Light Cone QCD Sum Rules and the Multipole Moments of Octet and Decuplet Baryons

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Abstract

In this talk, the application of the Light Cone QCD sum rules to the calculation of the magnetic moments of the octet and decuplet baryons and also to the calculation of the magnetic and quadrupole moments for the radiative transitions $\mathcal{D} \to \mathcal{O}\gamma$ is discussed. Due to limited space the results for the $\mathcal{D} \to \mathcal{O}\gamma$ transitions are presented only and they are compared with existing theoretical and experimental results.

Outline

- Introduction
- Light Cone QCD Sum Rules for Radiative Decays: General Overview
- Phenomenological Description of the Electromagnetic Interactions of the Octet and Decuplet Baryons
- Derivation of the Sum Rules: $\mathcal{D} \rightarrow \mathcal{O}$ case.
- Numerical Analysis and Conclusion

Introduction

- Electromagnetic interactions of hadrons provide unique insight into the structure of hadrons.
- They can be used to study the non-perturbative dynamics inside hadrons.
- They are also important testing grounds for non perturbative methods.
- Light Cone QCD Sum rules is one of the succesfull non-perturbative methods which is based on the fundamental QCD Lagrangian.

- That the nucleon could be deformed was proposed more than 20 years ago [1]
- It is still an intensive theoretical and experimental area of activity.
- The process $\Delta \to N\gamma$ can give us information on this aspect also.
- If the wave functions of both the initial and final states are spherical, the quadrupole moments for this decay should vanish.
- Recent experimental results show that the quadrupole moments are non zero.
- The spin-parity selection rules allow for magnetic dipole (M1), and electric (E2) or coulomb (C2) quadrupole moments.
- The moments have been studied using various models.

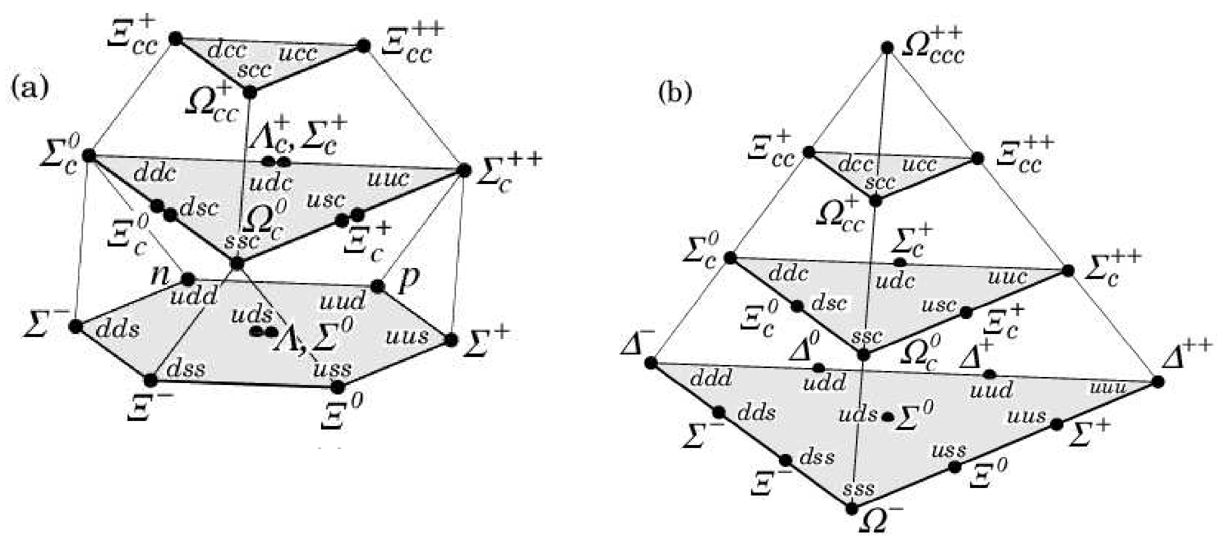
- In the naive (spherical) quark model of the nucleon, Δ is a pure spin flip (M1) transition, and $E2 = C2 = 0$
- Experimentally, indeed M1 dominates over the other moments.
- In other refined models, small $E2$ and $C2$ moments are predicted.
- In "QCD inspired" quark model, one introduces a tensor forces which introduce a d-state admixture to the nucleon.
- Stronger contributions are expected from pion clouds.

Octet and Decuplet Baryons:

- These are the baryons belonging to the octet and decuplet representation in the product [2]

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10 \tag{1}$$

of $SU(3)_f$.



Light Cone QCD Sum Rules for Radiative Decays: General Overview

- In QCD sum rules approach, properties of hadrons are expressed in terms of the vacuum properties through non zero condensates in the vacuum
- One starts by studying a correlation function of the form:

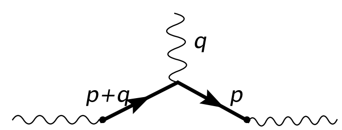
$$\Pi(p,q) = i \int d^4x e^{ipx} \langle \gamma(q) | \mathcal{T} \eta_{\mathcal{B}_1}(x) \bar{\eta}_{\mathcal{B}_2}(0) | 0 \rangle \tag{2}$$

  where $\eta_{\mathcal{B}}$ is an operator with the quantum numbers of the $\mathcal{B}$ baryon and $\mathcal{B}$ is the lightest baryon which it can create from the vacuum.
- For $p^2 > 0$, the correlation function is calculated in terms of hadronic parameters.
- In the deep Euclidean region, $p^2 << 0$ and $(p+q)^2 << 0$, the correlation function is calculated using the OPE in terms of QCD degrees of freedom.
- Sum rules are obtained by matching the two representation using spectral representation.

- For $p^2 > 0$, two complete sets of hadronic states can be inserted to get:

$$\Pi(p,q) = \frac{\langle 0|\eta_{\mathcal{B}_1}|\mathcal{B}_1(p)\rangle}{p^2 - m_1^2} \langle \mathcal{B}_1|\mathcal{B}_2\rangle_\gamma \frac{\langle \mathcal{B}_2(p+q)|\eta_{\mathcal{B}_2}|0\rangle}{(p+q)^2 - m_2^2} + \cdots \tag{3}$$

where $\cdots$ stands for the contribution of higher states and continuum.



- The matrix elements of the currents between the single baryon state and the vacuum are defined as

$$\langle 0|\eta_{\mathcal{B}}|\mathcal{B}(p)\rangle \quad = \quad \lambda_{\mathcal{B}} u(p,s) \tag{4}$$

where λ's are the residues and $u(p,s)$ is the function which describes the baryon with momentum p and spin s.

Phenomenological Description of the Electromagnetic Interactions of the Octet and Decuplet Baryons

- The photon vertex $\langle \mathcal{B}_2(p)|\mathcal{B}_1(p+q)\rangle_\gamma$ can be written as:

$$\langle \mathcal{B}_2(p)|\mathcal{B}_1(p+q)\rangle_\gamma = J_\mu \varepsilon^\mu \tag{5}$$

  where J_μ is the transition current and ε^μ is the polarization vector of the photon.

- Gauge invariance implies that

$$q_\mu J^\mu = 0 \tag{6}$$

- The most general current for the $\mathcal{O} \to \mathcal{O}$ transition currents consistant with the spin parity selection rules is

$$\begin{aligned} J_\mu^{1/2\to 1/2} &= \bar{u}(p)\left(f_1\gamma_\mu - i\frac{f_2}{2m}\sigma_{\mu\nu}q^\nu\right)u(p+q) \\ &= \bar{u}(p)\left[(f_1+f_2)\,\gamma_\mu - \frac{P_\mu}{2m}f_2\right]u(p+q) \end{aligned} \tag{7}$$

  where $P = p + (p+q) = 2p + q$, f_i are form factors which in general depend on q^2, and u is a Dirac spinor.

- To obtain a more intuitive picture of what these formfactors represent, consider this current in the Breit frame, i.e. the frame in which $\vec{P} = 0$

- In this frame, the zeroth component becomes:

$$
\begin{aligned}
J_0^{1/2\to 1/2} &= (f_1+f_2)\bar{u}(p)\gamma_0 u(p+q) - \frac{P_0}{2m} f_2 \bar{u}(p) u(p+q) \\
&= f_1 \bar{u}(p)\gamma_0 u(p+q). \qquad (8)
\end{aligned}
$$

- The point particle case, corresponds to $f_1 = 1$. Hence f_1 describes the spatial distribution of the electric charge of the hadron. Hence f_1 can be identified with the electric form factor.

- For the vector part we get

$$
\vec{J}^{1/2\to 1/2} = \frac{1}{2m}(f_1+f_2) i\vec{q} \times \left(\bar{u}(p)\vec{\Sigma} u(p+q)\right) \qquad (9)
$$

where $\vec{\Sigma}$ is the three dimensional spin operator [3]:

$$
\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \qquad (10)
$$

This can be correlated with the current density

$$
\vec{J}(r) = \vec{\bigtriangledown} \times \vec{\mu}(r) \qquad (11)
$$

Hence $f_1 + f_2$ can be identified by the magnetic form factor.

- For the interaction of higher spin particles, the structure of the current is richer.
- The current for the $\mathcal{D} \to \mathcal{D}$ transition is:

$$J_\rho = \bar{u}_\mu(p)\mathcal{O}^{\mu\rho\nu}(p,q)u_\nu(p+q)\,, \tag{12}$$

where the Lorenz tensor $\mathcal{O}^{\mu\rho\nu}$ is given by:

$$\begin{aligned}\mathcal{O}^{\mu\rho\nu}(p,q) &= -g^{\mu\nu}\left[\gamma_\rho(f_1+f_2)+\frac{P_\rho}{2M}f_2+q_\rho f_3\right]- \\ &- \frac{q_\mu q_\nu}{(2M)^2}\left[\gamma_\rho(G_1+G_2)+\frac{P_\rho}{2M}G_2+q_\rho G_3\right]\end{aligned} \tag{13}$$

where M is the mass of the decuplet baryon.

- $u_\mu(p)$ is the Rarita-Schwinger spin vector satisfying $(\not{p}-m)u_\mu(p)$, $\gamma^\mu u_\mu(p)=0$ and $p^\mu u_\mu(p)=0$
- The magnetic moment of the decuplet baryon is given by $g_M = 3(f_1+f_2)$.

- Finally for the mixed $\mathcal{D} \to \mathcal{O}$ transition, the current can be written as:

$$\begin{aligned} J_\mu^{3/2\to 1/2} &= eu(p)\left\{G_1\left(q_\rho \gamma_\mu - g_{\mu\rho}\ \not{q}\right)\gamma_5\right. \\ &+ G_2\left(P_\mu q_\rho - (Pq) g_{\mu\rho}\right)\gamma_5 \\ &+ \left. G_3\left(q_\mu q_\rho - q^2 g_{\mu\rho}\right)\gamma_5\right\} u^\rho(p+q) \end{aligned} \tag{14}$$

where $P = \frac{1}{2}\left(p + (p+q)\right)$.

- The magnetic dipole moment, G_M, and the electric quadrupole moment G_E are defined as(at $q^2 = 0$): [4]:

$$\begin{aligned} G_E &= \frac{m(M-m)}{3}\left(\frac{G_1}{M} + G_2\right) \\ G_M &= \frac{m}{3}\left[(3M+m)\,\frac{G_1}{M} + (M-m)G_2\right] \end{aligned} \tag{15}$$

where M and m are the masses of the decuplet and the octet baryon respectively.

- The decay width for the decay $\mathcal{D} \to \mathcal{O}\gamma$ in terms of the multipole moments G_E and G_M:

$$\Gamma_\gamma = 3\frac{\alpha}{32}\frac{(M^2-m^2)^3}{M^3m^2}\left(G_M^2+3G_E^2\right) \tag{16}$$

- The helicity amplitudes are:

$$\begin{aligned} A_{1/2} &= -\eta\left(G_M-3G_E\right) \\ A_{3/2} &= -\sqrt{3}\eta\left(G_M+G_E\right) \end{aligned} \tag{17}$$

where

$$\eta = \frac{1}{2}\sqrt{\frac{3}{2}}\left(\frac{M^2-m^2}{2m}\right)^{1/2}\frac{e}{2m}$$

Derivation of the Sum Rules: $\mathcal{D} \to \mathcal{O}$ case

Hadronic Representation:

- Using the matrix elements for the currents and the vertex, and

$$\sum_s u_\alpha(p,s)\bar{u}_\beta(p,s) = -\left(\not p + M\right)\left\{g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2p_\alpha p_\beta}{3M^2} + \frac{p_\alpha\gamma_\beta - p_\beta\gamma_\alpha}{3M}\right\} \tag{18}$$

one can obtain an expression for the correlation function in terms of the form factors.

- Schematically, we have

$$\begin{aligned} T_\mu \quad &= \quad e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\frac{1}{p^2-m^2}\frac{1}{(p+q)^2-M^2}\left[G_2(\varepsilon p)q_\mu \not q \not p\gamma_5 + \right. \\ &+ \quad M\left(2G_1 - G_2(M-m)\right)(\varepsilon p)q_\mu\gamma_5 \\ &- \quad \left(2G_1 - G_2(M-m)\right)(\varepsilon p)q_\mu \not p\gamma_5 - \left(2G_1 + G_2 m\right)(\varepsilon p)q_\mu \not q\gamma_5\left.\right] + \text{other structures with } \not\varepsilon \text{ at} \\ &\quad\quad \text{the beginning and } \gamma_\mu,\ (p+q)_\mu \text{ or } \varepsilon_\mu \text{ at the end} \end{aligned} \tag{19}$$

where we have chosen the ordering $\not\varepsilon \not q \not p\gamma_\mu$

- The reason for choosing this ordering and the structure $\propto q_\mu$ is that, spin-1/2 particles do not contribute to this structure.

$$\langle 0|\eta_{\frac{3}{2}\mu}|\frac{1}{2}(p+q)\rangle = \left(A'(p+q)_\mu + B'\gamma_\mu\right)\gamma_5 u(p) \tag{20}$$

- Using the matrix element, one can not create a structure $\propto q_\mu$ with this ordering.
- In our study, we choose the structures $(\varepsilon p)q_\mu\gamma_5$ and $q_\mu\, \not{\varepsilon}\gamma_5$.
- The coefficients of these structures in the hadronic representation are:

For the $(\varepsilon p)q_\mu\gamma_5$ structure:

$$\Pi_2 \quad = \quad e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\frac{1}{p^2-m^2}\frac{1}{(p+q)^2-M^2}M\left(2G_1+G_2(m-M)\right) \tag{21}$$

For the $q_\mu\, \not{\varepsilon}\gamma_5$ structure:

$$\Pi_4 = -e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\frac{1}{p^2-m^2}\frac{1}{(p+q)^2-M^2}\left(2G_1+G_2 m\right) \tag{22}$$

- Both can be written in the form

$$\Pi_i = \int_0^\infty ds_1 ds_2 \frac{\rho_i^{phen}(s_1, s_2)}{(s_1 - p^2)(s_2 - (p+q)^2)} + \cdots \tag{23}$$

where $\cdots$ are polynomials in p^2 and $(p+q)^2$, and

$$\begin{aligned} \rho_2^{phen}(s_1, s_2) &= e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}M\left(2G_1 + G_2(m-M)\right)\delta(s_1 - m^2)\delta(s_2 - M^2) + \cdots \\ \rho_4^{phen}(s_1, s_2) &= -e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\left(2G_1 + G_2 m\right)\delta(s_1 - m^2)\delta(s_2 - M^2) + \cdots \end{aligned} \tag{24}$$

where $\cdots$ represent the contributions of the excited states and continuum.

QCD Representation

- For $p^2 << 0$ and $(p+q)^2 << 0$, the main contributions come from small distances, hence one can use the OPE.
- One must choose appropriate currents.

  For the Decuplet:

$$
\begin{aligned}
\eta_\mu^{\Sigma^{*0}} &= \sqrt{\frac{2}{3}}\epsilon^{abc}\left[\left(u^{aT}C\gamma_\mu d^b\right)s^c + \left(d^{aT}C\gamma_\mu s^b\right)u^c + \left(s^{aT}C\gamma_\mu u^b\right)d^c\right] \\
\eta_\mu^{\Sigma^{*+}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(d \to u) \\
\eta_\mu^{\Sigma^{*-}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(u \to d) \\
\eta_\mu^{\Delta^{*+}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(s \to u) \\
\eta_\mu^{\Delta^{*0}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(s \to d) \\
\eta_\mu^{\Xi^{*0}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(d \to s) \\
\eta_\mu^{\Xi^{*-}} &= \frac{1}{\sqrt{2}}\eta_\mu^{\Sigma^{*0}}(u \to s)
\end{aligned} \tag{25}
$$

For the Octet:

$$
\begin{aligned}
\eta^{\Sigma^0} &= \sqrt{\frac{1}{2}}\epsilon^{abc}\left[-\left(u^{aT}Cs^b\right)\gamma_5 d^c + \left(u^{aT}C\gamma_5 s^b\right)d^c + \left(s^{aT}Cd^b\right)\gamma_5 u^c - \left(s^{aT}C\gamma_5 d^b\right)u^c\right] \\
\eta^{\Sigma^+} &= \frac{1}{\sqrt{2}}\eta^{\Sigma^0}(d \to u) \\
\eta^{\Sigma^-} &= \frac{1}{\sqrt{2}}\eta^{\Sigma^0}(u \to d) \\
\eta^{p} &= -\sqrt{2}\eta^{\Sigma^0}(s \to u) \\
\eta^{n} &= -\sqrt{2}\eta^{\Sigma^0}(s \to d) \\
\eta^{\Xi^0} &= -\sqrt{2}\eta^{\Sigma^0}(d \to s) \\
\eta^{\Xi^-} &= -\sqrt{2}\eta^{\Sigma^0}(u \to s) \\
\eta^{\Lambda} &= \sqrt{\frac{1}{6}}\epsilon^{abc}\left[2\left(u^{aT}Cd^b\right)\gamma_5 s^c - 2\left(u^{aT}C\gamma_5 d^b\right)s^c + \left(u^{aT}Cs^b\right)\gamma_5 d^c\right. \\
&- \left.\left(u^{aT}C\gamma_5 s^b\right)d^c + \left(s^{aT}Cd^b\right)\gamma_5 u^c - \left(s^{aT}C\gamma_5 d^b\right)u^c\right] \qquad (26)
\end{aligned}
$$

- Accept for the correlation function for $\Sigma^{*0} \to \Lambda$, the others can be obtained from the correlation function for $\Sigma^{*0} \to \Sigma^0$:

$$
\begin{aligned}
\Pi^{\Sigma^{*+} \to \Sigma^+} &= \Pi^{\Sigma^{*0} \to \Sigma^0}(d \to u) \\
\Pi^{\Sigma^{*-} \to \Sigma^-} &= \Pi^{\Sigma^{*0} \to \Sigma^0}(u \to d) \\
\Pi^{\Delta^+ \to p} &= -2\Pi^{\Sigma^{*0} \to \Sigma^0}(s \to u) \\
\Pi^{\Delta^0 \to n} &= -2\Pi^{\Sigma^{*0} \to \Sigma^0}(s \to d) \\
\Pi^{\Xi^{*0} \to \Xi^0} &= -2\Pi^{\Sigma^{*0} \to \Sigma^0}(d \to s) \\
\Pi^{\Xi^{*-} \to \Xi^-} &= -2\Pi^{\Sigma^{*0} \to \Sigma^0}(u \to s)
\end{aligned}
\tag{27}
$$

- It is also possible to obtain the correlation function for $\Sigma^{*0} \to \Lambda$ from the correlation function for $\Sigma^{*0} \to \Sigma^0$ [5, 6].
- Note that:

$$2\eta^{\Sigma^0}(d \leftrightarrow s) = -\sqrt{3}\eta^{\Lambda} - \eta^{\Sigma^0} \tag{28}$$

- and hence

$$-\sqrt{3}\Pi^{\Sigma^{*0}\to\Lambda} = 2\Pi^{\Sigma^{*0}\to\Sigma^0}(d \leftrightarrow s) + \Pi^{\Sigma^{*0}\to\Sigma^0} \tag{29}$$

- Using Wick Theorem, one can express the correlation function in terms of diagrams like the ones in Fig. 1

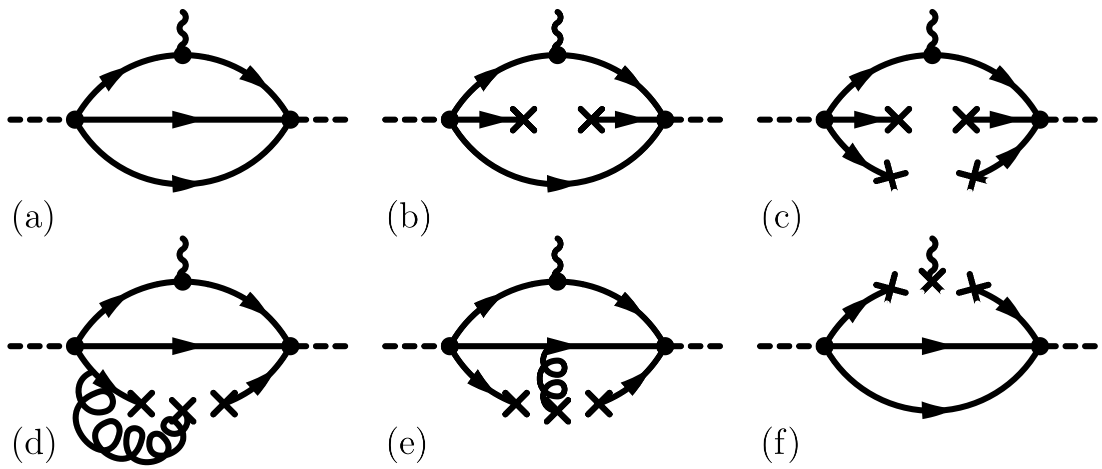
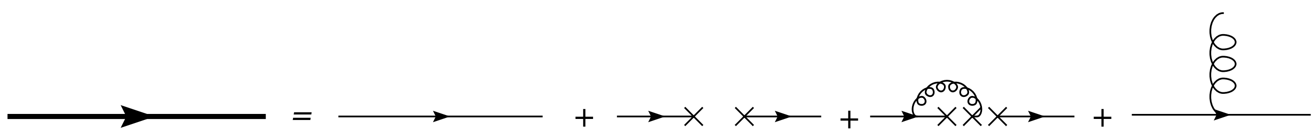


Figure 1: Some of the Feynman diagrams contributing to the correlation function

- The propagator for the light quarks are:

$$
\begin{aligned}
S_q(x) \quad = \quad & \frac{i\,\not{x}}{2\pi^2 x^4} - \frac{m_q}{4\pi^2 x^2} - \frac{\langle \bar{q}q \rangle}{12}\left(1 - i\frac{m_q}{4}\,\not{x}\right) - \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle \left(1 - i\frac{m_q}{6}\,\not{x}\right) \\
& - i g_s \int_0^1 du \left[\frac{\not{x}}{16\pi^2 x^2} G_{\mu\nu}(ux)\sigma_{\mu\nu} - u x^\mu G_{\mu\nu}(ux)\gamma^\nu \frac{i}{4\pi^2 x^2} \right. \\
& \left. - i \frac{m_q}{32\pi^2} G_{\mu\nu}\sigma^{\mu\nu}\left(\ln\left(\frac{-x^2\Lambda^2}{4}\right) + 2\gamma_E \right) \right]
\end{aligned}
\tag{30}
$$

where Λ is the energy cut off separating perturbative and non perturbative regimes.



- The emission of the photon can be both perturbative or non perturbative:
- To calculate the perturbative emission one uses the free quark propagator and the quark-photon vertex factor $-ie\gamma_\mu$.
- The non perturbative emission is described by matrix elements of the form $\langle\gamma(q)|\bar{q}(x)\Gamma q(0)|0\rangle$
- These matrix elements are expanded around the light cone $x^2=0$ and can be expressed in terms of photon wave functions: [7]:

$$
\begin{aligned}
&\langle\gamma(q)|\bar{q}(x)\sigma_{\mu\nu}q(0)|0\rangle = -ie_q\langle\bar{q}q\rangle(\varepsilon_\mu q_\nu-\varepsilon_\nu q_\mu)\int_0^1 du e^{i\bar{u}qx}\left(\chi\varphi_\gamma(u)+\frac{x^2}{16}\mathbb{A}(u)\right)\\
&-\frac{i}{2(qx)}e_q\langle\bar{q}q\rangle\left[x_\nu\left(\varepsilon_\mu-q_\mu\frac{\varepsilon x}{qx}\right)-x_\mu\left(\varepsilon_\nu-q_\nu\frac{\varepsilon x}{qx}\right)\right]\int_0^1 du e^{i\bar{u}qx}h_\gamma(u)\\
&\langle\gamma(q)|\bar{q}(x)\gamma_\mu q(0)|0\rangle = \cdots\\
&\langle\gamma(q)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle = \cdots\\
&\langle\gamma(q)|\bar{q}(x)g_s G_{\mu\nu}(vx)q(0)|0\rangle = \cdots\\
&\langle\gamma(q)|\bar{q}(x)g_s \tilde{G}_{\mu\nu}(vx)q(0)|0\rangle = \cdots\\
&\langle\gamma(q)|\bar{q}(x)g_s \tilde{G}_{\mu\nu}(vx)\gamma_\alpha\gamma_5 q(0)|0\rangle = \cdots\\
&\langle\gamma(q)|\bar{q}(x)g_s G_{\mu\nu}(vx)i\gamma_\alpha\gamma_5 q(0)|0\rangle = \cdots\\
&\langle\gamma(q)|\bar{q}(x)\sigma_{\alpha\beta}g_s G_{\mu\nu}(vx)q(0)|0\rangle = \cdots
\end{aligned}
\tag{31}
$$

- With these ingredients, the correlation function can be calculated in terms of the photon wave functions, condensates and QCD parameters.

- The correlation function can be written is the spectral representation:

$$\Pi_i = \int_0^\infty \frac{\rho_i^{OPE}(s_1, s_2)}{(s_1 - p^2)(s_2 - (p+q)^2)} + \cdots \tag{32}$$

  where $\cdots$ are polynomials in p^2 or $(p+q)^2$.

- For the spectral densities, on the phenomenological side we have:

$$\begin{aligned}\rho_2^{phen}(s_1,s_2) &= e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}M\left(2G_1+G_2(m-M)\right)\delta(s_1-m^2)\delta(s_2-M^2)+\cdots\\ \rho_4^{phen}(s_1,s_2) &= -e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\left(2G_1+G_2 m\right)\delta(s_1-m^2)\delta(s_2-M^2)+\cdots\end{aligned} \tag{33}$$

  and we also have calculated the spectral densities using QCD parameters: $\rho_2^{OPE}(s_1,s_2)$ and $\rho_4^{OPE}(s_1,s_2)$.

- The two problems that are to be solved in order to obtain the sum rules:

  The contributions of the higher states and the continuum are not known.

  The are unknown polynomials in the spectral representations of the correlation function

- To model the contributions of the higher states and the continuum, quark-hadron duality is used, i.e.

$$\begin{aligned}
\rho_2^{phen}(s_1,s_2) &= e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}M\left(2G_1+G_2(m-M)\right)\delta(s_1-m^2)\delta(s_2-M^2)+\rho_2^{OPE}\theta(s_1-s_0)\theta(s_2-s_0) \\
\rho_4^{phen}(s_1,s_2) &= -e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\left(2G_1+G_2 m\right)\delta(s_1-m^2)\delta(s_2-M^2)+\rho_4^{OPE}\theta(s_1-s_0)\theta(s_2-s_0) \qquad (34)
\end{aligned}$$

- To eliminate the unknown polynomials, the results are Borel transformed with respect to $p_1^2=p^2$ and $p_2^2=(p+q)^2$:

$$\begin{aligned}
\frac{1}{(p_i^2-m_i^2)^n} &\to \frac{(-1)^n}{\Gamma(n)}\frac{1}{M_i^{2(n-1)}}e^{-\frac{m^2}{M^2}} \\
p_i^{2n} &\to 0 \qquad (35)
\end{aligned}$$

  where M_i^2 are called the Borel parameters.

- Borel transformation also suppresses the contributions of the higher states and the continuum.

- The sum rules are obtained by equating both sides of the Borel transformed expressions:

$$\begin{aligned} e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}M\left(2G_1+G_2(m-M)\right)e^{-\frac{m^2}{M_1^2}-\frac{M^2}{M_2^2}} &= \int_0^{s_0}\rho_2^{OPE}(s_1,s_2)e^{-\frac{s_1}{M_1^2}-\frac{s_2}{M_2^2}} \\ -e\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\left(2G_1+G_2 m\right)e^{-\frac{m^2}{M_1^2}-\frac{M^2}{M_2^2}} &= \int_0^{s_0}\rho_4^{OPE}(s_1,s_2)e^{-\frac{s_1}{M_1^2}-\frac{s_2}{M_2^2}} \end{aligned} \qquad (36)$$

- For the sum rules we obtain:

$$\begin{aligned} \sqrt{3}\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}M\Sigma_2^{\Sigma^{*0}\to\Sigma^0}e^{-\frac{m^2}{M_1^2}-\frac{M^2}{M_2^2}} &= (e_d+e_u-2e_s)\frac{M^6}{32\pi^4}E_2(x)+\cdots \\ \sqrt{3}\lambda_{\mathcal{O}}\lambda_{\mathcal{D}}\Sigma_4^{\Sigma^{*0}\to\Sigma^0}e^{-\frac{m^2}{M_1^2}-\frac{M^2}{M_2^2}} &= (e_u m_u+e_d m_d-2e_s m_s)\frac{11M^4}{384\pi^2}E_1(x)+\cdots \end{aligned} \qquad (37)$$

where $\frac{1}{M^2}=\frac{1}{M_1^2}+\frac{1}{M_2^2}$, M_i^2 are the Borel parameters, $x=\frac{s_0}{M^2}$, s_0 is the continuum threshold, the functions $E_n(x)$ are used to subtract the contributions of the higher states and continuum and are defined as:

$$E_n(x)=1-e^{-x}\sum_{i=0}^{n}\frac{x^i}{i!} \qquad (38)$$

- In our numerical analysis, we set $M_1^2=M_2^2=2M^2$ and $u_0=\frac{M_1^2}{M_1^2+M_2^2}=\frac{1}{2}$

- An interesting limit to consider is the $SU(3)_f$ symmetry limit: i.e. the limit $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle = \langle \bar{q}q \rangle$ and $m_u = m_d = m_s = m_q$
- In this limit, we get

$$\Pi^{\Sigma^{*0} \to \Sigma^0} = (e_u + e_d - 2e_s)\mathcal{C} = \mathcal{C} \tag{39}$$

- Setting $s \to u$ and multiplying by -2, we get

$$\Pi^{\Delta^+ \to p} = -2(e_d - e_u)\mathcal{C} = 2\mathcal{C} = 2\Pi^{\Sigma^{*0} \to \Sigma^0} \tag{40}$$

- Similarly

$$2\Pi^{\Sigma^{*0} \to \Sigma^0} = \Pi^{\Delta^+ \to p} = -\Pi^{\Delta^0 \to n} = \Pi^{\Sigma^{*+} \to \Sigma^+} = -\Pi^{\Xi^{*0} \to \Xi^0} \tag{41}$$

and

$$\Pi^{\Sigma^{*-} \to \Sigma^-} = \Pi^{\Xi^{*-} \to \Xi^-} = 0 \tag{42}$$

- The last ingredients needed to obtain a prediction for Σ_2 and Σ_4, are the residues $\lambda_{\mathcal{O}}$ and $\lambda_{\mathcal{D}}$ which can be obtained using the mass sum rules.

- One considers the correlator

$$\Pi = i \int d^4 x e^{ipx} \langle 0 | \mathcal{T} \eta(x) \bar{\eta}(0) | 0 \rangle \tag{43}$$

- For the octet, on the phenomenological side it reduces to

$$\Pi = \lambda_{\mathcal{O}}^2 \frac{\not{p} + m}{p^2 - m^2} + \cdots \tag{44}$$

- For the decuplet

$$\Pi_{\mu\nu} = -\lambda_{\mathcal{D}}^2 \frac{\not{p} + M}{p^2 - M^2} \left(g_{\mu\nu} - \frac{1}{3}\gamma_\mu \gamma_\nu - \frac{2 p_\mu p_\nu}{3M^2} + \frac{p_\mu \gamma_\nu - p_\nu \gamma_\mu}{3M} \right) \tag{45}$$

- Note that the mass sum rules do not give us the sign of λ, hence LCQSR does not predict the sign of Σ_2 and Σ_4 separately, it only predicts their relative sign.

Numerical Analysis

For the numerical values of the input parameters, the following values are used: $\langle \bar{u}u\rangle(1\ GeV) = \langle \bar{d}d\rangle(1\ GeV) = -(0.243)^3\ GeV^3$, $\langle \bar{s}s\rangle(1\ GeV) = 0.8\langle \bar{u}u\rangle(1\ GeV)$, $m_0^2(1\ GeV) = 0.8$, $\chi(1\ GeV) = -4.4\ GeV^{-2}$, $\Lambda = 300\ MeV$ and $f_{3\gamma} = -0.0039\ GeV^2$.

- M^2 is a completely arbitrary parameter, and the predictions should be independent of its numerical value.
- An upper bound for M^2 is determined by requiring that the contributions of the higher states and the continuum are below a certain limit.
- A lower bound is obtained by requiring that the contributions of the term containing the highest power of $\frac{1}{M^2}$ is less then a limit.

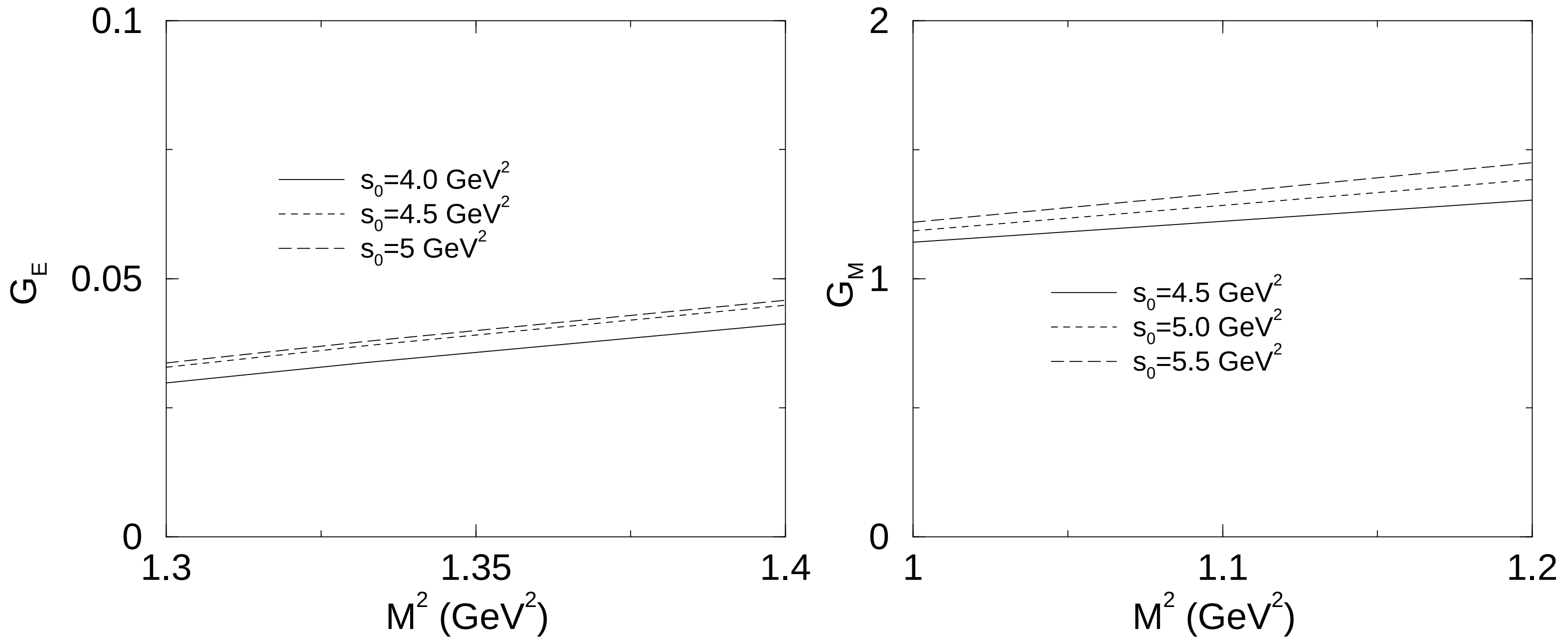


Figure 2: The M^2 dependence of $G_X^{\Delta^+\to p}$

Process	G_E	G_M	$\mathcal{R}_{EM}(\%)$
$\Delta^+ \to p$	0.037 ± 0.008	1.3 ± 0.2	-2.8
$\Delta^0 \to n$	-0.037 ± 0.008	-1.3 ± 0.2	-2.8
$\Sigma^{*+} \to \Sigma^+$	-0.017 ± 0.008	1.2 ± 0.1	1.4
$\Sigma^{*0} \to \Sigma^0$	-0.008 ± 0.008	0.55 ± 0.05	-1.4
$\Sigma^{*0} \to \Lambda$	0.023 ± 0.006	-1.48 ± 0.12	1.5
$\Sigma^{*-} \to \Sigma^-$	0.0087 ± 0.0006	-0.13 ± 0.01	6.7
$\Xi^{*0} \to \Xi^0$	0.031 ± 0.006	-1.5 ± 0.14	2.1
$\Xi^{*-} \to \Xi^-$	-0.0095 ± 0.0005	0.14 ± 0.02	6.8

Table 1: The predictions on the moments for various decays. The magnetic moments are given in terms of natural magnetons

Process	G_E	$G_E^{[8]}$	G_M	$G_M^{[8]}$	$\mathcal{R}_{EM}(\%)$	$\mathcal{R}_{EM}^{[8]}(\%)$
$\Delta^+ \to p$	0.037 ± 0.008	-0.04(11)	1.3 ± 0.2	2.01(33)	-2.8	3(8)
$\Delta^0 \to n$	-0.037 ± 0.008	0.04(11)	-1.3 ± 0.2	-2.01(33)	-2.8	3(8)
$\Sigma^{*+} \to \Sigma^+$	-0.017 ± 0.008	-0.06(8)	1.2 ± 0.1	2.13(16)	1.4	5(6)
$\Sigma^{*0} \to \Sigma^0$	-0.008 ± 0.008	-0.02(4)	0.55 ± 0.05	0.87(7)	-1.4	4(6)
$\Sigma^{*-} \to \Sigma^-$	0.0087 ± 0.0006	0.020(10)	-0.13 ± 0.01	-0.38(4)	6.7	8(4)
$\Xi^{*0} \to \Xi^0$	0.031 ± 0.006	0.03(4)	-1.5 $\pm$ 0.14	-2.26(14)	2.1	2.4(27)
$\Xi^{*-} \to \Xi^-$	-0.0095 $\pm$ 0.0005	-0.018(7)	0.14 ± 0.02	0.38(3)	6.8	7.4(30)

Table 2: Our results together with the results from lattice [8]

Process	$A_{1/2}(GeV^{-1/2})$	$A_{3/2}(GeV^{-1/2})$	$\Gamma(MeV)$
$\Delta^+ \to p$	-0.07 ± 0.01	-0.14 ± 0.02	0.19 ± 0.06
$\Delta^0 \to n$	0.07 ± 0.01	0.14 ± 0.02	0.19 ± 0.06
$\Sigma^{*+} \to \Sigma^+$	-0.048 ± 0.008	-0.08 ± 0.01	0.038 ± 0.012
$\Sigma^{*0} \to \Sigma^0$	-0.044 ± 0.005	-0.074 ± 0.007	0.032 ± 0.005
$\Sigma^{*0} \to \Lambda$	0.072 ± 0.005	0.12 ± 0.01	0.14 ± 0.02
$\Sigma^{*-} \to \Sigma^-$	0.0056 ± 0.0003	0.0075 ± 0.0006	0.00038 ± 0.00006
$\Xi^{*0} \to \Xi^0$	0.055 ± 0.004	0.088 ± 0.008	0.06 ± 0.01
$\Xi^{*-} \to \Xi^-$	-0.0059 ± 0.0006	-0.0079 ± 0.0012	0.00054 ± 0.00015

Table 3: The predictions for the helicity amplitudes and the decay widths for various decays

	Particle Data Group	Our Work
$A_{1/2}(\times 10^{-3} GeV^{-1/2})$	-135 ± 6	-70 ± 10
$A_{3/2}(\times 10^{-3} GeV^{-1/2})$	-255 ± 8	-140 ± 20
$\mathcal{R}_{EM}(\%)$	-2.5 ± 0.5	-3 ± 1

Table 4: Comparison of our results with the experimental results for the decay $\Delta^+ \to p\gamma$

References

[1] S. L. Glashow, Physica (Amsterdam) **96A** (1979) 27

[2] S. Eidelman et al., Phys. Lett. **B592**, 1 (2004)

[3] L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics 4: Quantum Electrodynamics* (Pergamon Press, Oxford, 1982)

[4] H. F. Jones and M.D. Scadron, Ann. Phys. **81** (1973) 1

[5] A.Ozpineci, S.B. Yakovlev, V.S. Zamiralov, preprint, hep-ph/0311271 (2003)

[6] A.Ozpineci, S.B. Yakovlev, V.S. Zamiralov, preprint, hep-ph/0310345 (2003)

[7] P. Ball, V. M. Braun, N. Kivel, Nucl. Phys. **B649** (2003) 263

[8] D.B. Leinweber, T. Draper, R.M. Woloshyn, Phys.Rev. **D48** (1993) 2230

[9] H. Tanabe and K. Ohta, Phys. Rev. **C31** (1985) 1876