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December 18, 2007

Sum Rules for the Octet Baryons: Ioffe Current vs. the General Current

Outline

- Introduction
- $SU(3)_f$ Limit
- Light Cone QCD Sum Rules
- Relations Between Correlation Functions
- General Analysis and Conclusion

Introduction

- QCD has an $SU(3)_f$ symmetry when $m_u = m_d = m_s$
- Baryons: $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$

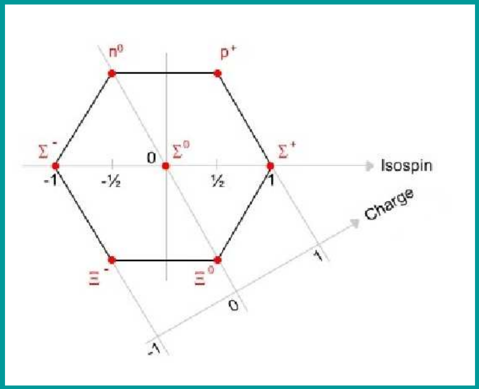


Figure 1: Octet of spin-1/2 baryons

- The octet baryons can be put in an $SU(3)_f$ multiplet:

$$B^\alpha_\beta = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

Light Cone QCD Sum Rules

- One starts with a correlation function of the form

$$\Pi = i \int d^4x e^{ipx} \langle \mathcal{M}(q) | \mathcal{T} \eta_{B_1}(x) \bar{\eta}_{B_2}(0) | 0 \rangle$$

  where $\eta_B(x)$ is an operator with the quantum numbers of the baryon B.

- Depending on the Lorentz structure of the currents chosen, the correlation function will in general contain more than one independent Lorentz structure, each of which will yield an independent sum rule.

- Inserting a complete set of hadronic states

$$\Pi^{phen} = \sum_{h,h'} \frac{\langle 0 | \eta_{B_1} | B_1^h \rangle}{p_1^{h2} - m_1^{h2}} \langle \mathcal{M} B_1^h | B_2^{h'} \rangle \frac{\langle B_2^{h'} | \bar{\eta}_{B_2} | 0 \rangle}{p_2^{h'2} - m_2^{h'2}}$$

- The strategy:

  Find the poles in the p_1^2 and p_2^2 complex plane.

  Identify the residue at the pole to get the relevant matrix element.

- The correlation function can be calculated using QCD when $p_1^2 << 0$ and $p_2^2 << 0$ since in this domain, the main contribution is from short distances.

- $$\mathcal{T}\eta_{B_1}(x)\bar{\eta}_{B_2}(0) \simeq \sum_t C_t(x^2) : \mathcal{O}_t(x) :$$

  where $: \cdots :$ stand for the normal product, and the first operators are of the form $\bar{q}(x)\Gamma q(0)$.

- The correlation function contains matrix elements of the form $\langle \mathcal{M} | : \mathcal{O}_t(x) : | 0 \rangle$

- The matrix elements $\langle \mathcal{M} | : \mathcal{O}_t(x) : | 0 \rangle$ are expanded around $x^2 \sim 0$ (hence the name light cone).

$$\langle \mathcal{M}(q) | : \bar{q}(x) \gamma_\mu \gamma_5 q(0) : | 0 \rangle =$$
$$-i f_{\mathcal{M}} q_\mu \int_0^1 du e^{i\bar{u}qx} \left(\varphi_{\mathcal{M}}(u) + \frac{1}{16} m_{\mathcal{M}}^2 x^2 \mathbb{A}(u) \right)$$

- With these inputs, one can calculate Π when $p_1^2 << 0$ and $p_2^2 << 0$.

- The two represantations are matched using spectral representations which become

$$\Pi = \int ds_1 ds_2 \rho(s_1, s_2) e^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}} \tag{1}$$

after Borel transformations.

- Ideally, an exact calculation would yield to the extraction of the poles and the residues exactly
- Hence, any current with $\langle B|\eta_B|0\rangle \neq 0$ can be used. And each one would give the same result.

- But, many approximations have to be made in order to calculate the correlation in terms of QCD parameter:

  The expansion in terms of non-local operators are truncated at a finite dimension

  The coefficients of the non-local operators are calculated upto a finite order in α_s (usually only the α_s^0 is considered.)

  The expansion of the matrix elements of the non-local operators are truncated at a finite twist

- The quality of the approximation will depend on the particular choice of the current and also on the Lorentz structure chosen.

- The modified strategy:
  1. Choose a current such that the approximations are justified
  2. Assume quark hadron duality:

$$\Pi^{phen\prime}(M_1^2, M_2^2) = \int_{s_0}^{\infty} ds_1 ds_2 \rho(s_1, s_2) e^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}} \tag{2}$$

     where $\Pi^{phen\prime}(M_1^2, M_2^2)$ is the constribution to the correlation function from the higher states and the continuum.
  3. The sum rule is given by

$$\begin{aligned} \langle 0|\eta_{B_1}|B_1\rangle\langle \mathcal{M}B_1|B_2\rangle\langle B_2|\bar{\eta}_{B_2}|0\rangle e^{-\frac{m_1^2}{M_1^2} - \frac{m_2^2}{M_2^2}} \\ = \int_0^{s_0} ds_1 ds_2 \rho(s_1, s_2) e^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}} \end{aligned} \tag{3}$$

Interpolating Currents

- To maximize the dominance of the lowest lying baryon, the current should have the same quantum numbers as the baryon under consideration.
- Any derivative will tend to increase the contribution of the excited states, and currents with least number of derivatives should be used.
- For the octet baryons, two independent currents are possible, and the most general form is a linear superposition of the two.

- $$
\begin{aligned}
\eta^{\Sigma^0} &= \sqrt{\frac{1}{2}}\epsilon^{abc}\left[\left(u^{aT}Cs^b\right)\gamma_5 d^c + t\left(u^{aT}C\gamma_5 s^b\right)d^c\right.\\
&- \left.\left(s^{aT}Cd^b\right)\gamma_5 u^c - t\left(s^{aT}C\gamma_5 d^b\right)u^c\right]\\
\eta^{\Sigma^+} &= -\frac{1}{\sqrt{2}}\eta^{\Sigma^0}(d\rightarrow u)\ ,\quad \eta^{\Sigma^-} = \frac{1}{\sqrt{2}}\eta^{\Sigma^0}(u\rightarrow d)\\
\eta^p &= \eta^{\Sigma^+}(s\rightarrow d)\ ,\quad \eta^n = \eta^{\Sigma^-}(s\rightarrow u)\\
\eta^{\Xi^0} &= \eta^n(d\rightarrow s)\ ,\quad \eta^{\Xi^-} = \eta^p(u\rightarrow s)\\
\eta^{\Lambda} &= -\sqrt{\frac{1}{6}}\epsilon^{abc}\left[2\left(u^{aT}Cd^b\right)\gamma_5 s^c + 2t\left(u^{aT}C\gamma_5 d^b\right)s^c + \left(u^{aT}Cs^b\right)\gamma_5 d^c\right.\\
&+ \left. t\left(u^{aT}C\gamma_5 s^b\right)d^c + \left(s^{aT}Cd^b\right)\gamma_5 u^c + t\left(s^{aT}C\gamma_5 d^b\right)u^c\right]
\end{aligned}
$$

- The Λ current can also be obtained from that of the Σ^0 by:

$$
\begin{aligned}
2\eta_{\Sigma^0}(d \leftrightarrow s) + \eta_{\Sigma^0} &= -\sqrt{3}\eta_\Lambda \\
2\eta_{\Sigma^0}(u \leftrightarrow s) - \eta_{\Sigma^0} &= -\sqrt{3}\eta_\Lambda
\end{aligned}
$$

Mass Sum Rules

- The chosen correlation function is

$$\Pi = i \int d^4x e^{ipx} \langle 0 | \mathcal{T} \{ \eta_B(x) \bar{\eta}_B(0) \} | 0 \rangle \quad (4)$$

- In terms of hadronic parameters

$$\Pi = \sum_h \lambda_h(t)^2 \frac{\not{p} + m_h}{p^2 - m_h^2} \quad (5)$$

where

$$\langle h | \eta_B | 0 \rangle = \lambda_h(t) u \quad (6)$$

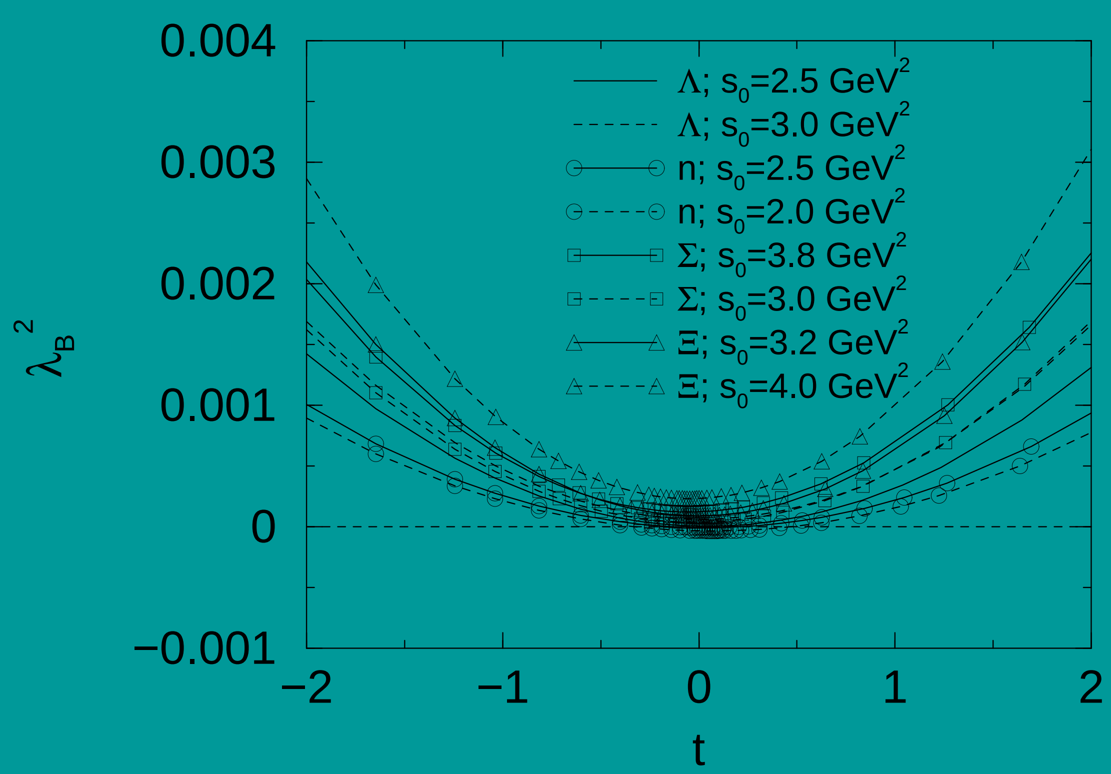


Figure 2: The dependence of the residue on the free parameter t

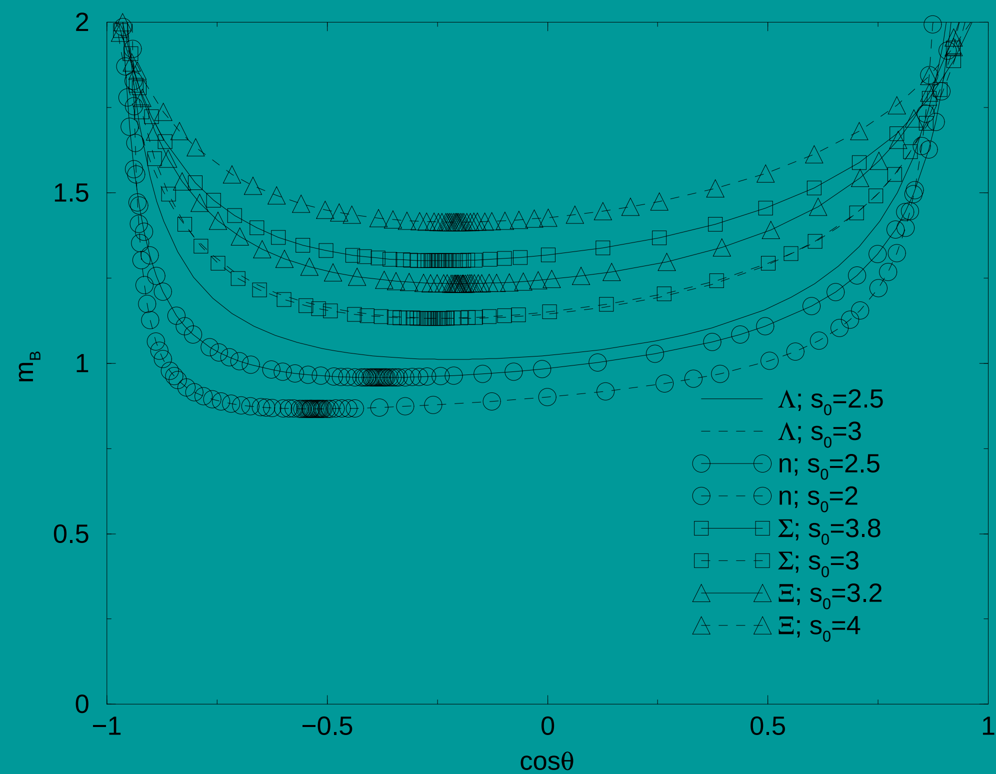


Figure 3: The dependence of the mass on $\cos\theta$ where $t = \tan\theta$

Magnetic Moments

- The chosen correlation function is:

$$\Pi = i \int d^4x e^{ipx} \langle \gamma(q) | \mathcal{T}\{\eta_B(x)\bar{\eta}_B(0)\} | 0 \rangle \tag{7}$$

- In the hadronic representation

$$\Pi = \sum_{h_1,h_2} \frac{\langle 0 \eta_B | h_1 \rangle}{p^2 - m_{h_1}^2} \langle \gamma(q) h_1(p) | h_2(p+q) \rangle \frac{\langle h_2 | \bar{\eta}_B | 0 \rangle}{(p+q)^2 - m_{h_2}^2} \tag{8}$$

where

$$\begin{aligned} &\langle \gamma(q) h_1(p) | h_2(p+q) \rangle \\ &= \bar{u}(p) \epsilon^\mu \left((f_1 + f_2)\gamma_\mu + \frac{2p_\mu + q_\mu}{m_{h_1} + m_{h_2}} f_2 \right) u(p+q) \end{aligned} \tag{9}$$

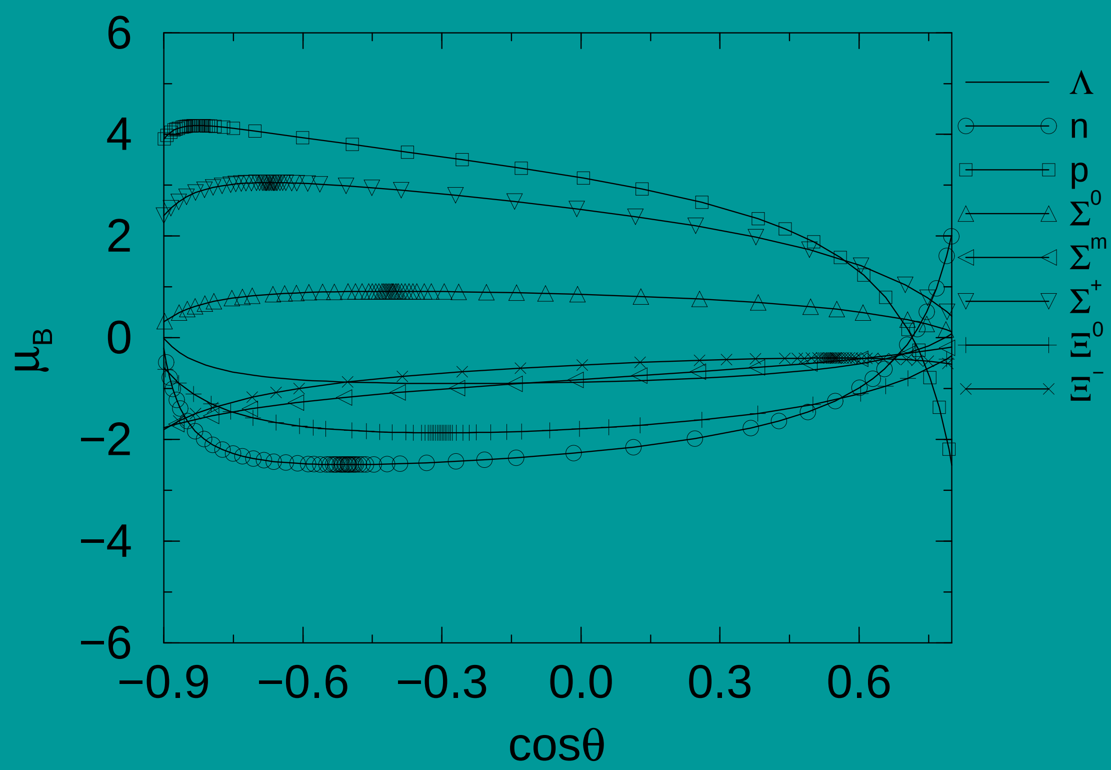


Figure 4: The dependence of the magnetic moments on $\cos\theta$

	NQM	QCDSR		χPT	LCQSR		EXP
		$\chi = -3.3$	$\chi = -4.4$		$\chi = -3.3$	$\chi = -4.4$	
μ_p	2.87	2.72	3.55	2.793	2.7 ± 0.5	3.5 ± 0.5	2.79
μ_n	-1.91	-1.65	-2.06	-1.913	-1.8 ± 0.35	-2.3 ± 0.4	-1.91
μ_{Σ^+}	2.62	2.52	3.30	2.458	2.2 ± 0.4	2.9 ± 0.4	2.46 ± 0.01
μ_{Σ^-}	-1.20	-1.13	-1.38	-1.16	-0.8 ± 0.2	-1.1 ± 0.3	-1.16 ± 0.03
μ_{Ξ^0}	-0.63	-0.89	-0.98	-1.25	-1.3 ± 0.3	-1.3 ± 0.4	-1.25 ± 0.01
μ_{Ξ^-}	-1.44	-1.18	-1.27	-0.6531	-0.7 ± 0.2	-1.0 ± 0.2	-0.65
μ_Λ	-0.63	-0.50	-0.80	-0.613	-0.7 ± 0.2	-0.9 ± 0.2	-0.61

Table 1: Predictions of various approaches for the octet baryon magnetic moments: naive quark model (NQM); QCD sum rules (QCDSR); chiral perturbation theory (χPT); present work (LCQSR). For completeness we present the experimental values of the octet baryons. All the values in the table are given in units of nuclear magneton μ_N.

Decuplet-Octet Transitions

- The correlation funcion:

$$\Pi = i \int d^4 x e^{ipx} \langle \gamma(q) | \mathcal{T} \{ \eta_{\mathcal{O}}(x) \bar{\eta}_{\mathcal{D}}(0) \} | 0 \rangle \qquad (10)$$

- Note the correlation function is linear in the unknown parameter t, and hence it is zero at some value of t.
- In principle, the residue should also go to zero at the same point, but due to the approximations used, they do not coincide exactly.

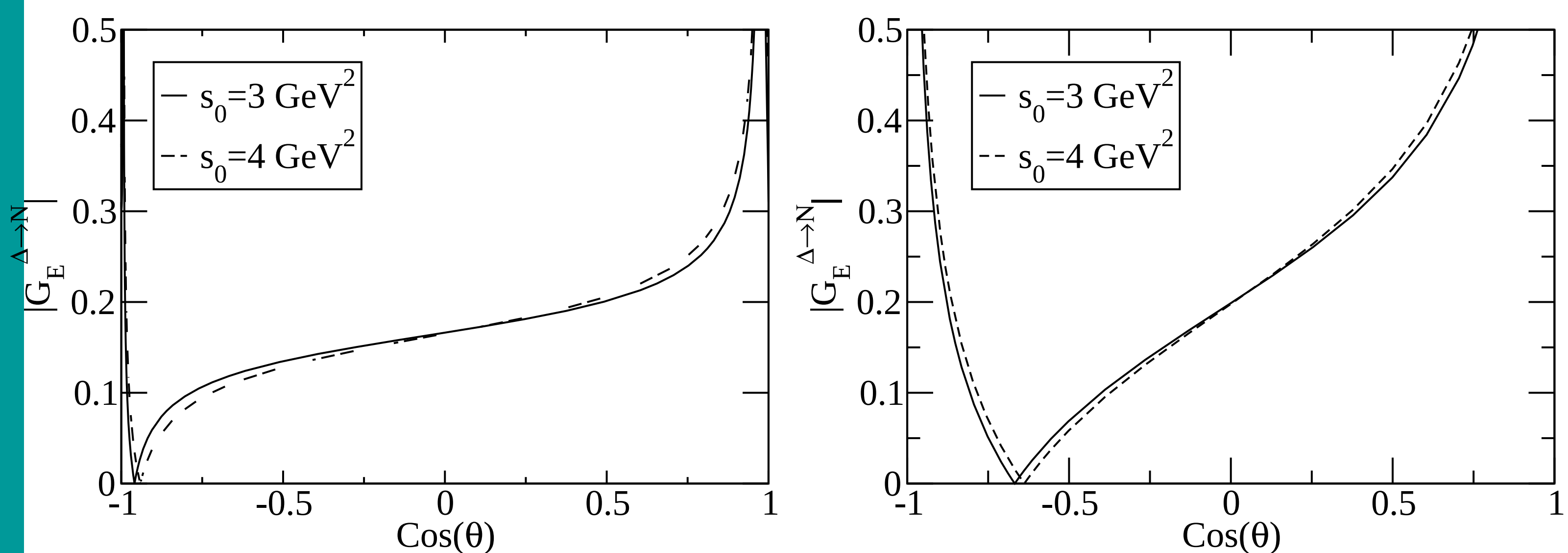


Figure 5: G_E for the transition $\Delta^+ \to p\gamma$

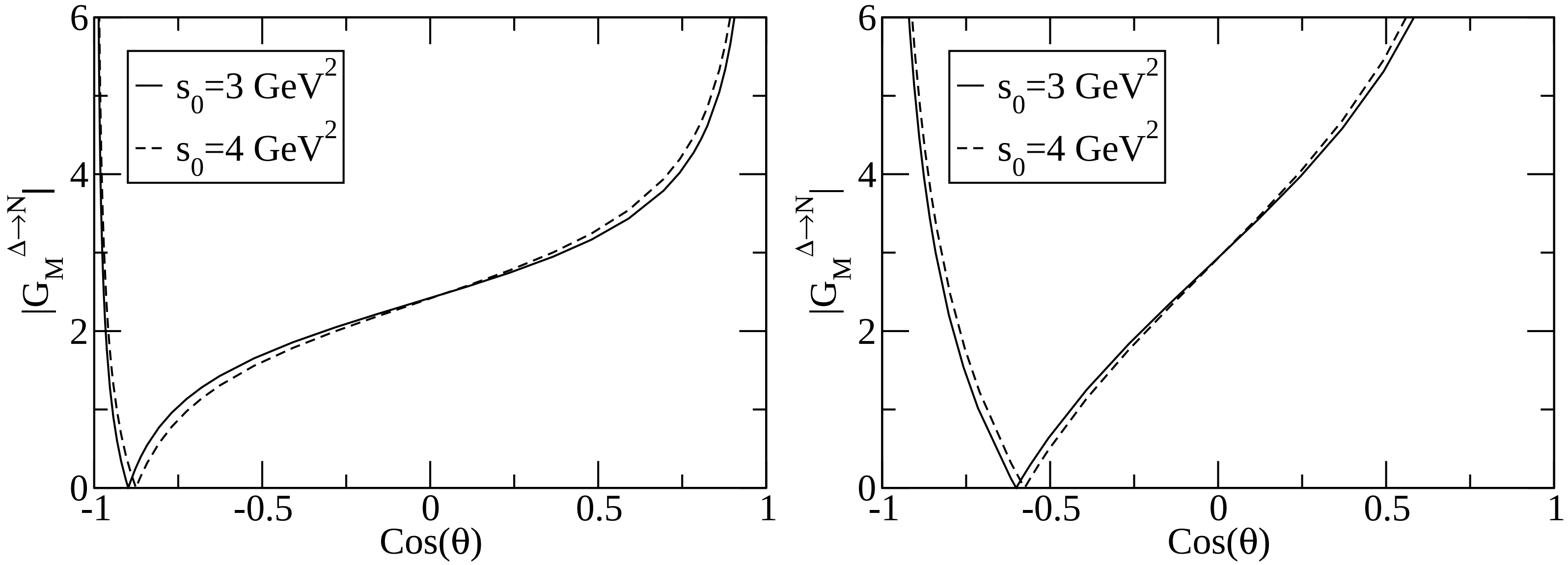


Figure 6: G_E for the transition $\Delta^+ \to p\gamma$

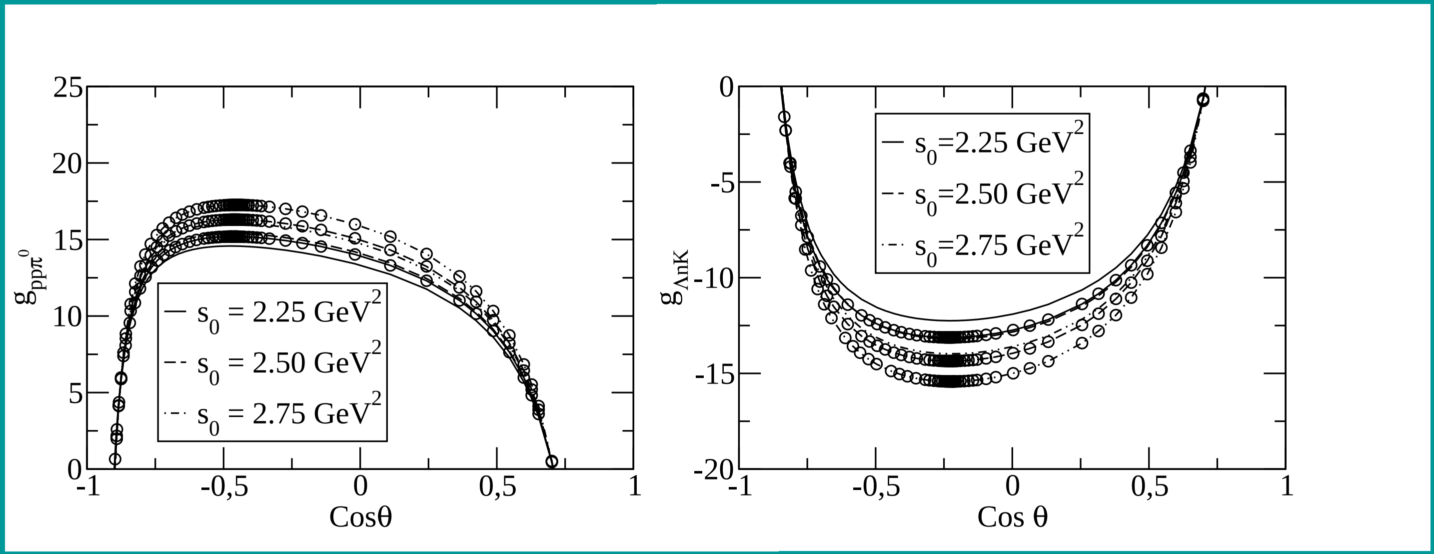
Meson-Baryon Coupling Constants

- The correlation funcion:

$$\Pi = i \int d^4x e^{ipx} \langle \mathcal{M}(q) | \mathcal{T}\{\eta_{\mathcal{O}}(x) \bar{\eta}_{\mathcal{O}}(0)\} | 0 \rangle \tag{11}$$

- The coupling constant is defined through:

$$\langle \mathcal{M} B_2 | B_1 \rangle = g_{B_1 B_2 \mathcal{M}} \bar{u}_{B_1} i \gamma_5 u_{B_2} \tag{12}$$


25
20
15
10
5
0
-1
-0,5
0
0,5
1
Cosθ
$g_{pp\pi^0}$
s_0 = 2.25 GeV^2
s_0 = 2.50 GeV^2
s_0 = 2.75 GeV^2
0
-5
-10
-15
-20
Cos θ
$g_{\Lambda nK}$
s_0=2.25 GeV^2
s_0=2.50 GeV^2
s_0=2.75 GeV^2

Channel	Gen. Current	$t=-1$	$SU(3)_f$	QSR*	QSR†	Exp.
$\Lambda \to nK$	-13 ± 3	-9.5 ± 1	-14.3	-2.37	-2.49	-13.5
$\Lambda \to \Sigma^+\pi^-$	10 ± 3	12 ± 1	10.0			
$\Lambda \to \Xi^0 K^0$	4.5 ± 2	-2.5 ± 0.5	4.25			
$n \to p\pi^-$	21 ± 4	20 ± 2	19.8			21.2
$n \to \Sigma^0 K^0$	-3.2 ± 2.2	-9.5 ± 0.5	-3.3	-0.025	-0.40	-4.25
$p \to \Lambda K^+$	-13 ± 3	-10 ± 1	-14.25	-2.37	-2.49	-13.5
$p \to p\pi^0$	14 ± 4	15 ± 1	Input	13.5		14.9
$p \to \Sigma^+ K^0$	4 ± 3	14 ± 1	5.75			
$\Sigma^0 \to nK^0$	-4 ± 3	-9.5 ± 1	-3.32	-0.025	-0.40	-4.25
$\Sigma^0 \to \Lambda\pi^0$	11 ± 3	12 ± 1.5	10.0	6.9		
$\Sigma^0 \to \Xi^0 K^0$	-13 ± 3	-13.5 ± 1	-14			
$\Sigma^- \to nK^-$	5 ± 3	15 ± 2	4.7			
$\Sigma^+ \to \Lambda\pi^+$	10 ± 3.5	12.5 ± 1	Input			
$\Sigma^+ \to \Sigma^0\pi^+$	-9 ± 2	-7.5 ± 0.7	-10.7	-11.9		
$\Xi^0 \to \Lambda K^0$	4.5 ± 1	-2.6 ± 0.3	4.25			
$\Xi^0 \to \Sigma^0 K^0$	-12.5 ± 3	-13.5 ± 1	-14			
$\Xi^0 \to \Sigma^+ K^-$	18 ± 4	19 ± 2	19.8			
$\Xi^0 \to \Xi^0\pi^0$	10 ± 2	0.3 ± 0.6	-3.32	-1.60		

Conclusion

- All the strong coupling constants are expressed in terms of four analytic functions
- The predicted coupling constants respect the $SU(3)_f$ symmetry
- There is good agreement with the experimental results.