transformed Reduced Stiffness Matrix
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> *restart* :
> *with*(*LinearAlgebra*) :

```
> # Define [T], inverse [T], [Q], and [R] matrices
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$$> T := \begin{bmatrix} (\cos(th))^2 & (\sin(th))^2 & 2\cdot\sin(th)\cdot\cos(th) \\ (\sin(th))^2 & (\cos(th))^2 & -2\cdot\sin(th)\cdot\cos(th) \\ -\sin(th)\cdot\cos(th) & \sin(th)\cdot\cos(th) & (\cos(th))^2-(\sin(th))^2 \end{bmatrix};$$

$$T := \begin{bmatrix} \cos(th)^2 & \sin(th)^2 & 2\sin(th)\cos(th) \\ \sin(th)^2 & \cos(th)^2 & -2\sin(th)\cos(th) \\ -\sin(th)\cos(th) & \sin(th)\cos(th) & \cos(th)^2-\sin(th)^2 \end{bmatrix} \quad (1)$$

$$> Tinverse := \begin{bmatrix} (\cos(th))^2 & (\sin(th))^2 & -2\cdot\sin(th)\cdot\cos(th) \\ (\sin(th))^2 & (\cos(th))^2 & 2\cdot\sin(th)\cdot\cos(th) \\ \sin(th)\cdot\cos(th) & -\sin(th)\cdot\cos(th) & (\cos(th))^2-(\sin(th))^2 \end{bmatrix};$$

$$Tinverse := \begin{bmatrix} \cos(th)^2 & \sin(th)^2 & -2\sin(th)\cos(th) \\ \sin(th)^2 & \cos(th)^2 & 2\sin(th)\cos(th) \\ \sin(th)\cos(th) & -\sin(th)\cos(th) & \cos(th)^2-\sin(th)^2 \end{bmatrix} \quad (2)$$

$$> Q := \begin{bmatrix} Q11 & Q12 & 0 \\ Q12 & Q22 & 0 \\ 0 & 0 & Q66 \end{bmatrix};$$

$$Q := \begin{bmatrix} Q11 & Q12 & 0 \\ Q12 & Q22 & 0 \\ 0 & 0 & Q66 \end{bmatrix} \quad (3)$$

$$> R := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix};$$

$$R := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad (4)$$

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> # Find the elements of Qbar matrix
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> *Qbar* := *Multiply*(*Multiply*(*Multiply*(*Multiply*(*Tinverse*, *Q*), *R*), *T*), *MatrixInverse*(*R*)) :

> $Qbar11 := expand(\ Qbar[1, 1]\ );$

$$Qbar11 := \cos(th)^4\ Q11 + 2\cos(th)^2 \sin(th)^2\ Q12 + \sin(th)^4\ Q22 + 4\sin(th)^2 \cos(th)^2\ Q66 \quad \textbf{(5)}$$

> $Qbar12 := expand(\ Qbar[1, 2]\ );$

$$Qbar12 := \sin(th)^2 \cos(th)^2\ Q11 + \sin(th)^4\ Q12 + \cos(th)^4\ Q12 + \cos(th)^2 \sin(th)^2\ Q22 - 4\sin(th)^2 \cos(th)^2\ Q66 \quad \textbf{(6)}$$

> $Qbar16 := expand(\ Qbar[1, 3]\ );$

$$Qbar16 := \sin(th)\cos(th)^3\ Q11 + \sin(th)^3 \cos(th)\ Q12 - \sin(th)\cos(th)^3\ Q12 - \sin(th)^3\cos(th)\ Q22 - 2\sin(th)\cos(th)^3\ Q66 + 2\sin(th)^3\cos(th)\ Q66 \quad \textbf{(7)}$$

> $Qbar22 := expand(\ Qbar[2, 2]\ );$

$$Qbar22 := \sin(th)^4\ Q11 + 2\cos(th)^2 \sin(th)^2\ Q12 + \cos(th)^4\ Q22 + 4\sin(th)^2 \cos(th)^2\ Q66 \quad \textbf{(8)}$$

> $Qbar26 := expand(\ Qbar[2, 3]\ );$

$$Qbar26 := \sin(th)^3\cos(th)\ Q11 + \sin(th)\cos(th)^3\ Q12 - \sin(th)^3\cos(th)\ Q12 - \sin(th)\cos(th)^3\ Q22 + 2\sin(th)\cos(th)^3\ Q66 - 2\sin(th)^3\cos(th)\ Q66 \quad \textbf{(9)}$$

> $Qbar66 := expand(\ Qbar[3, 3]\ );$

$$Qbar66 := \sin(th)^2\cos(th)^2\ Q11 - 2\cos(th)^2\sin(th)^2\ Q12 + \cos(th)^2\sin(th)^2\ Q22 + Q66\cos(th)^4 - 2\sin(th)^2\cos(th)^2\ Q66 + Q66\sin(th)^4 \quad \textbf{(10)}$$

```
> # Show that Qbar matrix is symmetric
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> $simplify(\ Qbar[2, 1] - Qbar[1, 2]\ );$

$$0 \quad \textbf{(11)}$$

> $simplify(\ Qbar[3, 1] - Qbar[1, 3]\ );$

$$0 \quad \textbf{(12)}$$

> $simplify(\ Qbar[3, 2] - Qbar[2, 3]\ );$

$$0 \quad \textbf{(13)}$$

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