Breaking symmetries

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Outline

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Nobel Laureates

Half of the Nobel prize is given to

- YOICHIRO NAMBU(1921) for *the discovery of the mechanism of spontaneous broken symmetry in subatomic physics*

... and the other half is divided equally between

- MAKOTO KOBAYASHI(1944) and TOSHIHIDE MASKAWA(1940) for *the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature*


Symmetries in Physics

- Symmetries play a crucial Law in Physics
- Noether's Teorem:
  If there is a continuous symmetry in nature, then there exists a corresponding conserved quantity and vice verse
  - Energy Conservation $\leftrightarrow$ time translation invariance
  - Momentum conservation $\leftrightarrow$ position translation invariance
  - Angular momentum conservation $\leftrightarrow$ rotational invariance
  - Electric charge conservation $\leftrightarrow$ $U(1)$ gauge invariance

Outline

A Toy Model with Z_2 Symmetry

- Consider the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

  where $\phi(x)$ is a scalar field and $g_{\mu\nu} = diag(1, -1, -1, -1)$ is the flat Minkowsky metric
- $\mathcal{L}$ describes a self-interacting particle with spin zero.
- Under the transformation $\phi \to -\phi$, $\mathcal{L}$ is invariant. $\mathcal{L}$ has a Z_2 symmetry.

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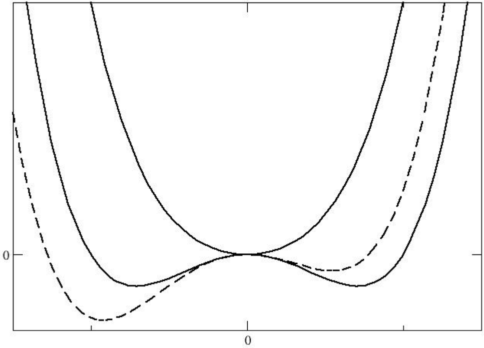
Breaking the Symmetry

- Explicitly Breaking the Symmetry:
  - If the term $\mathcal{L}^{br} = \alpha\phi^3$ is added to $\mathcal{L}$, the symmetry is said to be explicitly broken, i.e. the total $\mathcal{L}_t = \mathcal{L} + \mathcal{L}^{br}$ does not have a Z_2 symmetry.

- Spontaneously Breaking the Symmetry
  - All physical systems behave in a way to minimize the potential energy:

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

  - $\lambda > 0$ since in the other case, $V(\phi)$ does not have a minimum and the physical system is not stable.
  - The sign of μ^2 is not fixed.
    - Case 1: If $\mu^2 > 0$, then the minimum of the potential is at $\phi = 0$, and μ is the mass of the scalar particle
    - Case 2: If $\mu^2 < 0$, then the minimum of the potential is at $\phi = \pm\phi_0 \equiv \pm\sqrt{\frac{-\mu^2}{\lambda}}$.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- The nature around us is an excitation around the minimum of the potential.
- Let $\phi = \phi_0 + \psi(x)$
- Them the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}\partial_\mu\psi\partial^\mu\psi - \frac{1}{2}m^2\psi^2 - \frac{m^2}{2\phi_0}\psi^3 - \frac{m^2}{8\phi_0^2}\psi^4 + \frac{m^2\phi_0^2}{8} \quad (1)$$

where $m^2 = -2\mu^2 > 0$ is the square of the mass of the scalar particle described by the field ψ

- Due to the ψ^3 term, the Z_2 symmetry is not obvious, it is hidden. Z_2 symmetry is said to be spontaneously broken.
- The minimum of the energy of the system does not show the symmetry of the Lagrangian density

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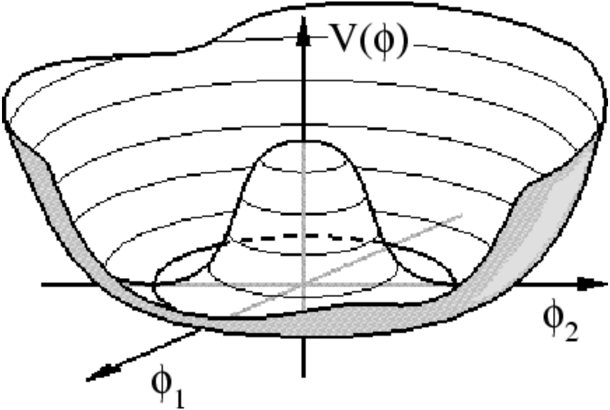
Breaking a Continuous Symmetry

- Consider a system described by the Lagrangian density

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \lambda(\phi^* \phi - v^2)^2$$

  where λ and v are positive constants and ϕ is a complex field.
- This Lagrangian is invariant under the transformation $\phi \to e^{i\alpha}\phi$ where α is an arbitrary real constant.
- The potential energy of the system has a minimum for $|\phi| = v$, i.e. on every point of the circle of radius v in the complex plane.


V(ϕ)
ϕ2
ϕ1

- Expand the complex field ϕ around the minimum in terms of two real fields as:

$$\phi = e^{i\psi_1/v}(v + \frac{\psi_2}{\sqrt{2}})$$

- In terms of the new real fields, the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}\partial_\mu\psi_2\partial^\mu\psi_2 + \frac{1}{2}\partial_\mu\psi_1\partial^\mu\psi_1 - \frac{1}{2}m_1^2\psi_1^2 - \frac{1}{2}m_2^2\psi_2^2 + \cdots$$

  where $m_1 = 0$ and $m_2 = 2\lambda v$. This Lagrangian describes two scalar particles one of which is massless.

- ψ_1 is called the Goldstone Boson. There is one Goldstone boson because the symmetry $U(1)$, which has a single generator, is broken down to nothing.

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Standard Model

- Standard Model is a local gauge theory with the gauge symmetry group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ and with fermion content

$$\text{Quarks:} \left\{ \begin{pmatrix} \text{u} \\ \text{d} \end{pmatrix}_L \begin{pmatrix} \text{c} \\ \text{s} \end{pmatrix}_L \begin{pmatrix} \text{t} \\ \text{b} \end{pmatrix}_L u_R,\ d_R,\ c_R,\ s_R,\ t_R,\ b_R \right.$$

$$\text{Leptons:} \left\{ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L e_R,\ \mu_R,\ \tau_R \right.$$

- The quantum numbers of the fermions are:

$$\begin{aligned} Q_L &= (3,\ 2, 1/3),\ u_R = (3,\ 1,\ 4/3),\ d_R = (3,\ 1,\ -2/3) \\ L_L &= (1,\ 2,\ -1),\ e_R = (1,\ 1,\ -2),\ \nu_R = (1, 1, 0) \end{aligned}$$

- If ν_R exists and the neutrinos are massless, then there is no way to produce the neutrinos.

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- The Lagrangian for the Standard Model is given by

$$\begin{aligned}\mathcal{L} &= \sum \bar{Q}_L \not{D} Q_L + \sum \bar{u}_R \not{D} u_R + \sum \bar{d}_R \not{D} d_R \\ &+ \sum \bar{L}_L \not{D} L_L + \sum \bar{e}_R \not{D} e_R \\ &+ -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \sum_{i=1}^{3} \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} \sum_{a=1}^{8} G^a_{\mu\nu} G^{a\mu\nu}\end{aligned}$$

where $\not{D} = \gamma_\mu \mathcal{D}^\mu$, γ_μ are the 4 Dirac matrices satisfying the algebra $\gamma_\mu, \gamma_\nu = 2g_{\mu\nu}$, $\mathcal{D}_\mu$ is the covariant derivative defined as

$$\mathcal{D}_\mu = \partial_\mu - ig_1 \frac{\lambda^a}{2} G^a_\mu - ig_2 \frac{\sigma^i}{2} W^i_\mu - ig_3 \frac{Y}{2} B_\mu \qquad (2)$$

and

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g^{abc} A^b_\mu A^c_\nu \qquad (3)$$

- Under a gauge transformation, the fermions transform as the fundamental representation and the gauge bosons as the adjoint representations

$$\begin{aligned} \Psi &\rightarrow U_g(x)\Psi \\ A^a_\mu \tau^a &\rightarrow U_g(x) A^a_\mu \tau^a U_g^{-1}(x) + (\partial_\mu U_g(x)) U_g^{-1}(x) \end{aligned}$$

- The chiral fermions are define in terms of the $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ matrix as $\psi_{L(R)} = \frac{1}{2}(1-(+)\gamma_5)\psi$
- The Lagrangian describes massless fermions and massless gauge bosons interacting by gauge forces.

- The mass term for a fermion is $\mathcal{L}_{mass} = m\bar{\psi}\psi = m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R)$
- Since the mass term mixes the left and the right fermions, it is not invariant under $SU(2)_L$.
- Since the left and the right handed fermions have different hyper charges Y, the mass terms is also not invariant under $U(1)_Y$
- The mass term for the gauge bosons should be $m_G^2 A_\mu A^\mu$ which is not gauge invariant.
- Explicitly breaking the gauge symmetry by adding the mass terms by hand also does not work because this also spoils renormalizability.

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The Higgs Mechanism

- The resolution is to break the gauge symmetry spontaneously: the masses will be generated without spoiling renormalizability
- Assign a complex Higgs doublet with the quantum numbers $(1,\ 2,\ 1)$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- Add $\mathcal{L}' = \mathcal{L}_\Phi + \mathcal{L}_Y$ to the Lagrangian:

$$\begin{aligned} \mathcal{L}_\Phi &= (\mathcal{D}_\mu \Phi)^\dagger \mathcal{D}^\mu \Phi - \frac{\lambda}{4}(\Phi^\dagger \Phi - v^2)^2 \\ \mathcal{L}_\mathcal{Y} &= y^u_{qij} \bar{Q}^i_L \Phi u^j_R + y^d_{qij} \bar{Q}^i_L \Phi^c d^j_R + y_{lij} L^i_L \Phi^c e^j_R + h.c. \end{aligned}$$

- In the minimum of the Higgs potential

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{4}$$

$\mathcal{L}_Y$ becomes

$$\mathcal{L}_Y = y^u_{qij} v \bar{u}^i_L u^j_R + y^d_{qij} v \bar{d}^i_L d^j_R + y_{lij} v \bar{e}^i_L e^j_R + h.c. \tag{5}$$

- The mass matrix is not diagonal, but can be diagonalized by a biunitary transformation
- Diagonalizing the mass matrix, the kinetic terms on the complete Lagrangian remain diagonal.

- The interaction terms with the $W^{\pm}$ bosons become

$$\mathcal{L}_W = V_{ij}\bar{u}^i\gamma_\mu(1-\gamma_5)d^j W^{+\mu} + h.c. \qquad (6)$$

where $W^{\pm} = \frac{1}{\sqrt{2}}(W^1 \mp iW^2)$

- V is a unitary matrix and is called the Cabibbo Matrix for 2 generations and CKM matrix for 3 generations.

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CP Violation

- C(charge conjugation): replace particles by anti particles
- P(parity): $\vec{x} \to -\vec{x}$
- CP violation is first discovered in kaon systems in 1964 by James Cronin and Val Fitch (Nobel Prize in Physics 1980)
- CP violation is a key ingredient to explain the observed matter-anti-matter asymmetry of the universe.
- Combined CP transformation transforms each operator in the SM by its hermitian conjugate
- Equivalently, CP transformation replaces every parameter by its complex conjugate
- Only parameter in the SM that might be complex are the elements of the mixing matrix V.

- For n families, The $n \times n$ matrix V contains n^2 complex, or $2n^2$ real parameters.
- unitarity, $V^\dagger V = 1$ gives n^2 real constraints
- $2n - 1$ unphysical phases can be eliminated by field phase redefinitions.
- $\frac{n(n-1)}{2}$ of the parameters are rotation angles
- The remaining $\frac{(n-1)(n-2)}{2}$ are physical complex phases.
- If there is CP Violation,n has to be at least 3 (1972)
- The top quark (the last member of the third family) is discovered in 1994.

Thank you for your interest!