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Strong Coupling Constants of the Octet Baryons to K and π Mesons
in Light Cone QCD Sum Rules ^a:
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Outline

- Introduction
- $SU(3)_f$ Limit
- Light Cone QCD Sum Rules
- Relations Between Correlation Functions
- General Analysis and Conclusion

Introduction

- QCD has an $SU(3)_f$ symmetry when $m_u = m_d = m_s$
- Mesons: $3 \otimes \bar{3} = 1 \oplus 8$
- Baryons: $3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$

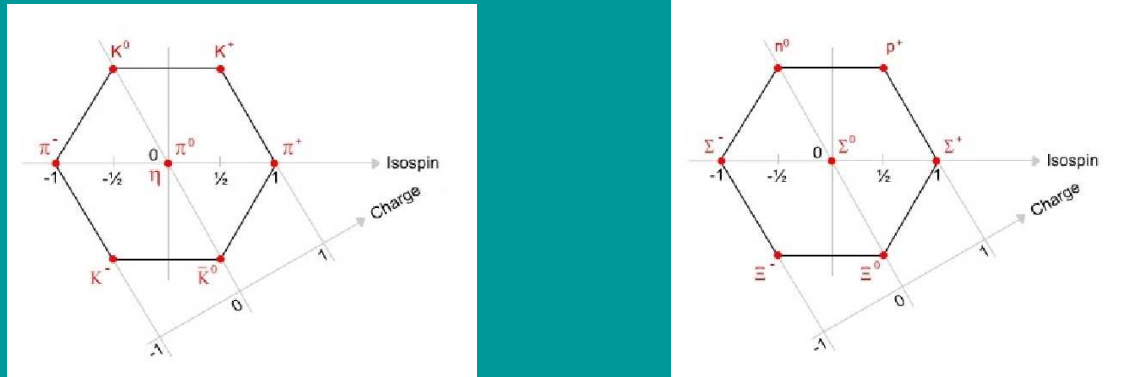


Figure 1: Octet of pseudoscalar mesons and spin-1/2 baryons

- $SU(3)_f$ Limit

- In the $SU(3)_f$ limit, the baryons can be grouped in the octet representation as:

$$B_{\beta}^{\alpha} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

- The Mesons as:

$$P_{\beta}^{\alpha} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

- The most general $SU(3)_f$ interactions can be described by two terms:

$$- \text{Tr} \bar{B} \{P, B\}$$

$$- \text{Tr} \bar{B} [P, B]$$

- The most general interaction lagrangian:

$$\mathcal{L} = \sqrt{2} (D \text{Tr} \bar{B} \{P, B\} + F \text{Tr} \bar{B} [P, B])$$

- The coupling constants can be read off as:

$$g_{\pi^0 pp} = F + D, \quad g_{\pi^0 \Sigma^+ \Sigma^+} = 2F, \quad g_{\pi^- \Sigma^+ \Sigma^0} = -2F$$

$$g_{\pi^0 \Xi^0 \Xi^0} = F - D, \quad g_{\pi^+ \Xi^+ \Xi^0} = -\sqrt{2}(F - D), \quad \text{etc.}$$

Light Cone QCD Sum Rules

- One starts with a correlation function of the form

$$\Pi = i \int d^4x e^{ipx} \langle \mathcal{M}(q) | \mathcal{T} \eta_{B_1}(x) \bar{\eta}_{B_2}(0) | 0 \rangle$$

- Inserting a complete set of hadronic states

$$\Pi^{phen} = \sum_{h,h'} \frac{\langle 0 | \eta_{B_1} | B_1^h \rangle \langle \mathcal{M} B_1^h | B_2^{h'} \rangle \langle B_2^{h'} | \eta_{B_2} | 0 \rangle}{p_1^{h^2} - m_1^{h^2} p_2^{h'^2} - m_2^{h'^2}}$$

- The correlation function contains different Dirac structures:

$$\begin{aligned} & \Pi^{B_2 \rightarrow B_1 \mathcal{M}}(p_1^2, p_2^2) \\ &= i \frac{g_{B_2 B_1 \mathcal{M}} \lambda_{B_1} \lambda_{B_2}}{(p_1^2 - M_1^2)(p_2^2 - M_2^2)} (-\not{p} \not{q} \gamma_5 - M_1 \not{q} \gamma_5 \\ &+ (M_2 - M_1) \not{p} \gamma_5 + (M_1 M_2 - p^2) \gamma_5) + \dots \end{aligned}$$

- Each structure gives a different sum rule having different convergence properties.
- The contributions of higher states are parameterized by quark hadron duality, introducing an auxiliary parameter s_0 .

- When $p_1^2 \rightarrow -\infty$ and $p_2^2 \rightarrow -\infty$, the main contribution is from small distances and times.
- Perturbative techniques can be applied:

$$\mathcal{T} \eta_{B_1}(x) \bar{\eta}_{B_2}(0) \simeq \sum_t C_t(x^2) : \mathcal{O}_t(x) :$$

where $:\dots:$ stand for the normal product, and the first operators are of the form $\bar{q}(x)\Gamma q(0)$.

- The correlation function contains matrix elements of the form $\langle \mathcal{M} | : \mathcal{O}_t(x) : | 0 \rangle$

- The matrix elements $\langle \mathcal{M} | : \mathcal{O}_t(x) : | 0 \rangle$ are expanded around $x^2 \sim 0$ (hence the name light cone).

$$\langle \mathcal{M}(q) | : \bar{q}(x) \gamma_\mu \gamma_5 q(0) : | 0 \rangle = -i f_{\mathcal{M}} q_\mu \int_0^1 du e^{i\bar{u}qx} \left(\varphi_{\mathcal{M}}(u) + \frac{1}{16} m_{\mathcal{M}}^2 x^2 \Delta(u) \right)$$

- With these inputs, one can calculate Π when $p_1^2 \ll 0$ and $p_2^2 \ll 0$.
- The two representations are matched using spectral representations which become

$$\Pi = \int ds_1 ds_2 \rho(s_1, s_2) e^{-\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2}} \quad (1)$$

after Borel transformations.

Relations Between Correlation Functions

- To obtain the correlation function, first suitable currents have to be chosen:

$$\eta^{\Sigma^0} = \sqrt{\frac{1}{2}} \epsilon^{abc} [(u^{aT} C s^b) \gamma_5 d^c + t (u^{aT} C \gamma_5 s^b) d^c - (s^{aT} C d^b) \gamma_5 u^c - t (s^{aT} C \gamma_5 d^b) u^c]$$

$$\eta^{\Sigma^+} = -\frac{1}{\sqrt{2}} \eta^{\Sigma^0} (d \rightarrow u), \quad \eta^{\Sigma^-} = \frac{1}{\sqrt{2}} \eta^{\Sigma^0} (u \rightarrow d)$$

$$\eta^p = \eta^{\Sigma^+} (s \rightarrow d), \quad \eta^n = \eta^{\Sigma^-} (s \rightarrow u)$$

$$\eta^{\Xi^0} = \eta^n (d \rightarrow s), \quad \eta^{\Xi^-} = \eta^p (u \rightarrow s)$$

$$\eta^\Lambda = -\sqrt{\frac{1}{6}} \epsilon^{abc} [2 (u^{aT} C d^b) \gamma_5 s^c + 2t (u^{aT} C \gamma_5 d^b) s^c + (u^{aT} C s^b) \gamma_5 d^c + t (u^{aT} C \gamma_5 s^b) d^c + (s^{aT} C d^b) \gamma_5 u^c + t (s^{aT} C \gamma_5 d^b) u^c]$$

- The Λ current can also be obtained from that of the Σ^0 by:

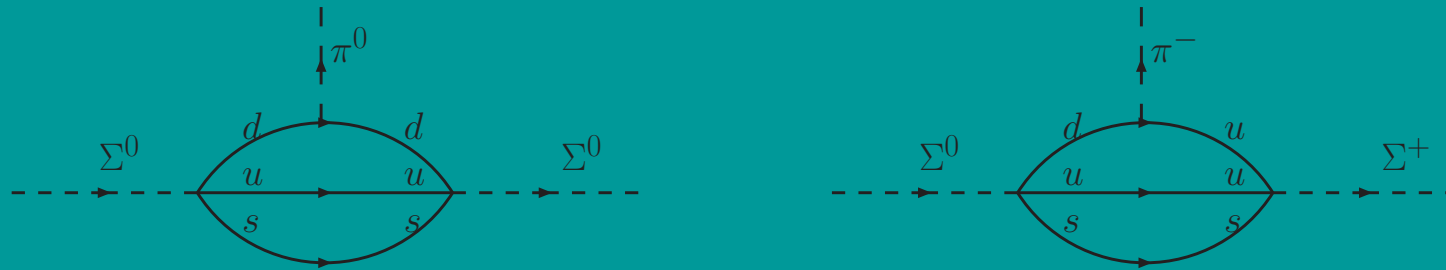
$$2\eta_{\Sigma^0}(d \leftrightarrow s) + \eta_{\Sigma^0} = -\sqrt{3}\eta_{\Lambda}$$

$$2\eta_{\Sigma^0}(u \leftrightarrow s) - \eta_{\Sigma^0} = -\sqrt{3}\eta_{\Lambda}$$

- Convention: Let $\pi^0 = g_{\pi uu}\bar{u}u + g_{\pi dd}\bar{d}d + g_{\pi ss}\bar{s}s$
- Denote $\Pi^{B_2 \rightarrow B_1 \mathcal{M}} = \langle \mathcal{M} | B_1 \bar{B}_2 | 0 \rangle$ and
 $\Pi^{\Sigma^0 \rightarrow \Sigma^0 \pi^0} = g_{\pi uu} \langle \bar{u}u | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle + g_{\pi dd} \langle \bar{d}d | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle + g_{\pi ss} \langle \bar{s}s | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle$
- Let $\Pi_1(u, d, s) = \langle \bar{u}u | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle$, $\Pi_2(u, d, s) = \langle \bar{s}s | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle$
- $\langle \bar{d}d | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle = \Pi_1(d, u, s)$ since $\Sigma^0(u \leftrightarrow d) = \Sigma^0$

- Consider $\Pi^{\Sigma^+ \rightarrow \Sigma^+ \pi^0} = g_{\pi \bar{u}u} \langle \bar{u}u | \Sigma^+ \bar{\Sigma}^+ | 0 \rangle + g_{\pi \bar{s}s} \langle \bar{s}s | \Sigma^+ \bar{\Sigma}^+ | 0 \rangle$
- Note that $\Sigma^0(d \rightarrow u) = -\sqrt{2}\Sigma^+$
- Hence $\langle \bar{u}u | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle (d \rightarrow u) = 2\langle \bar{u}u | \Sigma^+ \bar{\Sigma}^+ | 0 \rangle'$
- $\langle \bar{u}u | \Sigma^+ \bar{\Sigma}^+ | 0 \rangle = 4\langle \bar{u}u | \Sigma^+ \bar{\Sigma}^+ | 0 \rangle'$
 $= 2\langle \bar{u}u | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle (d \rightarrow u) = 2\Pi_1(u, u, s)$
- Hence $\Pi^{\Sigma^+ \rightarrow \Sigma^+ \pi^0} = \sqrt{2}\Pi_1(u, u, s)$

- What about charged mesons? Consider $\Pi^{\Sigma^0 \rightarrow \Sigma^+ \pi^-}$



- Hence it is natural to expect that $\langle \bar{d}d | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle$ to be proportional to $\langle \bar{u}d | \Sigma^+ \bar{\Sigma}^0 | 0 \rangle$
- Indeed

$$\Pi^{\Sigma^0 \rightarrow \Sigma^+ \pi^-} = \langle \bar{u}d | \Sigma^+ \bar{\Sigma}^0 | 0 \rangle = -\sqrt{2} \langle \bar{d}d | \Sigma^0 \bar{\Sigma}^0 | 0 \rangle = -\sqrt{2} \Pi_1(d, u, s)$$
- Exchanging u and d quarks, one also obtains

$$\Pi^{\Sigma^0 \rightarrow \Sigma^- \pi^+} = \langle \bar{d}u | \Sigma^- \bar{\Sigma}^0 | 0 \rangle = \sqrt{2} \Pi_1(u, d, s)$$

- Similarly

$$\Pi^{\Xi^0 \rightarrow \Xi^- \pi^+} = \langle \bar{d}u | \Xi^- \Xi^0 | 0 \rangle = -\sqrt{2} \langle \bar{u}u | \Xi^0 \bar{\Xi}^0 | 0 \rangle = -\Pi_2(s, s, u)$$



- Exchanging u and d , one obtains $\Pi^{\Xi^- \rightarrow \Xi^+ \pi^-} = -\Pi_2(s, s, d)$

- What about Λ ?
- Using $2\Sigma^0(d \leftrightarrow s) = -\sqrt{3}\Lambda - \Sigma^0$ and the result $\Pi^{\Sigma^0 \rightarrow \Sigma^- \pi^+} = \sqrt{2}\Pi_1(u, d, s)$, we have $2\sqrt{2}\Pi_1(u, s, d) = \sqrt{3}\Pi^{\Lambda \rightarrow \Xi^- K^+} + \Pi^{\Sigma^0 \rightarrow \Xi^- K^+}$
- How to separate the two correlation functions? Define $\Pi_3(u, d, s) = -\Pi^{\Sigma^0 \rightarrow \Xi^- K^+} = -\langle \bar{s}u | \Xi^- \bar{\Sigma}^0 | 0 \rangle$ and $\Pi_4(u, d, s) = -\Pi^{\Xi^- \rightarrow \Sigma^0 K^-} = -\langle \bar{u}s | \Sigma^0 \bar{\Xi}^- | 0 \rangle$

- How are the functions related to the F and D constants in the $SU(3)_f$ limit?
- In the $SU(3)_f$ limit, $\Pi_1 \propto \sqrt{2}F$, $\Pi_2 \propto \sqrt{2}(F - D)$, and $\Pi_4 = \Pi_3 \propto -(F + D)$

Explicit Expressions:

$$\begin{aligned}
 \Pi_1(u, d, s) = & \\
 & \frac{f_{\mathcal{M}}}{64\pi^2} M^4 (m_s(1-t)^2 - m_d(1-t^2)) i_2(\phi_{\mathcal{M}}) \\
 - & \frac{\mu_{\mathcal{M}}}{64\pi^2} M^4 (1 - \tilde{\mu}_{\mathcal{M}}^2) (1 - t^2) i_2(\phi_{\sigma}) \\
 - & \frac{f_{\mathcal{M}}}{32\pi^2} m_{\mathcal{M}}^2 M^2 (m_s(1-t)^2 + 3m_d(1-t^2)) i_1(\mathcal{V}, 1) \left(\gamma_E - \ln \frac{M^2}{\Lambda^2} \right) \\
 + & \frac{f_{\mathcal{M}}}{16\pi^2} m_{\mathcal{M}}^2 M^2 (m_s(1-t)^2 - m_d(1-t^2)) i_1(\mathcal{V}_{\perp}, 1) \left(\gamma_E - \ln \frac{M^2}{\Lambda^2} \right) \\
 + & \frac{f_{\mathcal{M}}}{768\pi^2} \langle g_s^2 GG \rangle (m_s(1-t)^2 - m_d(1-t^2)) i_2(\phi_{\mathcal{M}}) \left(\gamma_E - \ln \frac{M^2}{\Lambda^2} \right) \\
 + & \dots
 \end{aligned}$$

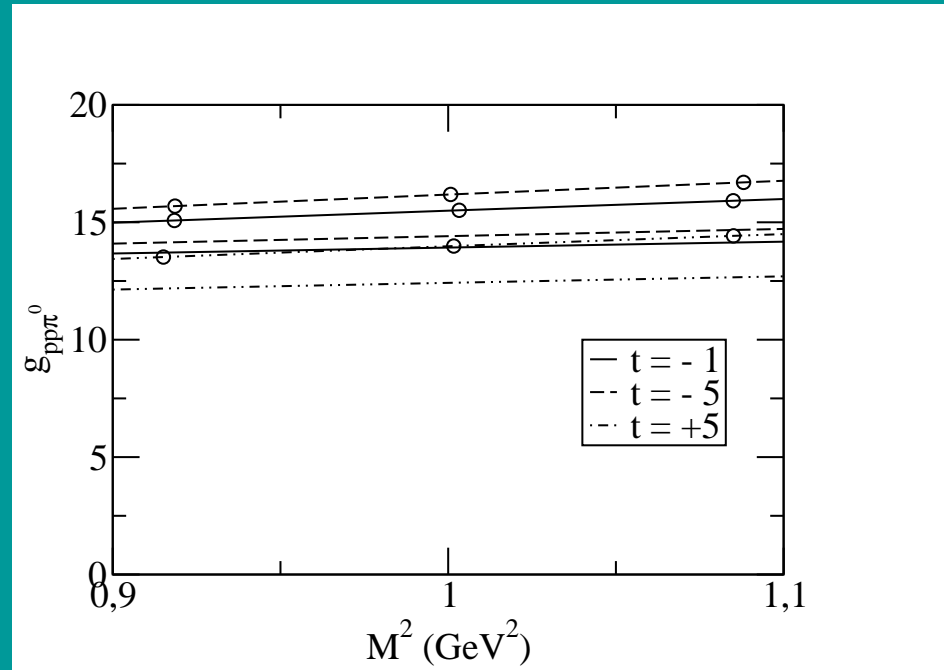


Figure 2: The M^2 dependence of the $g_{pp\pi}$ coupling constant.

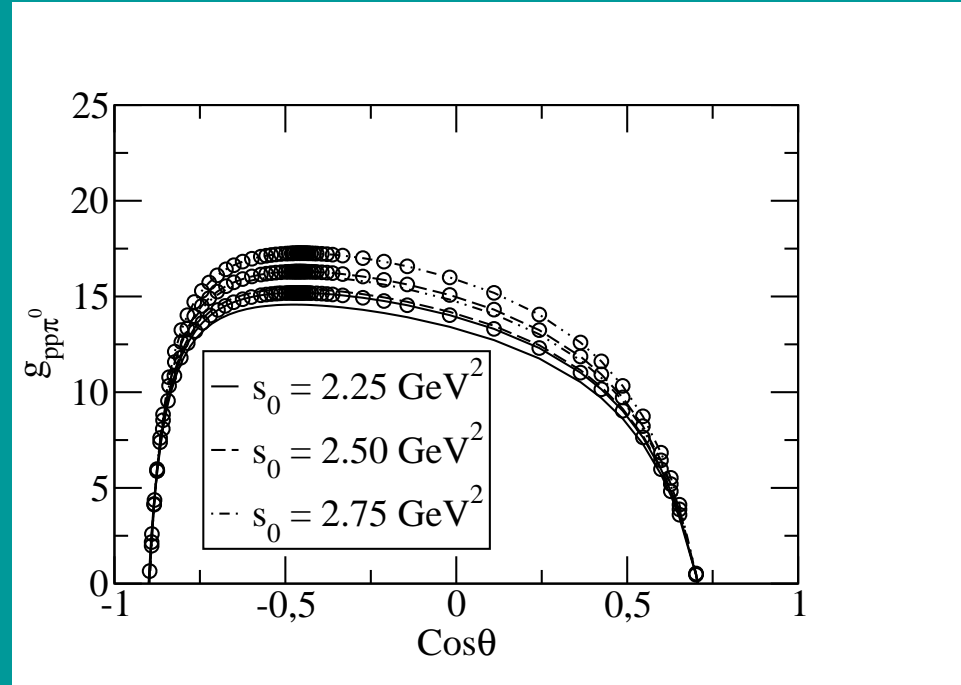


Figure 3: The θ dependence of the $g_{pp\pi}$ where $\tan \theta = t$

Channel	Gen. Current	$t = -1$	$SU(3)_f$	QSR*	QSR [†]	Exp.
$\Lambda \rightarrow nK$	-13 ± 3	-9.5 ± 1	-14.3	-2.37	-2.49	-13.5
$\Lambda \rightarrow \Sigma^+ \pi^-$	10 ± 3	12 ± 1	10.0			
$\Lambda \rightarrow \Xi^0 K^0$	4.5 ± 2	-2.5 ± 0.5	4.25			
$n \rightarrow p\pi^-$	21 ± 4	20 ± 2	19.8			21.2
$n \rightarrow \Sigma^0 K^0$	-3.2 ± 2.2	-9.5 ± 0.5	-3.3	-0.025	-0.40	-4.25
$p \rightarrow \Lambda K^+$	-13 ± 3	-10 ± 1	-14.25	-2.37	-2.49	-13.5
$p \rightarrow p\pi^0$	14 ± 4	15 ± 1	Input	13.5		14.9
$p \rightarrow \Sigma^+ K^0$	4 ± 3	14 ± 1	5.75			
$\Sigma^0 \rightarrow nK^0$	-4 ± 3	-9.5 ± 1	-3.32	-0.025	-0.40	-4.25
$\Sigma^0 \rightarrow \Lambda\pi^0$	11 ± 3	12 ± 1.5	10.0	6.9		
$\Sigma^0 \rightarrow \Xi^0 K^0$	-13 ± 3	-13.5 ± 1	-14			
$\Sigma^- \rightarrow nK^-$	5 ± 3	15 ± 2	4.7			
$\Sigma^+ \rightarrow \Lambda\pi^+$	10 ± 3.5	12.5 ± 1	Input			
$\Sigma^+ \rightarrow \Sigma^0 \pi^+$	-9 ± 2	-7.5 ± 0.7	-10.7	-11.9		
$\Xi^0 \rightarrow \Lambda K^0$	4.5 ± 1	-2.6 ± 0.3	4.25			
$\Xi^0 \rightarrow \Sigma^0 K^0$	-12.5 ± 3	-13.5 ± 1	-14			
$\Xi^0 \rightarrow \Sigma^+ K^-$	18 ± 4	19 ± 2	19.8			
$\Xi^0 \rightarrow \Xi^0 \pi^0$	10 ± 2	0.3 ± 0.6	-3.32	-1.60		

Conclusion

- All the strong coupling constants are expressed in terms of four analytic functions
- The predicted coupling constants respect the $SU(3)_f$ symmetry
- There is good agreement with the experimental results.