Semileptonic $B \to \eta \ell \nu$ decay in light cone QCD

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Outline

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Motivation

- $B \to \eta \ell \nu$ decay can be used to extract the $B \to \eta$ transition formfactors.
- Together with the $B \to \eta' \ell \nu$, give information on the mixing angle between the η and η'
- Could lead to an alternative determination of the CKM matrix element V_{ub}
- This decay could be extensively studied in *Babar* and Belle.
- Theoretical calculation requires a nonperturbative method one of which is the LCQSR.
- In QSR, the nonperturbative aspects are parametrized in terms of vacuum condensates.

$B \to \eta\ell\nu$ Decay

- The amplitude for the decay $B \to \eta l\nu$ can be written as:

$$\mathcal{M}(B \to \eta l\nu) = \frac{G_F}{\sqrt{2}} V_{ub}\ \bar{l}\gamma_\mu(1-\gamma_5)\nu\ \langle \eta(p)|\bar{u}\gamma^\mu(1-\gamma_5)b|B(p+q)\rangle,$$

- The formfactors f_+ and f_- defined as

$$\langle \eta(p)|\bar{u}\gamma_\mu(1-\gamma_5)b|B(p+q)\rangle = 2f_+^\eta p_\mu + (f_+^\eta + f_-^\eta)q_\mu\,.$$

- The branching ratio is obtained as:

$$\Gamma(B \to \eta\ell\nu) = \frac{G^2|V_{ub}|^2}{192\pi^3 m_B^3}\int_0^{(m_B-m_\eta)^2} dq^2 |f_+^P(q^2)|^2 \lambda^{3/2}(m_B^2, m_\eta^2, q^2)$$

where $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ac$

LCQSR for the Formfactors

- The following correlator is studied to calculate the formfactors in LCQSR.

  $$\Pi_\mu(p,q) = i \int d^4x e^{iqx} \langle \eta(p) | \mathcal{T} \left\{ \bar{u}(x)\gamma_\mu(1-\gamma_5)b(x)j^B(0) \right\} |0\rangle$$

  where j^B is a current with the B meson quantum numbers.

- We choose $j^B = -\bar{b}i(1-\gamma_5)u$
- The correlator can be calculated in two different kinematical regions: $q^2 >> 0$ and $q^2 << 0$.
- The two different regions are matched dispersion relations of the form:

  $$\Pi(q^2,(p+q)^2) = \int ds \frac{\rho(s)}{s-(p+q)^2} + subtraction\ terms$$

- For $q^2 >> 0$, the correlation function can be calculated by inserting a complete set of hadronic states having the same quantum numbers as the current j^B.

- In terms of hadronic parameters, we obtain:

$$\begin{aligned}\Pi_\mu(p,q) &= \frac{\langle\eta|\bar{u}\gamma_\mu b|B\rangle\langle B|\bar{b}i\gamma_5 u|0\rangle}{m_B^2-(p+q)^2} \\ &- \sum_h \frac{\langle\eta|\bar{u}\gamma_\mu b|h\rangle\langle h|\bar{b}i(1-\gamma_5)u|0\rangle}{m_h^2-(p+q)^2} \\ &= \Pi_1(q^2,(p+q)^2)p_\mu + \Pi_2(q^2,(p+q)^2)q_\mu\end{aligned}$$

- Both pseudoscalar($J^P = 0^-$) and scalar ($J^P = 0^+$) hadrons contribute to the sums. If one had chosen $j^B = \bar{b}i\gamma_5 u$, then only the pseudoscalar hadrons would have contributed.

- Using

$$\langle B|\bar{b}i\gamma_5 d|0\rangle = \frac{m_B^2 f_B}{m_b}$$

one obtains

$$\begin{aligned}\rho_1(s) &= 2f_+^\eta(q^2)\frac{m_B^2 f_B}{m_b}\delta(s-m_B^2) + \rho_1^h(s) \\ \rho_2(s) &= (f_+^\eta + f_-^\eta)\frac{m_B^2 f_B}{m_b}\delta(s-m_B^2) + \rho_2^h(s)\end{aligned}$$

- In the other kinematical region, $q^2 << 0$, the main contribution is from the small distances and hence one can use the OPE to calculate the correlation function.

- Contracting the b-quark fields, one obtains:

$$\Pi_\mu(p,q) = -i\int d^4x e^{iqx}\langle\eta(p)|\bar{u}(x)\gamma_\mu(1-\gamma_5)S_b(x)(1-\gamma_5)u(0)|0\rangle$$

where

$$\begin{aligned} iS_b(x) &= iS_b^0(x) \\ &- ig_s\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 dv\left\{\frac{1}{2}\frac{\not{k}+m_b}{(m_b^2-k^2)^2}G^{\mu\nu}(vx)\sigma_{\mu\nu}\right. \\ &\left.+\frac{1}{m_b^2-k^2}vx_\mu G^{\mu\nu}(vx)\gamma_\nu\right\} \end{aligned}$$

- Due to the choice of the current, the contributions from the twist 3 wave functions are eliminated.

- For the u quark propogator:

$$\begin{aligned} S_u &= \frac{i\not{x}}{2\pi^2x^4}-\frac{\langle\bar{q}q\rangle}{12}+\frac{x^2}{192}m_0^2\langle\bar{q}q\rangle - \\ &- ig_s\int_0^1 dv\left[\frac{\not{x}}{16\pi^2x^2}G_{\mu\nu}(vx)\sigma_{\mu\nu}-vx_\mu G_{\mu\nu}\gamma_\nu\frac{i}{4\pi^2x^2}\right] \end{aligned}$$

- In order to calculate the correlation function within OPE, all the contributing terms can be expressed in terms of the following matrix elements:

$$
\begin{aligned}
&\langle \eta(p)|\bar{q}\gamma_\mu\gamma_5 q|0\rangle \\
&= -if_\eta p_\mu \int_0^1 du e^{-iupx}\left[\varphi_\eta(u) + \frac{1}{16}m_\eta^2 x^2 A(u)\right] \\
&-\frac{i}{2} f_\eta m_\eta^2 \frac{x_\mu}{px}\int_0^1 du e^{-iupx} B(u) \\
&\langle \eta(p)|\bar{q}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx) q(0)|0\rangle = \\
&f_\eta m_\eta^2 \left[p_\beta\left(g_{\alpha\mu} - \frac{x_\alpha p_\mu}{px}\right) - p_\alpha\left(g_{\beta\mu} - \frac{x_\beta p_\mu}{px}\right)\right] \\
&\times \int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) e^{-ipx(\alpha_1 + u\alpha_3)} \\
&+ f_\eta m_\eta^2 \frac{p_\mu}{px}(p_\alpha x_\beta - p_\beta x_\alpha)\int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i) e^{-ipx(\alpha_1 + u\alpha_3)} \\
&\langle \eta(p)|\bar{q}(x) g_s \tilde{G}_{\alpha\beta}(vx)\gamma_\mu q(0)|0\rangle = \\
&if_\eta m_\eta^2 \left[p_\beta\left(g_{\alpha\mu} - \frac{x_\alpha p_\mu}{px}\right) - p_\alpha\left(g_{\beta\mu} - \frac{x_\beta p_\mu}{px}\right)\right] \\
&\times \int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) e^{-ipx(\alpha_1 + u\alpha_3)} \\
&+ if_\eta m_\eta^2 \frac{p_\mu}{px}(p_\alpha x_\beta - p_\beta x_\alpha)\int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) e^{-ipx(\alpha_1 + u\alpha_3)}
\end{aligned}
$$

where $\overline{q}\Gamma q = \frac{1}{\sqrt{6}}(\overline{u}\Gamma u + \overline{d}\Gamma d - 2\overline{s}\Gamma s)$, $F_\eta = \frac{f_\eta}{\sqrt{6}}$, and $\langle 0|\overline{q}\gamma_\mu\gamma_5 q|\eta(q)\rangle = -if_\eta q_\mu$.

- Putting everything together, one obtains the following expression for the correlation function:

$$\Pi_\mu^{th} = i\int d^4x e^{iqx}\Bigg[\int_0^1 du e^{iupx}\Bigg\{-\frac{1}{32\pi^2}F_\eta m_b^2(16\varphi_\eta + Am_\eta^2 x^2)\frac{K_1(m_b\sqrt{-x^2})}{\sqrt{-x^2}}p_\mu$$

$$-\frac{1}{4\pi^2}F_\eta m_b^2 m_\eta^2 B\frac{K_1(m_b\sqrt{-x^2})}{\sqrt{-x^2}}\frac{x_\mu}{px}\Bigg\}$$

$$+\int \mathcal{D}\alpha\int_0^1 du e^{i(\alpha_2+u\alpha_3)px}\Bigg\{\frac{1}{12\pi^2}F_\eta m_\eta^2 m_b(\varphi_\parallel + \tilde{\varphi}_\parallel)K_0(m_b\sqrt{-x^2})\frac{x_\mu p^2 - p_\mu(px)}{px}$$

$$+\frac{1}{12\pi^2}F_\eta m_\eta^2 m_b(\varphi_\perp + \tilde{\varphi}_\perp)K_0(m_b\sqrt{-x^2})\frac{p^2 x_\mu + 2(px)p_\mu}{px}\Bigg\}\Bigg]$$

- Now we have two representation of the correlation function which can be matched using dispersion relations.
- The contribution of the higher states and continuum are modeled using the quark hadron duality, i.e.

$$\rho_i^h(s) = \rho_i^{th}(s)\theta(s - s_0)$$

- The contributions of the higher states are suppressed and the subtraction terms are eliminated by applying Borel transformations wrt the variable $(p+q)^2$. After the Borel transformation, the spectral representation becomes:

$$\Pi(s, M^2) = \int ds\rho(s)e^{-\frac{s}{M^2}}$$

- Our final results for the formfactors are:

$$
\begin{aligned}
f_+^\eta &= \frac{m_b F_\eta}{\sqrt{2} m_B^2 f_B} e^{\frac{m_B^2}{M^2}} \left[\frac{m_\eta^2}{3} I_1^1(\varphi_\parallel + \tilde{\varphi}_\parallel - 2\varphi_\perp - 2\tilde{\varphi}_\perp) \right. \\
&+ \frac{m_\eta^4}{3m_b} \tilde{I}_2^2(\varphi_\parallel + \tilde{\varphi}_\parallel + \varphi_\perp + \tilde{\varphi}_\perp) + 2m_b J_1^0(\varphi_\eta) \\
&\left. - \frac{m_b m_\eta^2}{16} J_1^2(A) - \tilde{J}_2^1(B) \right] \\
f_+^\eta + f_-^\eta &= \frac{\sqrt{2} F_\eta m_b}{m_B^2 f_B} e^{\frac{m_B^2}{M^2}} \left[-J_2^1(B) + \right. \\
&\left. \frac{m_\eta^4}{3m_b} I_2^2(\varphi_\parallel + \tilde{\varphi}_\parallel + \varphi_\perp + \tilde{\varphi}_\perp) \right]
\end{aligned}
$$

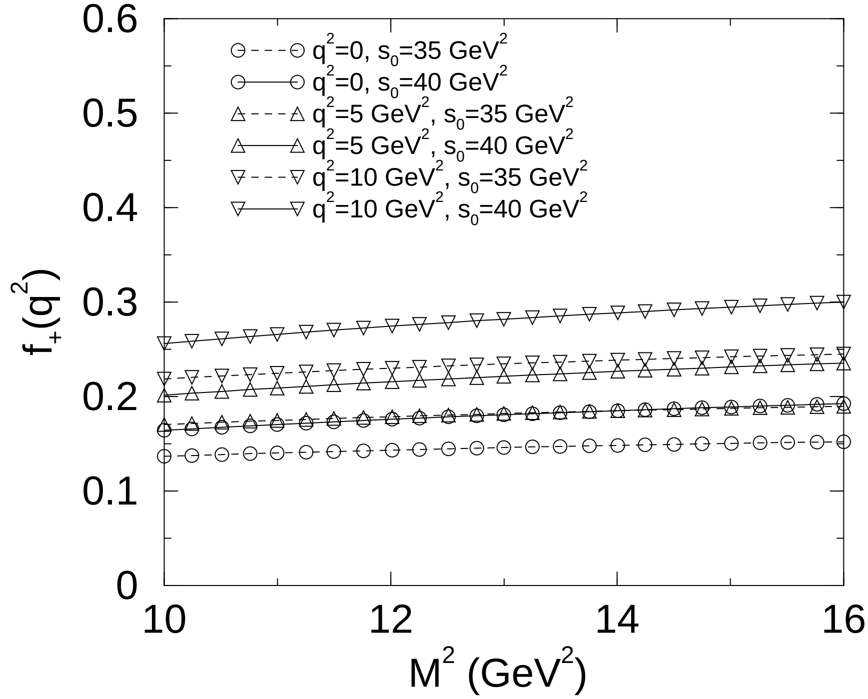

Figure 1: The dependence of the formfactor $f_+^\eta(q^2)$ on the Borel parameter M^2.

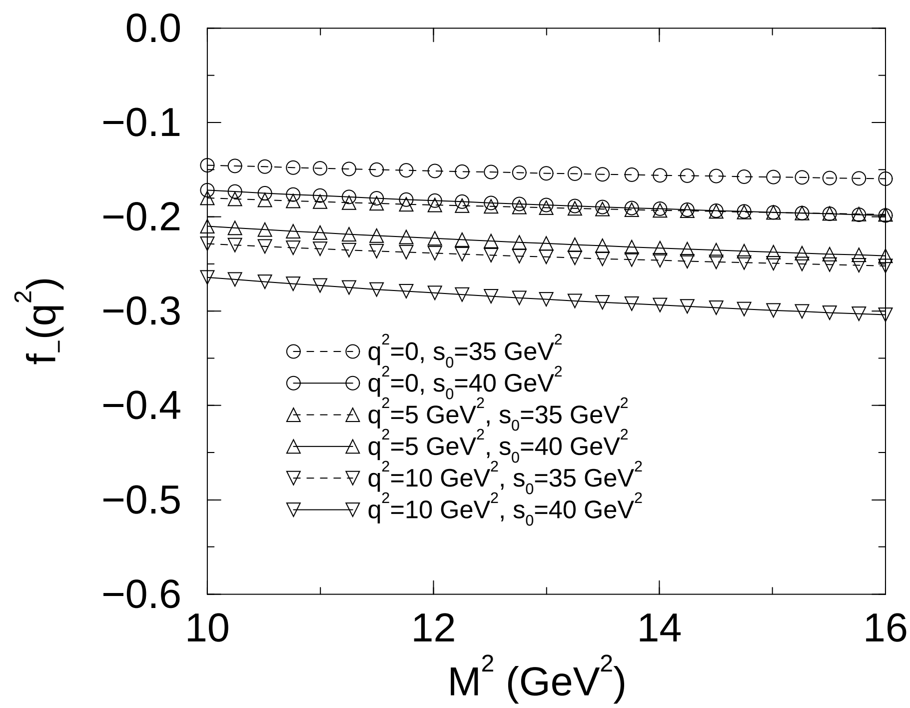

Figure 2: The same as Fig. (1) but for the formfactor $f_-^\eta(q^2)$.

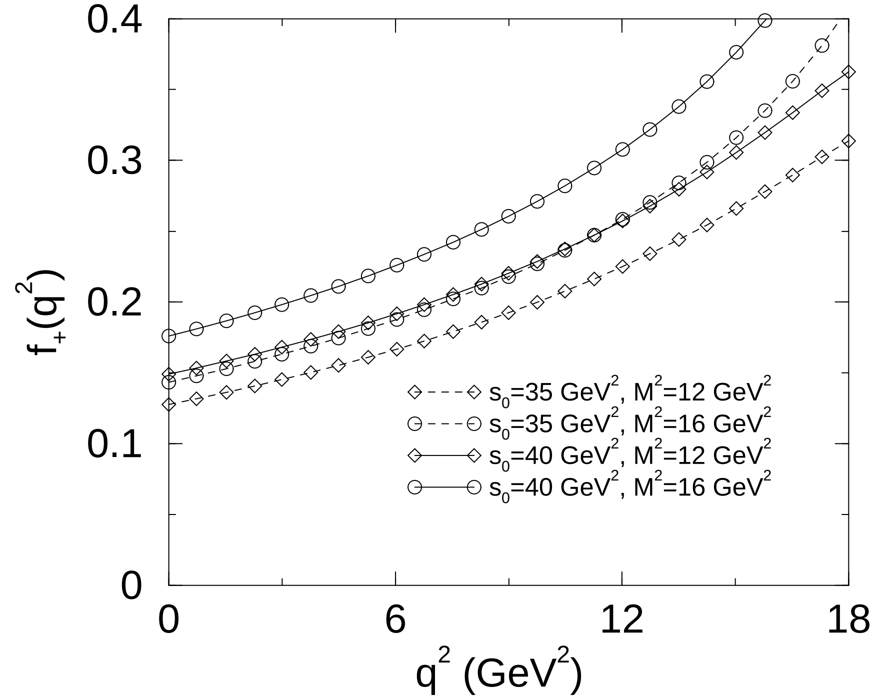

Figure 3: The dependence of the formfactor $f_+^\eta(q^2)$ on q^2.

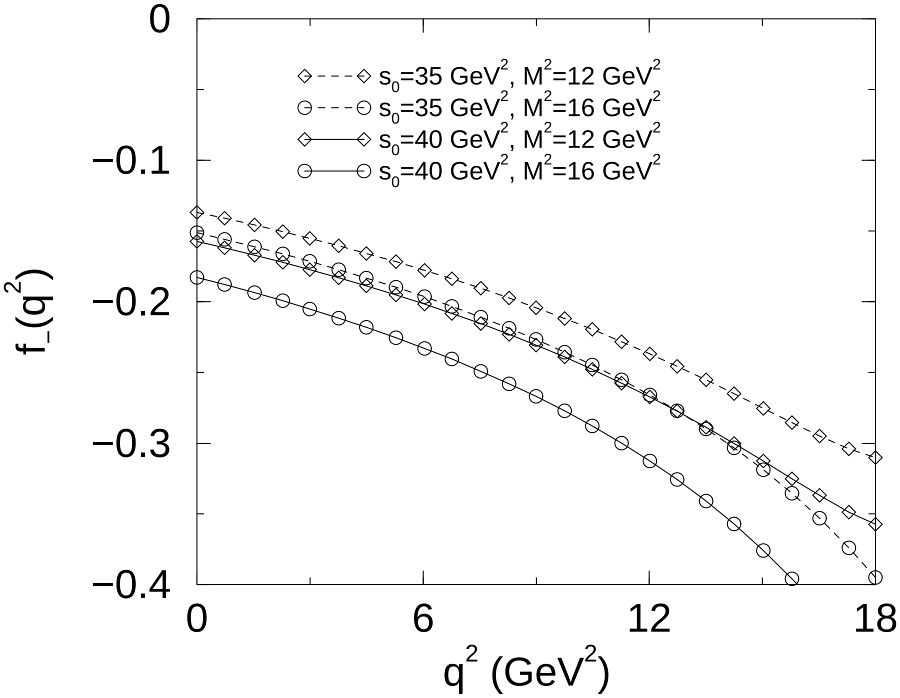

Figure 4: The same as Fig. (3) but for the formfactor $f_-^\eta(q^2)$.

- Sum rules valid only for $q^2 < m_b^2 - 2m_b\chi \simeq 18\ GeV^2$
- They are extrapolated to the whole region of interest using the extrapolation formula:

$$f_i(q^2) = \frac{f_i(0)}{1 - a_F\left(\frac{q^2}{m_B^2}\right) + b_F\left(\frac{q^2}{m_B^2}\right)^2}$$

-

f_i	$f_i(0)$	a_F	b_F
f_-	-0.16 ± 0.2	1.08 ± 0.11	0.22 ± 0.12
f_+	0.15 ± 0.03	1.08 ± 0.06	0.22 ± 0.12

- With these values, one obtains

$$B(B^- \to \eta\ell\nu) = 1.6 \pm 0.8\ 10^{-5}$$

Conclusion

- The transition formfactors for the $B \to \eta$ transition are calculated.
- The formfactors are stable with respect to the variations of the parameters.
- Using the formfactors, the branching ratio is obtained to be:

$$B(B^- \to \eta \ell \nu) = 1.6 \pm 0.8 \; 10^{-5}$$