Application of QCD Sum Rules to Heavy Baryons

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Outline

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Phys. Dept. of METU

- Found in 1960 as Department of Physics and Dept. of Theoretical Physics.
- United in 1970
- 37 Prof.'s, 14 Assoc. Prof.'s, 4 Assis. Prof.'s, 44 TA's
- Fields: Astrophysics, Atomic and Molecular Physics, HEP, Mathematical Physics, Nuclear Physics, Plasma Physics, Solid State Physics
- http://www.physics.metu.edu.tr

Phenomenology Group

- E. O. Iltan: Models Beyond the Standard Model, Multi Higgs Doublet Models
- N. K. Pak, and T. M. Aliev: Unparticle Physics, Models Beyond Standard Model
- G. Turan, T. M. Aliev: B-physics
- T. M. Aliev, A. Ozpineci: QCD Sum Rules, Hadron Physics, B decays
- A. Gokalp, O. Yilmaz: Nuclear Physics, Sum Rules for (scalar) mesons

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   - Introduction to QCD Sum Rules
   - Interpolating Currents for Heavy Baryons
3. Results
   - Magnetic Moment Predictions for Ξ_Q
   - Mass Postdictions for the Ξ_Q
   - $g_{\pi\Xi_Q}$ Coupling Constant Predictions(PRELIMINARY)
   - $g_{\Xi \ldots}$ Coupling Constants

ISSCSMB

- Annual school on “INTERNATIONAL SUMMER SCHOOL AND CONFERENCE ON HIGH ENERGY PHYSICS: STANDARD MODEL AND BEYOND” in Mugla

ISSCSMB

- A school for graduate students
- Organizers: T. M. Aliev (METU), S. Oktik (MU), M. Serin (METU)
- 27 August-4 September 2009
- http://milonga.physics.metu.edu.tr/schools/mugla_2009/index.html (under constructions)

TROIA'09

- A biannual conference on hadron physics held in Canakkale (where the ancient cities of Troy are)

TROIA'09

- Organizers: A. Kucukarslan (COMU), A. Ozpineci (METU)
- 10-14 September 2009
- Special emphasis is given to Sum Rules, Chiral Perturbation Theory, and Lattice Results
- http://milonga.physics.metu.edu.tr/hep-th/troia09 // (available in SPIRES)

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Experimental Status

- Review in PDG, by C. G. Wohl, on charmed baryons state that (Revised March 2008):

  > *There are 17 known charmed baryons, and four other candidates not well enough established to be promoted to the Summary Tables.*

  (23 are listed in the listings. 1 has status *, 3 are omitted from the listings)
- One double charmed state Ξ_{cc} is seen by SELEX, but no evidence from BABAR or BELLE (omitted from listings)

- In PDG, four bottom baryons: Λ_b, Σ_b, Σ_b^* and Ξ_b, are listed
- In 2007, D0 and CDF claimed to have observed Ξ_b^- directly.
- Ξ_b^- and Ξ_c had been observed indirectly by DELPHI Collaboration in 2004 in the channels $\Xi_b^- \to \Xi^- \ell^- \bar{\nu} X$ and $\Xi_c^0 \to \Xi_c^- \pi^+$

- 10% of the b-quarks form b-baryons.
- charmed baryons can also be produced by B meson decays with a branching fraction of about 5%

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Motivation

- Lowest lying heavy quark mesons are $I = 0$ and $s = 0$ or $s = 1$ states: $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle$, i.e. do not carry any information about the heavy quark spin.
- If heavy baryons are $I = 0$ states formed from a scalar diquark orbiting the heavy quark, the spin of the baryon is entirely due to the spin of the heavy quark, i.e. spin properties of heavy quark transitions might be studied.
- In this report, I will present the applications of QCD sum rules on heavy baryons.

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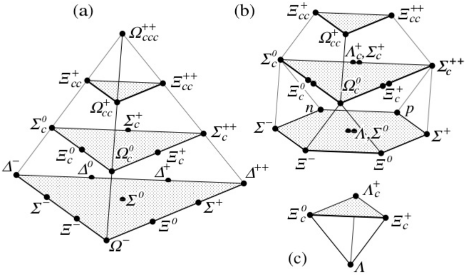
SU(4) Classification

- In group theoretical notation: $4 \otimes 4 \otimes 4 = 20 \oplus 20'_1 \oplus 20'_2 \oplus 4$



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- In QCD Sum Rules, hadronic properties are expressed in terms of the properties of vacuum.
- One starts with a correlation function of the form:

$$\Pi = i \int d^4p e^{ipx} \langle \mathcal{M} | \mathcal{T} \eta_1(x) \eta_2(0) | \Omega \rangle \qquad (1)$$

  where $\mathcal{M}$ is the QCD vacuum for mass, or can be a hadronic state if one is interested in coupling constants or form factors.
- η_i are composite operators made up of quark fields that have the same quantum numbers as the hadrons under consideration.
- The correlation functions can be expressed in terms of the properties of hadrons and also in terms of the properties of the vacuum.

- By inserting complete sets of hadronic states, the correlation function can be written as:
  - if $\mathcal{M} = \Omega$ and $\eta_1 = \eta_2 = \eta$:

$$\Pi = \sum_h \frac{\langle\Omega|\eta|h(p)\rangle\langle h(p)|\eta|\Omega\rangle}{p^2 - m_h^2} \tag{2}$$

  - if $\mathcal{M}(q)$ is another hadron,

$$\Pi = \sum_h \frac{\langle\mathcal{M}(q)|\eta_1|h(p+q)\rangle\langle h(p+q)|\eta_2|\Omega\rangle}{(p+q)^2 - m_h^2} \tag{3}$$

    or by inserting another complete set

$$\Pi = \sum_{h_1,h_2} \frac{\langle\Omega|\eta_1|h_1(p)\rangle\langle h_1(p)\mathcal{M}(q)|h_2(p+q)\rangle\langle h_2(p+q)|\eta_2|\Omega\rangle}{(p^2 - m_{h_1}^2)((p+q)^2 - m_{h_2}^2)} \tag{4}$$

- The matrix elements $\langle\Omega|\eta|h(p)\rangle = \lambda u(p)$ where $u(p)$ is the wavefunction (a spinor in our case) and λ is called the corresponding residue.
- The matrix elements $\langle\mathcal{M}(q)|\eta|h(p+q)\rangle$ and $\langle\mathcal{M}(q)h_1(p)|h_2(p+q)\rangle$ can be expressed in terms of coupling constants or form factors.

- The correlation function can also be calculated in the deep Euclidean region using OPE:

$$\mathcal{T}\eta_1(x)\eta_2(0) = \sum_d C_d(x^2)\mathcal{O}_d(x) \quad (5)$$

- In the case of mass sum rules or traditional sum rules, $\mathcal{O}_d(x)$ are local operators. After Fourier transform, the correlation function becomes:

$$\Pi = \sum_d C_d(p^2)\langle\Omega|\mathcal{O}_d|\Omega\rangle \quad (6)$$

where $\langle\Omega|\mathcal{O}_d|\Omega\rangle$ are called the vacuum condensates.
- Some of the well known vacuum condensates include:

$$\begin{aligned} d &= 3 \qquad \langle\bar{q}q\rangle \\ d &= 4 \qquad \langle g_s^2 G^2\rangle \\ d &= 5 \qquad \langle\bar{q}G\sigma q\rangle \end{aligned}$$

- In case of light cone sum rules, matrix elements of the form $\langle \mathcal{M}(q)|\mathcal{O}_d(x)|\Omega\rangle$ are needed.
- The matrix elements are expanded around $x^2 \simeq 0$ in terms of distribution amplitudes. The distribution amplitudes describes the parton content of $\mathcal{M}$

- Two expressions for the correlation function is matched using spectral representation.

$$\Pi(p^2) = \int ds \frac{\rho(s)}{s-p^2} + \text{polynomials in } p^2 \tag{8}$$

- To subtract the contributions of higher states and continuum, quark hadron duality is assumed:

$$\rho^{hadron}(s) = \rho^{OPE}(s) \quad \text{for } s > s_0 \tag{9}$$

- Contribution of higher states and continuum are further suppressed by Borel transformation:

$$B_{M^2}(p^2)^n \to 0 \quad \text{for } n > 0$$

$$B_{M^2}\left(\frac{1}{(m^2-p^2)^n}\right) = \frac{1}{\Gamma(n)}\frac{e^{\frac{-m^2}{M^2}}}{(M^2)^{n-1}} \qquad (10)$$

- The sum rules is obtained through

$$\Pi^{\text{lowest lying}}(M^2) = \int_0^{s_0} ds\rho(s)e^{-\frac{s}{M^2}} \qquad (11)$$

- There are two auxiliary parameters of the sum rules: M^2 and s_0

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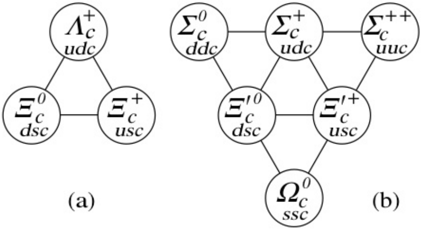
Interpolating Currents

- Any operator η can be used in the sum rules as long as $\langle\Omega|\eta|B_Q\rangle \neq 0$
- For the Σ_Q and Λ_Q baryons, one can simply replace the strange quark field operator in the light baryon interpolating current with the heavy quark field operator:

$$
\begin{aligned}
\eta^{\Sigma_Q^0} &= \sqrt{\frac{1}{2}}\epsilon^{abc}\left[\left(u^{aT}CQ^b\right)\gamma_5 d^c + \beta\left(u^{aT}C\gamma_5 Q^b\right)d^c\right. \\
&\quad \left. - \left(Q^{aT}Cd^b\right)\gamma_5 u^c - \beta\left(Q^{aT}C\gamma_5 d^b\right)u^c\right] \\
\eta^{\Sigma_Q^+} &= -\frac{1}{\sqrt{2}}\eta^{\Sigma_Q^0}(d \to u) \qquad \eta^{\Sigma_Q^-} = \frac{1}{\sqrt{2}}\eta^{\Sigma_Q^0}(u \to d)
\end{aligned}
$$

Currents for Ξ_Q baryons

- Ξ baryon contains two *s*-quarks, one of which is replaced by a heavy quark: hence there are two heavy Ξ_Q baryons. The lighter one being denoted by Ξ_Q and the heavier one by Ξ'_Q. $3 \otimes 3 = \bar{3} \oplus 6$



- Since Ξ_Q is in the same multiplet as Λ_Q, the currents can be chosen as $\eta^{\Xi_Q} = \eta^{\Lambda_Q}(q \to s)$
- Another alternative is to ignore the $SU(4)$ classification and choose the currents from the diquark-heavy-quark picture as:

$$\eta_{\Xi_Q} = \varepsilon_{abc}[(q^{a^T} C s^b)\gamma_5 + \beta(q^{a^T} C\gamma_5 s^b)]Q^c, \qquad (12)$$

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- The correlation function can be expressed as:

$$\begin{aligned}\Pi &= -\lambda_Q^2\epsilon^\mu \frac{\not{p}_f + m_{\Xi_Q}}{p_f^2 - m_{\Xi_Q}^2}\left[(f_1+f_2)\gamma_\mu + \frac{(p_i-p_f)_\mu}{2m_{\Xi_Q}} f_2\right]\frac{\not{p}_i + m_{\Xi_Q}}{p_i^2 - m_{\Xi_Q}^2} \\ &= -\mu_{\Xi_Q}\frac{\lambda_Q^2}{(p_i^2-m_{\Xi_Q}^2)(p_f^2-m_{\Xi_Q}^2)}\not{p}_f \not{\epsilon} \not{p}_i + \cdots \end{aligned} \quad (13)$$

where $\mu = f_1 + f_2$

- The magnetic moment is μ evaluated at $q^2 = 0$

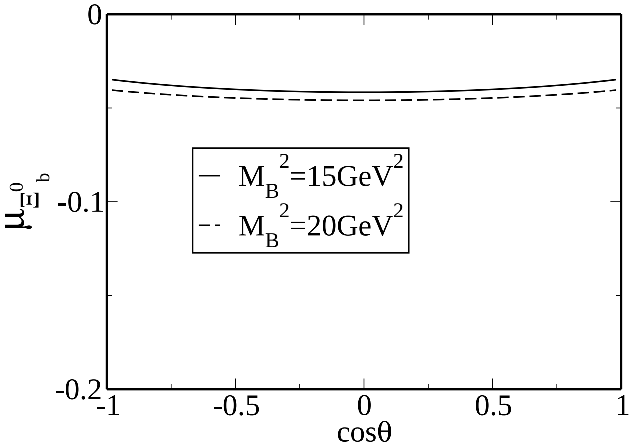

Figure: Variation of μ_{Ξ_b} with respect to $\cos\theta$ ($\beta = \tan\theta$) using the diquark-quark picture of the current

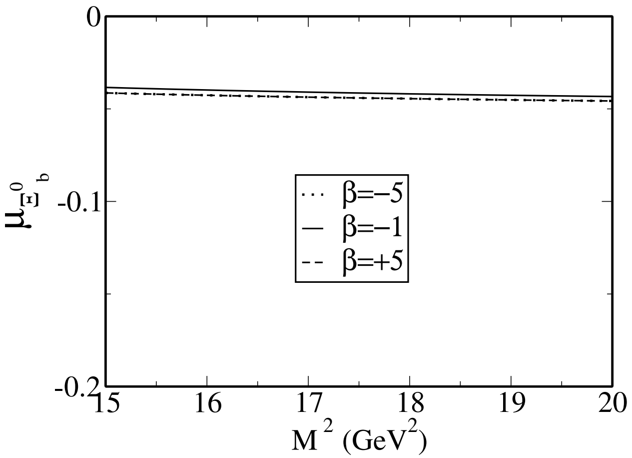

Figure: Variation of μ_{Ξ_b} with respect to M^2

	$\mu_{\Xi_b^0}$	$\mu_{\Xi_b^-}$	$\mu_{\Xi_c^0}$	$\mu_{\Xi_c^+}$
Our results	-0.045 ± 0.005	-0.08 ± 0.02	0.35 ± 0.05	0.50 ± 0.05
RQM	-0.06	-0.06	0.39	0.41
NQM	-0.06	-0.06	0.37	0.37
B. Patel, *et al.*	-	-	$-1.02 \div -1.06$	$0.45 \div 0.48$
M. Savage	-	-	0.32	0.42
D. O. Riska	-	-	0.38	0.38
Y. Oh, *et al*	-	-	0.28	0.28
C. S. An	-	-	$0.28 \div 0.34$	$0.39 \div 0.46$

Table: Results for the magnetic moments of Ξ_Q baryons in different approaches.

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Mass Postdictions for the Ξ_Q

- The correlation function can be written as

$$\Pi(p^2) = \sum_h \lambda_h^2 \frac{\not p + m_h}{p^2 - m_h^2} = \Pi_1(p^2)\ \not p + \Pi_2(p^2) \qquad (14)$$

where $\langle \Omega | \eta_{\Xi_Q} | h \rangle = \lambda_h u_h(p)$

- After borel trasformation

$$\Pi_1^{subtracted}(M^2) = \lambda_{\Xi_Q}^2 e^{\frac{-m_{\Xi_Q}^2}{M^2}} \qquad (15)$$

- Mass of the Ξ_Q baryons can be extracted using:

$$m_{\Xi}^2 = M^4 \frac{\partial}{\partial M^2} \ln \Pi_1^{subtracted} \qquad (16)$$

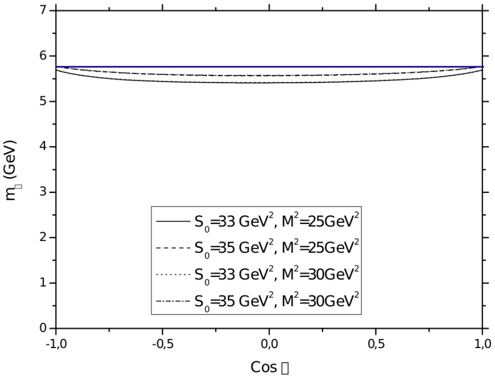

Figure: Variation of the mass of Ξ_b with respect to $\cos\theta$ where $t = \tan\theta$ and using the current motivated by $SU(4)$ symmetry

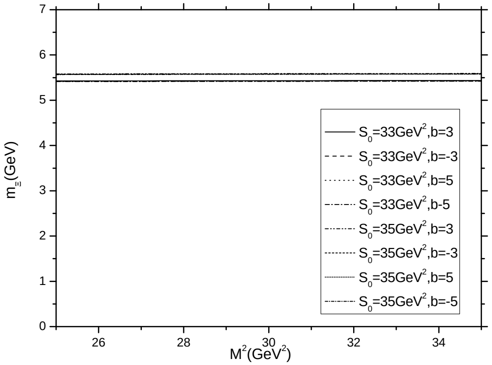

Figure: Variation of m_{Ξ_b} with respect to M^2

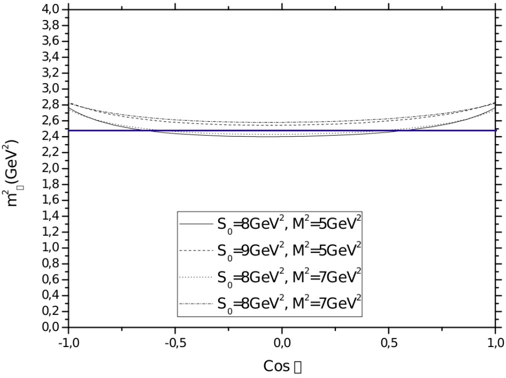

Figure: Variation of m_{Ξ_c} with respect to $\cos\theta$

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$g_{\pi\Xi_Q}$ Coupling Constant Predictions

- The coupling is defined as :

$$\langle \Xi_Q(p)\pi(q)|\Xi_Q(p+q)\rangle = g_{\Xi_Q\pi}\bar{u}(p)i\gamma_5 u(p+q)$$

- The hadronic representation of the correlation function:

$$\Pi(p^2,(p+q)^2) = i\frac{\lambda^2_{\Xi_Q} g_{\pi\Xi_Q}}{(p^2-m^2_{\Xi_q})((p+q)^2-m^2_{\Xi_Q})} \times (-\not{p}\not{q}\gamma_5 - M_Q\not{q}\gamma_5) \qquad (17)$$

- Two possible sum rules from the structures $\not{p}\not{q}\gamma_5$ and $\not{q}\gamma_5$. We used the $\not{p}\not{q}\gamma_5$ structure

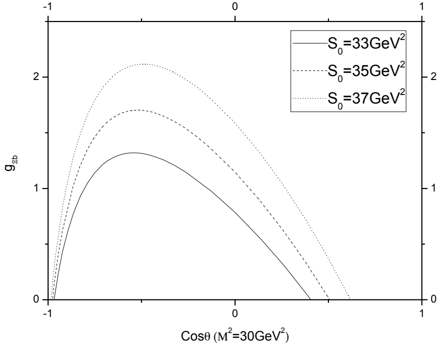

Figure: Dependence of $g_{\Xi_b\pi}$ on $\cos\theta$ for the current from the $SU(4)$ symmetry and $M^2 = 30\ GeV^2$

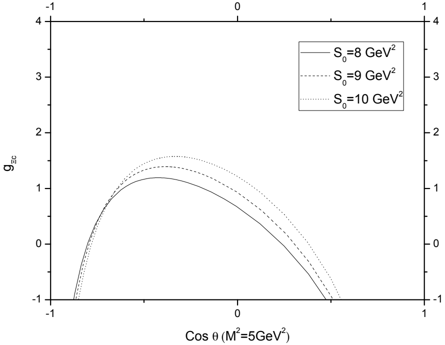

Figure: Dependence of $g_{\Xi_c\pi}$ on $\cos\theta$ for the current from the $SU(4)$ symmetry and $M^2 = 5\ GeV^2$

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$g_{\Sigma_Q\Lambda_Q\pi}$ Coupling Constants

- $\langle\Lambda_Q(p)\pi(q)|\Sigma_Q(p+q)\rangle = g_{\Sigma_Q\Lambda_Q\pi}\bar{u}_{\Lambda_Q}(p+q)i\gamma_5 u_{\Sigma_Q}$
- $g_{\Sigma_b\Lambda_b\pi} = 25.5 \pm 4.9$
- $g_{\Sigma_c\Lambda_c\pi} = 10.8 \pm 2.2$
-

	$\Gamma(\Sigma_c \longrightarrow \Lambda_c\pi)$	$\Gamma(\Sigma_b \longrightarrow \Lambda_b\pi)$
Our Result	2.16 ± 0.62	3.93 ± 1.18
RTQM [Ivanov, 2002]	3.63 ± 0.27	-
LFQM [Tawfig, 1999]	1.555 ± 0.165	-
χPT [Cheng, 1992]	2.45	-
Exp. [Amsler, 2008]	2.21 ± 0.4	-

Table: Results for the decay rates of $\Sigma_Q \longrightarrow \Lambda_Q\pi$ in different approaches in MeV.

TÜBİTAK

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Mass and Magnetic Moments of Heavy Decuplet

	$m_{\Omega_b^*}$	$m_{\Omega_c^*}$	$m_{\Sigma_b^*}$	$m_{\Sigma_c^*}$	$m_{\Xi_b^*}$	$m_{\Xi_c^*}$
this work	6.08 ± 0.40	2.72 ± 0.20	5.85 ± 0.35	2.51 ± 0.15	5.97 ± 0.40	2.66 ± 0.18
[Liu, 2007]	$6.063^{+0.083}_{-0.082}$	$2.790^{+0.109}_{-0.105}$	$5.835^{+0.082}_{-0.077}$	$2.534^{+0.096}_{-0.081}$	$5.929^{+0.088}_{-0.079}$	$2.634^{+0.102}_{-0.094}$
[Ebert, 2005]	6.088	2.768	5.834	2.518	5.963	2.654
[Capstick, 1986]	-	-	5.805	2.495	-	-
[Ronaglia, 1995]	6.090	2.770	5.850	2.520	5.980	2.650
[Savage, 1995]	-	2.768	-	2.518	-	-
[Jenkins, 1996]	6.083	2.760	5.840	-	5.966	-
[Mathur, 2002]	6.060	2.752	5.871	2.5388	5.959	2.680
Exp [Amsler, 2008]	-	2.768	5.829	2.518	-	2.647

Table: Comparison of mass of the heavy flavored baryons from present work and other approaches and with experiment.

Mass and Magnetic Moments of Heavy Decuplet

	Our results	hyper central model[Patel, 2008]
$\mu_{\Omega_b^{*-}}$	-1.95 ± 0.45	$-$
$\mu_{\Omega_c^{*0}}$	-0.91 ± 0.25	$-0.87 \div -0.90$
$\mu_{\Sigma_b^{*-}}$	-1.50 ± 0.32	-
$\mu_{\Sigma_b^{*0}}$	0.50 ± 0.10	-
$\mu_{\Sigma_b^{*+}}$	2.52 ± 0.50	-
$\mu_{\Sigma_c^{*0}}$	-0.81 ± 0.20	$-1.05 \div -1.10$
$\mu_{\Sigma_c^{*+}}$	2.00 ± 0.50	$1.10 \div 1.16$
$\mu_{\Sigma_c^{*++}}$	4.81 ± 1.22	$3.28 \div 3.43$
$\mu_{\Xi_b^{*-}}$	-1.72 ± 0.40	-
$\mu_{\Xi_b^{*0}}$	0.20 ± 0.04	-
$\mu_{\Xi_c^{*0}}$	-0.83 ± 0.20	$-0.96 \div -1.00$
$\mu_{\Xi_c^{*+}}$	1.40 ± 0.29	$1.18 \div 1.24$

Table: The magnetic moment of the heavy flavored baryons.

Summary

- Sum rules have been applied to heavy baryons.
- Predictions on the properties of the heavy Σ_Q and Λ_Q baryons are consistent with experiment and other approaches.
- Predictions on Ξ_Q baryons are less reliable. More studies and experimental results are necessary.