# Light Cone QCD Sum Rules for the Magnetic Moments of Baryons<sup>\*</sup>

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# Outline

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## Introduction

- QCD sum rules [1] is a powerful tool for studying hadronic physics based on QCD.
- One of the important properties of hadrons is their magnetic moments.
- In QCDSR and its various extensions, non-perturbative contributions are parameterized in terms of vacuum condensates.
- In LCQSR the operator product is carried out in the light cone  $x^2 \simeq 0$  and not at small distances.
- The non-perturbative dynamics are described by the so called light cone wave functions.

#### LCQSR for the Magnetic Moments

• In order to obtain an expression for the magnetic moments, one starts from the following correlation function:

$$\Pi = i \int d^4x \, e^{ipx} \left\langle \gamma(q) | T\{\eta_B(x)\bar{\eta}_B(0)\} | 0 \right\rangle \,, \tag{1}$$

where B is the baryon under consideration and  $\eta^B$  is a current having the quantum numbers of the B hadron.

• For  $p^2 > 0$  the correlator can be calculated by inserting a complete set of hadronic states between the currents:

$$\Pi = \frac{\langle 0|\eta_B|B(p_1)\rangle}{p_1^2 - m_B^2} \langle \gamma(q)B(p_1)|B(p_2)\rangle \frac{\langle B(p_2)|\bar{\eta}_B|0\rangle}{p_2^2 - m_B^2} + \sum_h \frac{\langle 0|\eta_B|h(p_1)\rangle}{p_1^2 - m_h^2} \langle \gamma(q)h(p_1)|h(p_2)\rangle \frac{\langle h(p_2)|\bar{\eta}_B|0\rangle}{p_2^2 - m_h^2} , \qquad (2)$$

where  $p_2 = p_1 + q$  and  $\{|B\rangle, |h\rangle\}$  forms a complete set of hadronic states.

• The matrix elements appearing in the expression for the correlation can be expressed as:

$$\langle 0|\eta_B|B(p)\rangle = \lambda_B u_B(p) , \qquad (3)$$

$$\langle \gamma(q)B(p_1)|B(p_2)\rangle = \bar{u}_B(p_1) \left[ (f_1 + f_2)\gamma_\mu + \frac{(p_1 + p_2)_\mu}{2m_B} f_2 \right] u_B(p_2)\varepsilon^\mu , \qquad (4)$$

where  $\varepsilon$  is the polarization vector of the photon and the form factors  $f_i$  are functions of  $q^2$ 

- The value of  $f_1 + f_2$  at  $q^2 = 0$  gives the magnetic moment of the baryon in units of natural magneton, i.e.,  $e\hbar/2m_Bc$
- Substituting these expressions back into the correlator, one obtains the following expression for the correlation function:

$$\Pi = -\lambda_B^2 \varepsilon^\mu \frac{\not p_1 + m_B}{p_1^2 - m_B^2} \left[ (f_1 + f_2)\gamma_\mu + \frac{(p_1 + p_2)_\mu}{2m_B} f_2 \right] \frac{\not p_2 + m_B}{p_2^2 - m_B^2} + \cdots , \qquad (5)$$

where  $\cdots$  represents contributions from the higher states and of the continuum.

At q<sup>2</sup> = 0, rearranging the various Dirac structures and showing only the coefficient of the structure p<sub>1</sub> ∉ ∉, the correlation function can be written as:

$$\Pi = -\lambda_B^2 \frac{1}{p_1^2 - m_B^2} \mu \frac{1}{p_2^2 - m_B^2} + \cdots$$

$$\int_{-\infty}^{\infty} ds ds - \rho^B(s_1, s_2) + \int_{-\infty}^{\infty} ds ds - \rho^h(s_1, s_2) + \cdots + (m + 1) +$$

$$= \int_{0}^{\infty} ds_{1} ds_{2} \frac{r(1-2)}{(s_{1}-p_{1}^{2})(s_{2}-p_{2}^{2})} + \int_{0}^{\infty} ds_{1} ds_{2} \frac{r(1-2)}{(s_{1}-p_{1}^{2})(s_{2}-p_{2}^{2})} + subtraction \ terms \ (7)$$

where  $\mu = (f_1 + f_2)|_{q^2=0}$  and  $\rho^B(s_1, s_2) = -\lambda_B^2 \mu \delta(s_1 - m_B^2) \delta(s_1 - s_2)$ . This is the final expression for the correlation function in terms of the phenomenological parameters.

- In the other kinematical region, when  $p^2 \ll 0$ , the main contribution to the correlation function is from small distances and hence one can use the OPE.
- For this purpose, one needs explicit expression for the interpolating currents.
- For the octet baryons, there are two different currents for each baryon and the most general current is a linear combination of the two:

$$\begin{split} \eta_{p} &= 2\epsilon_{abc} \sum_{\ell=1}^{2} \left( u^{aT} C A_{1}^{\ell} d^{B} \right) A_{2}^{\ell} u^{c} ,\\ \eta_{n} &= n_{p} \left( u^{c} \to d^{c} \right) ,\\ \eta_{\Sigma^{0}} &= \frac{1}{\sqrt{2}} \epsilon_{abc} \sum_{\ell=1}^{2} \left[ \left( u^{aT} C A_{1}^{\ell} s^{b} \right) A_{2}^{\ell} d^{c} - \left( d^{cT} C A_{1}^{\ell} s^{b} \right) A_{2}^{\ell} u^{a} \right] ,\\ \eta_{\Lambda} &= \frac{2}{\sqrt{6}} \epsilon_{abc} \sum_{\ell=1}^{2} \left[ -2 \left( u^{aT} C A_{1}^{\ell} d^{c} \right) A_{2}^{\ell} s^{c} + \left( u^{aT} C A_{1}^{\ell} s^{b} \right) A_{2}^{\ell} d^{c} + \left( d^{cT} C A_{1}^{\ell} s^{b} \right) A_{2}^{\ell} u^{a} \right] ,\\ \eta_{\Xi^{0}} &= -2\epsilon_{abc} \sum_{\ell=1}^{2} \left( s^{aT} C A_{1}^{\ell} u^{b} \right) A_{2}^{\ell} s^{c} ,\\ \eta_{\Sigma^{+}} &= \eta_{\Sigma^{0}} \left( d \to u \right) ,\\ \eta_{\Xi^{-}} &= \eta_{\Xi^{0}} \left( u \to d \right) , \end{split}$$
(8)

where  $A_1^1 = 1, A_1^2 = t\gamma_5, A_2^1 = \gamma_5, A_2^2 = 1$  and t is arbitrary.

• Using these currents and the Wick theorem, one can write the correlation functions for the  $\Lambda$  and  $\Sigma^+$  baryons as:

$$\begin{aligned} \Pi^{\Lambda} &= -\frac{2}{3} \epsilon_{abc} \epsilon_{def} \int d^{4}x e^{ipx} \langle \gamma | \sum_{\ell=1}^{2} \sum_{k=1}^{2} \left\{ 4A_{2}^{\ell} S_{s}^{be}(x) A_{2}^{k} \operatorname{Tr} S_{d}^{cf}(x) CA_{1}^{k} S_{u}^{adT}(x) CA_{1}^{\ell} \right. \\ &+ 2A_{2}^{\ell} S_{s}^{be}(x) CA_{1}^{k} S_{u}^{adT}(x) CA_{1}^{\ell} S_{d}^{cf}(x) A_{2}^{k} - 2A_{2}^{\ell} S_{s}^{be}(x) CA_{1}^{k} S_{d}^{cfT}(x) (CA_{1}^{\ell})^{T} S_{u}^{ad}(x) A_{2}^{k} \\ &+ 2A_{2}^{\ell} S_{d}^{cf}(x) CA_{1}^{k} S_{u}^{adT}(x) CA_{1}^{\ell} S_{s}^{be}(x) A_{2}^{k} + A_{2}^{\ell} S_{d}^{cf}(x) A_{2}^{k} \operatorname{Tr} S_{s}^{be}(x) CA_{1}^{k} S_{u}^{adT}(x) CA_{1}^{\ell} \right. \end{aligned}$$
(9)  
$$&- A_{2}^{\ell} S_{d}^{cf}(x) (CA_{1}^{k})^{T} S_{s}^{beT}(x) (CA_{1}^{\ell})^{T} S_{u}^{ad}(x) A_{2}^{k} - 2A_{2}^{\ell} S_{u}^{ad}(x) (CA_{1}^{k})^{T} S_{d}^{cfT}(x) CA_{1}^{\ell} S_{s}^{be}(x) A_{2}^{k} \\ &- A_{2}^{\ell} S_{u}^{ad}(x) (CA_{1}^{k})^{T} S_{s}^{beT}(x) (CA_{1}^{\ell})^{T} S_{d}^{cf}(x) A_{2}^{k} + A_{2}^{\ell} S_{u}^{ad}(x) A_{2}^{k} \operatorname{Tr} S_{s}^{be}(x) CA_{1}^{k} S_{d}^{cfT}(x) CA_{1}^{\ell} \right\} |0\rangle , \end{aligned}$$

$$\Pi^{\Sigma^{+}} = -2\epsilon_{abc}\epsilon_{def} \int d^{4}x e^{ipx} \langle \gamma | \sum_{\ell=1}^{2} \sum_{k=1}^{2} \left\{ A_{2}^{\ell} S_{u}^{cf}(x) A_{2}^{k} \operatorname{Tr} S_{s}^{be}(x) C A_{1}^{k} S_{u}^{adT}(x) C A_{1}^{\ell} \right. \\ \left. + A_{2}^{\ell} \left[ S_{u}^{cf}(x) (CA_{1}^{k})^{T} S_{s}^{beT}(x) (CA_{1}^{\ell})^{T} S_{u}^{ad}(x) A_{2}^{k} + S_{u}^{ad}(x) (CA_{1}^{k})^{T} S_{s}^{beT}(x) (CA_{1}^{\ell})^{T} S_{u}^{cf}(x) A_{2}^{k} \right. \\ \left. + S_{u}^{ad}(x) A_{2}^{k} \operatorname{Tr} S_{s}^{be}(x) C A_{1}^{k} S_{d}^{cfT}(x) C A_{1}^{\ell} \right] \right\} |0\rangle , \qquad (10)$$

• The photon can be emitted by any one of the quarks either perturbatively

$$S_{q\,\alpha\beta}^{\ ab} \to 2ee_q \left( \int dy \,\mathcal{F}^{\mu\nu} y_{\nu} S_q^{free}(x-y) \gamma_{\mu} S_q^{free}(y) \right)_{\alpha\beta}^{ab} , \qquad (11)$$

where the Fock–Schwinger gauge  $x^{\mu}A_{\mu}(x) = 0$  is used

• ... or non perturbatively

$$S_{q\,\alpha\beta}^{\ ab} \to -\frac{1}{4}\bar{q}^a A_j q^b (A_j)_{\alpha\beta} , \qquad (12)$$

where  $A_j \in \{1, \gamma_5, \gamma_{\alpha}, i\gamma_5\gamma_{\alpha}, \sigma_{\alpha\beta}/\sqrt{2}\}$  and sum over  $A_j$  is implied.

• The contributions of the non-perturbative contributions can be written in terms of photon wave functions defined as [2, 3]:

$$\langle \gamma(q) | \bar{q} \gamma_{\alpha} \gamma_{5} q | 0 \rangle = \frac{f}{4} e_{q} \epsilon_{\alpha\beta\rho\sigma} \varepsilon^{\beta} q^{\rho} x^{\sigma} \int_{0}^{1} du \, e^{iuqx} \psi(u) ,$$

$$\langle \gamma(q) | \bar{q} \sigma_{\alpha\beta} q | 0 \rangle = i e_{q} \langle \bar{q}q \rangle \int_{0}^{1} du \, e^{iuqx} \left\{ (\varepsilon_{\alpha} q_{\beta} - \varepsilon_{\beta} q_{\alpha}) \left[ \chi \phi(u) + x^{2} \left( g_{1}(u) - g_{2}(u) \right) \right] \right\}$$

$$+ \left[ qx (\varepsilon_{\alpha} x_{\beta} - \varepsilon_{\beta} x_{\alpha}) + \varepsilon x (x_{\alpha} q_{\beta} - x_{\beta} q_{\alpha}) \right] g_{2}(u) \right\},$$

$$(13)$$

where  $\chi$  is the magnetic susceptibility of the quark condensate,  $\phi(u)$  and  $\psi(u)$  are the leading twist-2 photon wave functions, while  $g_1(u)$  and  $g_2(u)$  are the twist-4 functions, respectively.

• In our calculations the *s*-quark mass is considered only up to linear order, and as the quark propagator, we used:

$$S_{q}(x) = \langle 0 | T \{ \bar{q}(x)q(0) \} | 0 \rangle$$

$$= \frac{i \not x}{2\pi^{2}x^{4}} - \frac{m_{q}}{4\pi^{2}x^{2}} - \frac{\langle \bar{q}q \rangle}{12} \left( 1 - \frac{im_{q}}{4} \not x \right) - \frac{x^{2}}{192} m_{0}^{2} \langle \bar{q}q \rangle \left( 1 - \frac{im_{q}}{6} \not x \right)$$

$$- ig_{s} \int_{0}^{1} dv \left[ \frac{\not x}{16\pi^{2}x^{2}} G_{\mu\nu}(vx) \sigma^{\mu\nu} - vx^{\mu} G_{\mu\nu}(vx) \gamma^{\nu} \frac{i}{4\pi^{2}x^{2}} - \frac{im_{q}}{32\pi^{2}} G_{\mu\nu} \sigma^{\mu\nu} \left( \ln \frac{-x^{2}\Lambda^{2}}{4} + 2\gamma_{E} \right) \right],$$

$$(14)$$

• The expression for the coefficient of the structure  $\sim p_1 \notin q$  can be obtained after a straight forward but tedious calculation and then the spectral density can be extracted.

$$\Pi^{OPE}(p_1^2, p_2^2) = \int_0^\infty ds_1 ds_2 \frac{\rho^{OPE}(s_1, s_2)}{(s_1 - p_1^2)(s_2 - p_2^2)} + subtraction \ terms \tag{15}$$

- Now we have two representations of the correlation function in two different kinematical regions. These can be matched using dispersion relations.
- In order to eliminate the subtraction terms, the correlation functions are Borel transformed after which the sum rules can be written in the form:

$$-\lambda_B^2 \mu e^{-m_B^2 \left(\frac{1}{M_1^2} + \frac{1}{M_2^2}\right)} + \int_D \rho^h(s_1, s_2) e^{-\frac{s_1}{M_1^2}} e^{-\frac{s_2}{M_2^2}} = \int_0^\infty ds_1 ds_2 \rho^{OPE}(s_1, s_2) e^{-\frac{s_1}{M_1^2}} e^{-\frac{s_2}{M_2^2}}$$
(16)

• Besides eliminating the subtraction terms, Borel transformation also exponentially suppresses the contributions of the higher states and the continuum. • The contribution of the higher states and continuum are modeled using the quark hadron duality, i.e.

$$\rho^{h}(s_{1}, s_{2}) = \rho^{OPE}(s_{1}, s_{2})\theta(s_{1} - s_{0})\theta(s_{1} - s_{0}')$$
(17)

• And the final sum rules are obtained from

$$-\lambda_B^2 \mu e^{-\frac{m_B^2}{M^2}} = \int_0^{s_0} ds_1 ds_2 \rho^{OPE}(s_1, s_2) e^{-\frac{s_1}{M_1^2}} e^{-\frac{s_2}{M_2^2}}$$
(18)

where  $\frac{1}{M^2} = \frac{1}{M_1^2} + \frac{1}{M_2^2}$ 

•  $\lambda_B^2 e^{-\frac{m_B^2}{M^2}}$  can be calculated similarly using the QCDSR starting from the correlation function:

$$\Pi = i \int d^4x \, e^{ipx} \, \langle 0|T\{\eta_B(x)\bar{\eta}_B(0)\}|0\rangle \quad , \tag{19}$$

which has the phenomenological representation:

$$\Pi = i\lambda_B^2 \frac{\not p_B + m_B}{p_B^2 - m_B^2} + \cdots$$
(20)

which becomes

$$\Pi = i\lambda_B^2 e^{-\frac{m_B^2}{M^2}} \left( \not\!\!p_B + m_B \right) \tag{21}$$

after Borel transforming with respect to  $p_B^2$ .

$$\begin{split} \lambda_{\lambda\mu\Lambda}^{2} e^{-M^{2}/M_{\Lambda}^{2}} &= \\ \frac{1}{192\pi^{4}} M^{6} E_{2}(x) \left[ (-1+t^{2})(e_{u}+e_{d}) + (13+10t+13t^{2})e_{s} \right] \\ - \frac{m_{s}}{96\pi^{2}} M^{4} E_{1}(x)(-5+4t+t^{2})\chi\varphi(u_{0})\langle\bar{q}q\rangle(e_{u}+e_{d}) \\ - \frac{m_{s}}{96\pi^{2}} M^{2} E_{0}(x)(-1-4t+5t^{2})\langle\bar{q}q\rangle \left(\gamma_{E}-\ln\frac{M^{2}}{\Lambda^{2}}\right)e_{s} \\ - \frac{m_{s}}{288\pi^{2}} m_{0}^{2}\langle\bar{q}q\rangle \left[ 9(-1+t^{2})(e_{u}+e_{d}) + (-3-12t+16t^{2})e_{s} \right] \left(\gamma_{E}-\ln\frac{M^{2}}{\Lambda^{2}}\right)e_{s} \\ - \frac{m_{s}}{9} \left[ g_{1}(u_{0}) - g_{2}(u_{0}) \right]\langle\bar{q}q\rangle \left[ (-5+4t+t^{2})\langle\bar{q}x\rangle + (-1+t)^{2}\langle\bar{q}q\rangle \right](e_{u}+e_{d}) \\ + \frac{m_{s}}{6\pi^{2}}(-5+4t+t^{2}) \left[ g_{1}(u_{0}) - g_{2}(u_{0}) \right] M^{2} E_{0}(x)\langle\bar{q}q\rangle (e_{u}+e_{d}) \\ + \frac{m_{s}}{9} \left( -1-4t-5t^{2} \right) \left[ g_{1}(u_{0}) - g_{2}(u_{0}) \right] \langle\bar{s}s\rangle\langle\bar{u}u\rangle e_{s} \\ + \frac{m_{s}}{18} M^{2} E_{0}(x)\chi\varphi(u_{0})\langle\bar{q}q\rangle \left[ (-5+4t+t^{2})\langle\bar{s}x\rangle + (-1+t)^{2}\langle\bar{q}q\rangle \right] (e_{u}+e_{d}) \\ + \frac{1}{12} M^{2} E_{0}(x)\chi\varphi(u_{0})\langle\bar{q}q\rangle \left[ (-5+4t+t^{2})\langle\bar{s}z\rangle + (-3-2t+5t^{2})\langle\bar{q}q\rangle \right] (e_{u}+e_{d}) \\ + \frac{1}{144} m_{0}^{2} \chi\varphi(u_{0})\langle\bar{u}u\rangle \left[ 2(-5+4t+t^{2})\langle\bar{s}z\rangle + (-3-2t+5t^{2})\langle\bar{q}q\rangle \right] (e_{u}+e_{d}) \\ + \frac{1}{19} \langle\bar{q}q\rangle \left[ (-5+4t+t^{2})(e_{u}+e_{d}) + (-11-2t+13t^{2})\langle\bar{q}q\rangle \right] (e_{u}+e_{d}) \\ + \frac{1}{19} \frac{m_{s}}{2} m_{0}^{2} \left[ (-5+4t+t^{2})\langle\bar{s}z\rangle - 3(-5+4t+t^{2})\langle\bar{q}q\rangle \right] (e_{u}+e_{d}) \\ - \frac{288\pi^{2}}{288\pi^{2}} m_{0}^{2} \left[ (-1+t)^{2}\langle\bar{s}s\rangle - 3(-5+4t+t^{2})\langle\bar{q}q\rangle \right] (e_{u}+e_{d}) , \qquad (22)$$

where

$$u_0 = \frac{M_2^2}{M_1^2 + M_2^2} \ . \tag{23}$$

In our calculations we set  $M_1^2 = M_2^2 = 2M^2$ , hence  $u_0 = 1/2$ .

$$\begin{split} M_{\Lambda}\lambda_{\Lambda}^{2}e^{-M_{\lambda}^{2}/M^{2}} &= \\ \frac{m_{s}}{192\pi^{4}}(-13+2t+11t^{2})M^{6}E_{2}(x) \\ &+ \frac{1}{48\pi^{2}}(1-t)\left\{(13+11t)\langle\bar{s}s\rangle + (1+5t)(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)\right\}M^{4}E_{1}(x) \quad (24) \\ &- \frac{m_{0}^{2}}{32\pi^{2}}(1-t^{2})(4\langle\bar{s}s\rangle + \langle\bar{u}u\rangle + \langle\bar{d}d\rangle)M^{2}E_{0}(x) \\ &+ \frac{m_{s}}{36}\left\{(1+4t-5t^{2})\langle\bar{s}s\rangle(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle) + 6(5+2t+5t^{2})\langle\bar{u}u\rangle\langle\bar{d}d\rangle\right\} \end{split}$$

$$\begin{split} \lambda_{\Lambda}^{2} e^{-M_{\Lambda}^{2}/M^{2}} &= \\ \frac{1}{256\pi^{4}} (5+2t+5t^{2})M^{6}E_{2}(x) \\ &+ \frac{1}{72} (1-t)m_{0}^{2} \left\{ 4(1+2t)\langle\bar{s}s\rangle(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle) + (25+23t)\langle\bar{u}u\rangle\langle\bar{d}d\rangle \right\} \frac{1}{M^{2}} \\ &+ \frac{m_{s}}{96\pi^{2}} \left\{ 3(5+2t+5t^{2})\langle\bar{s}s\rangle + 2(1+4t-5t^{2})(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle) \right\} M^{2}E_{0}(x) \\ &+ \frac{m_{s}}{32\pi^{2}}m_{0}^{2}(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle)(1-t^{2}) \left\{ \gamma_{E} - \ln\left(\frac{M^{2}}{\Lambda^{2}}\right) \right\} \\ &- \frac{1}{18} (1-t) \left\{ (1+5t)\langle\bar{s}s\rangle(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle) + (13+11t)\langle\bar{u}u\rangle\langle\bar{d}d\rangle \right\} \\ &- \frac{m_{s}}{192\pi^{2}}m_{0}^{2} \left\{ 2(5+2t+5t^{2})\langle\bar{s}s\rangle + (-5+4t+t^{2})(\langle\bar{u}u\rangle+\langle\bar{d}d\rangle) \right\} \end{split}$$

#### Numerical Analysis

• In numerical calculations, the following forms of the photon wave functions are used [4, 3]:

$$\phi(u) = 6u(1-u) , \quad \psi(u) = 1 ,$$
  

$$g_1(u) = -\frac{1}{8}(1-u)(3-u) , \quad g_2(u) = -\frac{1}{4}(1-u)^2$$

- The values of the other input parameters that are used in the numerical analysis are:  $f = 0.028 \ GeV^2$ ,  $\chi = -4.4 \ GeV^{-2}$  [5](in [6] this quantity is estimated to be  $\chi = -3.3 \ GeV^{-2}$ ),  $\langle \bar{q}q \rangle (1 \ GeV) = -(0.243)^3 \ GeV^3$ ,  $m_0^2 = (0.8 \pm 0.2) \ GeV^2$  [7],  $m_s(1 \ GeV) = (150 \pm 50) \ MeV$  and  $\langle \bar{s}s \rangle (1 \ GeV) = 0.8 \langle \bar{q}q \rangle (1 \ GeV)$ .
- The analysis should be carried out in several steps:

\* Find the region of the Borel parameter  $M^2$  in which the sum rules for the magnetic moments are independent of  $M^2$ 

\* Find the region of t where the mass sum rules make sense, i.e. the mass sum rule for  $\lambda_B^2 e^{-\frac{m_B^2}{M^2}}$  is positive definite, and the predicted mass is stable with respect to the variations of this parameter.

\* Find the region of t where the predictions for the magnetic moments do not have a strong sensitivity on t

contributions of the higher states and the continuum. The Borel mass parameter  $M^2$  should be big enough to suppress the contributions of the higher dimensional operators in the OPE which are neglected and small enough to suppress the



- Figure 1: The dependence of  $\mu_p$  on  $M^2$  at  $s_0 = 2 \ GeV^2$ .
- 0.9The working region for the Borel mass squared parameter for the members of octet baryons are found to be:  $G_{
  ho}V^2$  $< M^2$  $< 1.3~G
  ho V^2$  for

.9 GeV 
$$\leq M^2 \leq 1.3$$
 GeV III  $p, m$ ;  
.3  $GeV^2 \leq M^2 \leq 1.7$   $GeV^2$  for  $\Sigma^-$ ,  $\Sigma^0$ ,  $\Sigma^+$  and  $\Lambda$ ;  
.7  $GeV^2 \leq M^2 \leq 2.1$   $GeV^2$  for  $\Xi^0$  and  $\Xi^-$ 



baryons Figure 3: The dependence of  $m_B$  on  $\cos \theta$ , where  $t = \tan \theta$ , for all the

Both criteria on t are satisfied in the region

(t) $\wedge$  $1.7(\cos\theta >$ is not in this region. -0.5) or t > $2.3(\cos\theta < 0.4)$ . Ioffe current



and A) and  $M^2 = 1.9 \ GeV^2$  (for  $\Xi$ ).  $s_0 = 2.0 \ GeV^2$  (for p and n),  $s_0 = 2.5 \ GeV^2$  (for A),  $s_0 = 3.0 \ GeV^2$ (for  $\Sigma$ ), and at  $M^2 = 1 \ GeV^2$  (for p and n),  $M^2 = 1.5 \ GeV^2$  (for  $\Sigma$ Figure 4: The dependence of  $\mu_B$  on  $\cos\theta$  at the continuum threshold

- value of t for  $t > 3.2(\cos \theta < 0.3)$  or t < -1.4 ( $\cos \theta > -0.6$ ). The predicted magnetic moments are not sensitive to the precise
- Combining with the previous analysis of the mass sum rules, the working region of t is found to be t < t-1.7 or t > 3.2.

	NQM SQM		QCDSR					LCQSR		
		$_{\rm SQM}$	$\chi = -3.3$	$\chi = -4.4$	QCDSA	$\chi PT$	SKRM	$\chi = -3.3$	$\chi = -4.4$	EXP
$\mu_p$	2.79	2.75	2.72	3.55	2.54	2.793	2.36	$2.7\pm0.5$	$3.5\pm0.5$	2.79
$\mu_n$	-1.91	-1.84	-1.65	-2.06	-1.69	-1.913	-1.87	$-1.8\pm0.35$	$-2.3\pm0.4$	-1.91
$\mu_{\Sigma^+}$	2.67	2.65	2.52	3.30	2.48	2.458	2.41	$2.2 \pm 0.4$	$2.9\pm0.4$	$2.46\pm0.01$
$\mu_{\Sigma^0}$	0.79			0.7[13]	0.8	0.649	0.66	$0.5\pm0.10$	$0.65\pm0.15$	
$\mu_{\Sigma^{-}}$	-1.09	-1.02	-1.13	-1.38	-0.90	-1.16	-1.10	$-0.8\pm0.2$	$-1.1\pm0.3$	$-1.16\pm0.03$
$\mu_{\pm 0}$	-1.43	-1.44	-1.18	-1.27	-1.49	-1.25	-1.96	$-1.3\pm0.3$	$-1.3\pm0.4$	$-1.25\pm0.01$
$\mu_{\Xi^{-}}$	-0.53	-0.52	-0.89	-0.98	-0.63	-0.6531	-0.84	$-0.7 \pm 0.2$	$-1.0 \pm 0.2$	-0.65
$\mu_{\Lambda}$	-0.61	-0.67	-0.50	-0.80	-0.69	-0.613	-0.60	$-0.7 \pm 0.2$	$-0.9 \pm 0.2$	-0.61

Table 1: Predictions of various approaches for the octet baryon magnetic moments: naive quark model (NQM, see ref. in [8]); static quark model (SQM) [9]; QCD sum rules (QCDSR) [10]; QCD string approach (QCDSA) [11]; chiral perturbation theory ( $\chi$ PT) [12]; skyrme model (SKRM) [13]; present work (LCQSR). For completeness we present the experimental values of the octet baryons. All the values in the table are given in units of nuclear magneton  $\mu_N$ .

## Conclusion

- The magnetic moments of the octet baryons are analyzed within the framework of LCQSR using the general form of the currents.
- It is shown that the predicted values for the magnetic moments are close to the experimental values for  $\chi = -3.3 \ GeV^2$
- It should be stressed that the predictions are not reliable for t = -1 which corresponds to the Ioffe current.

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